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MATHEMATICAL  
PRINCIPLES  
OF  
Natural Philosophy.

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By Sir *ISAAC NEWTON*.

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OF THE  
MOTION  
OF  
BODIES.  

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BOOK II.


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SECTION I.

*Of the Motion of Bodies that are resisted in the ratio of the Velocity.*

PROPOSITION I. THEOREM I.

*If a body is resisted in the ratio of its velocity, the motion lost by resistance is as the space gone over in its motion.*

 OR since the motion lost in each equal particle of time is as the velocity, that is, as the particle of space gone over; then, by composition, the motion lost in the whole time will be as the whole space gone over. Q.E.D.

Vol. II.

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COR.



COR. Therefore if the body, destitute of all gravity, move by its innate force only in free spaces, and there be given both its whole motion at the beginning, and also the motion remaining after some part of the way is gone over; there will be given also the whole space which the body can describe in an infinite time. For that space will be to the space now described, as the whole motion at the beginning is to the part lost of that motion.

### LEMMA I.

*Quantities proportional to their differences are continually proportional.*

Let  $A$  be to  $A-B$  as  $B$  to  $B-C$  and  $C$  to  $C-D$ , &c. and, by conversion,  $A$  will be to  $B$  as  $B$  to  $C$  and  $C$  to  $D$ , &c. *Q.E.D.*

### PROPOSITION II. THEOREM II.

*If a body is resisted in the ratio of its velocity, and moves, by its vis insita only, through a similar medium, and the times be taken equal; the velocities in the beginning of each of the times are in a geometrical progression, and the spaces described in each of the times are as the velocities.*

CASE I. Let the time be divided into equal particles; and if at the very beginning of each particle we suppose the resistance to act with one single impulse which is as the velocity; the decrement of the velocity in each of the particles of time will be as the same velocity. Therefore the velocities are proportional to their differences, and therefore (by Lem. 1. Book 2.)

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continually proportional. Therefore if out of an equal number of particles there be compounded any equal portions of time, the velocities at the beginning of those times will be as terms in a continued progression, which are taken by intervals, omitting every where an equal number of intermediate terms. But the ratio's of these terms are compounded of the equal ratio's of the intermediate terms equally repeated; and therefore are equal. Therefore the velocities, being proportional to those terms, are in geometrical progression. Let those equal particles of time be diminished, and their number increased *in infinitum*, so that the impulse of resistance may become continual; and the velocities at the beginnings of equal times, always continually proportional, will be also in this case continually proportional. *Q. E. D.*

CASE 2. And, by division, the differences of the velocities, that is, the parts of the velocities lost in each of the times, are as the wholes: But the spaces described in each of the times are as the lost parts of the velocities, (by Prop. 1. Book 2.) and therefore are also as the wholes. *Q. E. D.*

COROL. Hence if to the rectangular asymptotes  $AC$ ,  $CH$ , the Hyperbola  $BG$  is described, and  $AB$ ,  $DG$  be drawn perpendicular to the asymptote  $AC$ , and both the velocity of the body, and the resistance of the medium, at the very beginning of the motion, be express'd by any given line  $AC$ , and after some time is elapsed, by the indefinite line  $DC$ ; the time may be express'd by the area  $ABGD$ , and the space described in that time by the line  $AD$ . For if that area, by the motion of the point  $D$ , be uniformly increased in the same manner as the time, the right line  $DC$  will decrease in a geometrical ratio in the same manner as the velocity, and the parts of the right line  $AC$ , described in equal times, will decrease in the same ratio.

## PROPOSITION III. PROBLEM I.

*To define the motion of a body which, in a similar medium, ascends or descends in a right line, and is resisted in the ratio of its velocity, and acted upon by an uniform force of gravity.*

The body ascending, let the gravity be expounded by any given rectangle  $BACH$ ; and the resistance of the medium, at the beginning of the ascent, by the rectangle  $BADE$ , taken on the contrary side of the right line  $AB$ . Through the point  $B$ , with the rectangular asymptotes  $AC, CH$ , describe an Hyperbola, cutting the perpendiculars  $DE, de$ , in  $G, g$ ; and the body ascending will in the time  $DGgd$  describe the space  $EGge$ ; in the time  $DGBA$ , the space of the whole ascent  $EGB$ ; in the time  $ABKI$ , the space of descent  $BFK$ ; and in the time  $IKki$  the space of descent  $KFfk$ ; and the velocities of the bodies (proportional to the resistance of the medium) in these periods of time, will be  $ABED, ABed, o, ABFI, ABfi$  respectively; and the greatest velocity which the body can acquire by descending, will be  $BACH$ .

For let the rectangle  $BACH$  be resolved into innumerable rectangles  $Ak, Kl, Lm, Mn, \&c.$  which shall be as the increments of the velocities produced in so many equal times; then will  $o, Ak, Al, Am, An, \&c.$  be as the whole velocities, and therefore (by supposition) as the resistances of the medium in the beginning of each of the equal times. Make  $AC$  to  $AK$ , or  $ABHC$  to  $ABkK$  as the force of gravity to the resistance in the beginning of the second time; then from the force of gravity subduct the resistances, and  $ABHC, KkHC, LlHC, MmHC, \&c.$  will be as the absolute forces with which the body is acted upon in the beginning of each of the times, and therefore

fore (by Law 2) as the increments of the velocities, that is, as the rectangles  $Ak$ ,  $Kl$ ,  $Lm$ ,  $Mn$ , &c. and therefore (by Lem. 1. Book 2.) in a geometrical progression. Therefore if the right lines  $Kk$ ,  $Ll$ ,  $Mm$ ,  $Nn$ , &c. are produced so as to meet the Hyperbola in  $q$ ,  $r$ ,  $s$ ,  $t$ , &c. the areas  $ABqK$ ,  $KqrL$ ,  $LrsM$ ,  $MstN$ , &c. will be equal, and therefore analogous to the equal times and equal gravitating forces. But the area  $ABqK$  (by Corol. 3. Lem. 7 & 8. Book 1.) is to the area  $Bkq$  as  $Kq$  to  $\frac{1}{2}kq$ , or  $AC$  to  $\frac{1}{2}AK$ , that is as the force of gravity to the resistance in the middle of the first time. And by the like reasoning the areas  $qKLR$ ,  $rLMs$ ,  $sMNt$ , &c. are to the areas  $qklr$ ,  $rlms$ ,  $smnt$ , &c. as the gravitating forces to the resistances in the middle of the second, third, fourth time, and so on. Therefore since the equal areas  $BAKq$ ,  $qKLR$ ,  $rLMs$ ,  $sMNt$ , &c. are analogous to the gravitating forces, the areas  $Bkq$ ,  $qklr$ ,  $rlms$ ,  $smnt$ , &c. will be analogous to the resistances in the middle of each of the times, that is (by supposition) to the velocities, and so to the spaces described. Take the sums of the analogous quantities, and the areas  $Bkq$ ,  $Blr$ ,  $Bms$ ,  $Bnt$ , &c. will be analogous to the whole spaces described; and also the areas  $ABqK$ ,  $ABrL$ ,  $ABsM$ ,  $ABtN$ , &c. to the times. Therefore the body, in descending, will in any time  $ABrL$ , describe the space  $Blr$ , and in the time  $LrtN$  the space  $rlnt$ . *Q. E. D.* And the like demonstration holds in ascending motion.

**COROL. 1.** Therefore the greatest velocity that the body can acquire by falling, is to the velocity acquired in any given time, as the given force of gravity which perpetually acts upon it, to the resisting force which opposes it at the end of that time.

**COROL. 2.** But the time being augmented in an arithmetical progression, the sum of that greatest velocity and the velocity in the ascent, and also their difference in the descent, decreases in a geometrical progression.

COROL. 3. Also the differences of the spaces, which are described in equal differences of the times, decrease in the same geometrical progression.

COROL. 4. The space described by the body is the difference of two spaces, whereof one is as the time taken from the beginning of the descent, and the other as the velocity; which [spaces] also at the beginning of the descent are equal among themselves.

#### PROPOSITION IV. PROBLEM II.

*Supposing the force of gravity in any similar medium to be uniform, and to tend perpendicularly to the plane of the horizon; to define the motion of a projectile therein, which suffers resistance proportional to its velocity.*

Let the projectile go from any place  $D$  in the direction of any right line  $DP$ , and let its velocity at the beginning of the motion be expounded by the length  $DP$ . From the point  $P$  let fall the perpendicular  $PC$  on the horizontal line  $DC$ , and cut  $DC$  in  $A$ , so that  $DA$  may be to  $AC$  as the resistance of the medium arising from its motion upwards at the beginning, to the force of gravity: or (which comes to the same) so that the rectangle under  $DA$  and  $DP$  may be to that under  $AC$  and  $CP$ , as the whole resistance at the beginning of the motion to the force of gravity. With the asymptotes  $DC$ ,  $CP$  describe any Hyperbola  $GTBS$  cutting the perpendiculars  $DG$ ,  $AB$  in  $G$  and  $B$ ; compleat the parallelogram  $DGKC$ , and let its side  $GK$  cut  $AB$  in  $Q$ . Take a line  $N$  in the same ratio to  $QB$  as  $DC$  is in to  $CP$ ; and from any point  $R$  of the right line  $DC$ , erect  $RT$  perpendicular to it, meeting the Hyperbola in  $T$ , and the right lines  $EH$ ,  $GK$ ,  $DP$  in  $I$ ,  $t$ , and  $V$ ; in that perpendicular

cular take  $Vr$  equal to  $\frac{tGT}{N}$ , or, which is the same

thing, take  $Rr$  equal to  $\frac{GTIE}{N}$ ; and the projectile in

the time  $DRTG$  will arrive at the point  $r$ , describing the curve line  $DraF$ , the locus of the point  $r$ ; thence it will come to its greatest height  $a$  in the perpendicular  $AB$ ; and afterwards ever approach to the asymptote  $PC$ . And its velocity in any point  $r$  will be as the tangent  $rL$  to the curve. *Q. E. I.*

For  $N$  is to  $QB$  as  $DC$  to  $CP$  or  $DR$  to  $RV$ , and therefore  $RV$  is equal to  $\frac{DR \times QB}{N}$ , and  $Rr$  (that is,

$RV - Vr$ , or  $\frac{DR \times QB - tGT}{N}$ ) is equal to  $\frac{DR \times AB - RDGT}{N}$ . Now let the time be expounded

by the area  $RDGT$ , and (by Laws Cor. 2.) distinguish the motion of the body into two others, one of ascent, the other lateral. And since the resistance is as the motion, let that also be distinguished into two parts proportional and contrary to the parts of the motion: and therefore the length described by the lateral motion, will be (by Prop. 2. Book 2.) as the line  $DR$ , and the height (by Prop. 3. Book 2.) as the area  $DR \times AB - RDGT$ , that is, as the line  $Rr$ . But in the very beginning of the motion the area  $RDGT$  is equal to the rectangle  $DR \times AQ$ , and therefore that line  $Rr$

(or  $\frac{DR \times AB - DR \times AQ}{N}$ ) will then be to  $DR$  as

$AB - AQ$  or  $QB$  to  $N$ , that is, as  $CP$  to  $DC$ ; and therefore as the motion upwards to the motion lengthwise at the beginning. Since therefore  $Rr$  is always as the height, and  $DR$  always as the length, and  $Rr$  is to  $DR$  at the beginning, as the height to the length: it

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follows,

follows, that  $Rr$  is always to  $DR$  as the height to the length; and therefore that the body will move in the line  $DraF$ , which is the locus of the point  $r$ .  $Q. E. D.$

COR. 1. Therefore  $Rr$  is equal to  $\frac{DR \times AB}{N}$  —  $\frac{RDGT}{N}$ ; and therefore if  $RT$  be produced to  $X$ , so that  $RX$  may be equal to  $\frac{DR \times AB}{N}$ , that is, if the parallelogram  $ACPT$  be completed, and  $DT$  cutting  $CP$  in  $Z$  be drawn, and  $RT$  be produced till it meets  $DT$  in  $X$ ;  $Xr$  will be equal to  $\frac{RDGT}{N}$ , and therefore proportional to the time.

COR. 2. Whence if innumerable lines  $CR$ , or, which is the same, innumerable lines  $ZX$ , be taken in a geometrical progression; there will be as many lines  $Xr$  in an arithmetical progression. And hence the curve  $DraF$  is easily delineated by the Table of Logarithms.

COR. 3. If a Parabola be constructed to the vertex  $D$ , and the diameter  $DG$ , produced downwards, and its latus rectum is to  $2DP$  as the whole resistance at the beginning of the motion to the gravitating force: the velocity with which the body ought to go from the place  $D$ , in the direction of the right line  $DP$ , so as in an uniform resisting medium to describe the curve  $DraF$ , will be the same as that with which it ought to go from the same place  $D$ , in the direction of the same right line  $DP$ , so as to describe a Parabola in a non-resisting medium. For the latus rectum of this Parabola, at the very beginning of the motion, is  $\frac{DV^2}{Vr}$ ; and  $Vr$  is  $\frac{tGT}{N}$  or  $\frac{DR \times Tt}{2N}$ . But a right  
line,

line, which, if drawn, would touch the Hyperbola  $GTS$  in  $G$ , is parallel to  $DK$ , and therefore  $Tt$  is  $\frac{CK \times DR}{DC}$ , and  $N$  is  $\frac{QB \times DC}{CP}$ : And therefore  $Vr$  is

equal to  $\frac{DR^2 \times CK \times CP}{2 DC^2 \times QB}$ , that is, (because  $DR$  and

$DC$ ,  $DV$  and  $DP$  are proportionals) to  $\frac{DV^2 \times CK \times CP}{2 DP^2 \times QB}$ ;

and the latus rectum  $\frac{DV^2}{Vr}$  comes out  $\frac{2 DP^2 \times QB}{CK \times CP}$ ,

that is, (because  $QB$  and  $CK$ ,  $DA$  and  $AC$  are proportional)  $\frac{2 DP^2 \times DA}{AC \times CP}$ , and therefore is to  $2 DP$ , as

$DP \times DA$  to  $CP \times AC$ ; that is, as the resistance to the gravity. *Q. E. D.*

COR. 4. Hence if a body be projected from any place  $D$ , with a given velocity, in the direction of a right line  $DP$  given by position; and the resistance of the medium, at the beginning of the motion, be given: the curve  $DraF$ , which that body will describe, may be found. For the velocity being given, the latus rectum of the parabola is given, as is well known. And taking  $2 DP$  to that latus rectum, as the force of gravity to the resisting force,  $DP$  is also given. Then cutting  $DC$  in  $A$ , so that  $CP \times AC$  may be to  $DP \times DA$  in the same ratio of the gravity to the resistance, the point  $A$  will be given. And hence the curve  $DraF$  is also given.

COR. 5. And on the contrary, if the curve  $DraF$  be given, there will be given both the velocity of the body, and the resistance of the medium in each of the places  $r$ . For the ratio of  $CP \times AC$  to  $DP \times DA$  being given, there is given both the resistance of the medium at the beginning of the motion, and the latus rectum of the parabola; and thence the velocity at the begin-



beginning of the motion is given also. Then from the length of the tangent  $rL$ , there is given both the velocity proportional to it, and the resistance proportional to the velocity in any place  $r$ .

COR. 6. But since the length  $2DP$  is to the latus rectum of the parabola as the gravity to the resistance in  $D$ ; and, from the velocity augmented, the resistance is augmented in the same ratio, but the latus rectum of the parabola is augmented in the duplicate of that ratio; it is plain that the length  $2DP$  is augmented in that simple ratio only; and is therefore always proportional to the velocity; nor will it be augmented or diminished by the change of the angle  $CDP$ , unless the velocity be also changed.

COR. 7. Hence appears the method of determining the curve  $DraF$ , nearly, from the phenomena, and thence collecting the resistance and velocity with which the body is projected. Let two similar and equal bodies be projected with the same velocity, from the place  $D$ , in different angles  $CDP$ ,  $CDp$ ; and let the places  $F, f$ , where they fall upon the horizontal plane  $DC$ , be known. Then taking any length for  $DP$  or  $Dp$ , suppose the resistance in  $D$  to be to the gravity in any ratio whatsoever, and let that ratio be expounded by any length  $SM$ . Then by computation, from that assumed length  $DP$ , find the lengths  $DF$ ,

$Df$ ; and from the ratio  $\frac{Ff}{DF}$ , found by calculation, sub-

duct the same ratio as found by experiment; and let the difference be expounded by the perpendicular  $MN$ . Repeat the same a second and a third time, by assuming always a new ratio  $SM$  of the resistance to the gravity, and collecting a new difference  $MN$ . Draw the affirmative differences on one side of the right line  $SM$ , and the negative on the other side; and through the points  $N, N, N$  draw a regular curve  $NNN$ , cutting the

the right line  $SMMM$  in  $X$ , and  $SX$  will be the true ratio of the resistance to the gravity, which was to be found. From this ratio the length  $DF$  is to be collected by calculation; and a length, which is to the assumed length  $DP$ , as the length  $DF$  known by experiment to the length  $DF$  just now found, will be the true length  $DP$ . This being known, you will have both the curve line  $DraF$  which the body describes, and also the velocity and resistance of the body in each place.

## S C H O L I U M.

But yet that the resistance of bodies is in the ratio of the velocity, is more a mathematical hypothesis than a physical one. In mediums void of all tenacity, the resistances made to bodies are in the duplicate ratio of the velocities. For by the action of a swifter body, a greater motion, in proportion to a greater velocity, is communicated to the same quantity of the medium, in a less time; and in an equal time, by reason of a greater quantity of the disturbed medium, a motion is communicated in the duplicate ratio greater; and the resistance (by Law 2 and 3.) is as the motion communicated. Let us therefore see what motions arise from this law of resistance.



S E C.



## SECTION II.

*Of the Motion of Bodies that are resisted  
in the duplicate ratio of their Velocities.*

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## PROPOSITION V. THEOREM III.

*If a body is resisted in the duplicate ratio of its velocity, and moves by its innate force only through a similar medium; and the times be taken in a geometrical progression, proceeding from less to greater terms: I say that the velocities at the beginning of each of the times are in the same geometrical progression inversely; and that the spaces are equal, which are described in each of the times.*

For since the resistance of the medium is proportional to the square of the velocity, and the decrement of the velocity is proportional to the resistance; if the time be divided into innumerable equal particles, the squares of the velocities at the beginning of each of the times will be proportional to the differences of the same velocities. Let those particles of time be  $AK, KL, LM, \&c.$  taken in the right line  $CD$ ; and erect the perpendiculars  $AB, Kk, Ll, Mm, \&c.$  meeting the Hyperbola  $BklmG$ , described with the centre  $C$ , and the rectangular asymptotes  $CD, CH$ , in  $B, k, l, m, \&c.$  then  $AB$  will be to  $Kk$ ,

as

as  $CK$  to  $CA$ , and, by division,  $AB - Kk$  to  $Kk$  as  $AK$  to  $CA$ , and, alternately,  $AB - Kk$  to  $AK$  as  $Kk$  to  $CA$ , and therefore as  $AB \times Kk$  to  $AB \times CA$ . Therefore since  $AK$  and  $AB \times CA$  are given,  $AB - Kk$  will be as  $AB \times Kk$ ; and lastly, when  $AB$  and  $Kk$  coincide, as  $AB^2$ . And, by the like reasoning,  $Kk - Ll$ ,  $Ll - Mm$ , &c. will be as  $Kk^2$ ,  $Ll^2$ , &c. Therefore the squares of the lines  $AB$ ,  $Kk$ ,  $Ll$ ,  $Mm$ , &c. are as their differences; and therefore, since the squares of the velocities were shewn above to be as their differences, the progression of both will be alike. This being demonstrated, it follows also that the areas described by these lines are in a like progression with the spaces described by these velocities. Therefore if the velocity at the beginning of the first time  $AK$  be expounded by the line  $AB$ , and the velocity at the beginning of the second time  $KL$  by the line  $Kk$ , and the length described in the first time by the area  $AKkB$ ; all the following velocities will be expounded by the following lines  $Ll$ ,  $Mm$ , &c. and the lengths described, by the areas  $Kl$ ,  $Lm$ , &c. And, by composition, if the whole time be expounded by  $AM$ , the sum of its parts, the whole length described will be expounded by  $AMmB$  the sum of its parts. Now conceive the time  $AM$  to be divided into the parts  $AK$ ,  $KL$ ,  $LM$ , &c. so that  $CA$ ,  $CK$ ,  $CL$ ,  $CM$ , &c. may be in a geometrical progression; and those parts will be in the same progression, and the velocities  $AB$ ,  $Kk$ ,  $Ll$ ,  $Mm$ , &c. will be in the same progression inversely, and the spaces described  $Ak$ ,  $Kl$ ,  $Lm$ , &c. will be equal. *Q. E. D.*

COR. 1. Hence it appears, that if the time be expounded by any part  $AD$  of the asymptote, and the velocity in the beginning of the time by the ordinate  $AB$ ; the velocity at the end of the time will be expounded by the ordinate  $DG$ ; and the whole space described, by the adjacent hyperbolic area  $ABGD$ ; and

and the space which any body can describe in the same time  $AD$ , with the first velocity  $AB$ , in a non-resisting medium, by the rectangle  $AB \times AD$ .

COR. 2. Hence the space described in a resisting medium is given, by taking it to the space described with the uniform velocity  $AB$  in a non-resisting medium, as the hyperbolic area  $ABGD$  to the rectangle  $AB \times AD$ .

COR. 3. The resistance of the medium is also given, by making it equal, in the very beginning of the motion, to an uniform centripetal force, which could generate, in a body falling thro' a non-resisting medium, the velocity  $AB$ , in the time  $AC$ . For if  $BT$  be drawn touching the hyperbola in  $B$ , and meeting the asymptote in  $T$ ; the right line  $AT$  will be equal to  $AC$ , and will express the time, in which the first resistance uniformly continued, may take away the whole velocity  $AB$ .

COR. 4. And thence is also given the proportion of this resistance to the force of gravity, or any other given centripetal force.

COR. 5. And *vice versa*, if there is given the proportion of the resistance to any given centripetal force; the time  $AC$  is also given, in which a centripetal force equal to the resistance may generate any velocity as  $AB$ ; and thence is given the point  $B$ , through which the hyperbola, having  $CH$ ,  $CD$  for its asymptotes, is to be described; as also the space  $ABGD$ , which a body, by beginning its motion with that velocity  $AB$ , can describe in any time  $AD$ , in a similar resisting medium.

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## PROPOSITION VI. THEOREM IV.

*Homogeneous and equal spherical bodies, oppos'd by resistances that are in the duplicate ratio of the velocities, and moving on by their innate force only, will, in times which are reciprocally as the velocities at the beginning, describe equal spaces, and lose parts of their velocities proportional to the wholes.*

To the rectangular asymptotes  $CD$ ,  $CH$  describe any hyperbola  $BbEe$ , cutting the perpendiculars  $AB$ ,  $ab$ ,  $DE$ ,  $de$ , in  $B$ ,  $b$ ,  $E$ ,  $e$ ; let the initial velocities be expounded by the perpendiculars  $AB$ ,  $DE$ , and the times by the lines  $Aa$ ,  $Dd$ . Therefore as  $Aa$  is to  $Dd$ , so (by the hypothesis) is  $DE$  to  $AB$ , and so (from the nature of the hyperbola) is  $CA$  to  $CD$ ; and, by composition, so is  $Ca$  to  $Cd$ . Therefore the areas  $ABba$ ,  $DEed$ , that is, the spaces described, are equal among themselves, and the first velocities  $AB$ ,  $DE$  are proportional to the last  $ab$ ,  $de$ ; and therefore, by division, proportional to the parts of the velocities lost,  $AB - ab$ ,  $DE - de$ .  $Q. E. D.$

## PROPOSITION VII. THEOREM V.

*If spherical bodies are resisted in the duplicate ratio of their velocities, in times which are as the first motions directly and the first resistances inversely, they will lose parts of their motions proportional to the wholes, and will describe spaces proportional to those times and the first velocities conjunctly.*

For the parts of the motions lost are as the resistances and times conjunctly. Therefore, that those parts may be

be proportional to the wholes, the resistance and time conjunctly ought to be as the motion. Therefore the time will be as the motion directly and the resistance inversely. Wherefore the particles of the times being taken in that ratio, the bodies will always lose parts of their motions proportional to the wholes, and therefore will retain velocities always proportional to their first velocities. And because of the given ratio of the velocities, they will always describe spaces, which are as the first velocities and the times conjunctly. *Q. E. D.*

COR. 1. Therefore if bodies equally swift are resisted in a duplicate ratio of their diameters: Homogeneous globes moving with any velocities whatsoever, by describing spaces proportional to their diameters, will lose parts of their motions proportional to the wholes. For the motion of each globe will be as its velocity and mass conjunctly, that is, as the velocity and the cube of its diameter; the resistance (by supposition) will be as the square of the diameter and the square of the velocity conjunctly; and the time (by this proposition) is in the former ratio directly and in the latter inversely, that is, as the diameter directly and the velocity inversely; and therefore the space, which is proportional to the time and velocity, is as the diameter.

COR. 2. If bodies equally swift are resisted in a sesquiplicate ratio of their diameters: Homogeneous globes, moving with any velocities whatsoever, by describing spaces that are in a sesquiplicate ratio of the diameters, will lose parts of their motions proportional to the wholes.

COR. 3. And universally, if equally swift bodies are resisted in the ratio of any power of the diameters: the spaces, in which homogeneous globes, moving with any velocity whatsoever, will lose parts of their motions proportional to the wholes, will be as the cubes of the diameters applied to that power. Let those di-  
ameters







ameters applied to that power. Let those diameters be D and E; and if the resistances, where the velocities are supposed equal, are as  $D^n$  and  $E^n$ : the spaces in which the globes, moving with any velocities whatsoever, will lose parts of their motions proportional to the wholes, will be as  $D^{3-n}$  and  $E^{3-n}$ . And therefore homogeneous globes, in describing spaces proportional to  $D^{3-n}$  and  $E^{3-n}$ , will retain their velocities in the same ratio to one another as at the beginning.

COR. 4. Now if the globes are not homogeneous, the space described by the denser globe must be augmented in the ratio of the density. For the motion, with an equal velocity, is greater in the ratio of the density, and the time (by this Prop.) is augmented in the ratio of motion directly, and the space described in the ratio of the time.

COR. 5. And if the globes move in different mediums, the space, in a medium which, *ceteris paribus*, resists the most, must be diminished in the ratio of the greater resistance. For the time (by this Prop.) will be diminished in the ratio of the augmented resistance, and the space in the ratio of the time.

## LEMMA II.

*The moment of any Genitum is equal to the moments of each of the generating sides drawn into the indices of the powers of those sides, and into their coefficients continually.*

I call any quantity a *Genitum*, which is not made by addition or subduction of divers parts, but is generated or produced in arithmetic by the multiplication, division, or extraction of the root of any terms whatsoever; in geometry by the invention of contents and sides, or of the extremes and means of proportionals. Quantities of

this kind are products, quotients, roots, rectangles, squares, cubes, square and cubic sides, and the like. These quantities I here consider as variable and indetermin'd, and increasing or decreasing as it were by a perpetual motion or flux; and I understand their momentaneous increments or decrements by the name of Moments; so that the increments may be esteem'd as added, or affirmative moments; and the decrements as subducted, or negative ones. But take care not to look upon finite particles as such. Finite particles are not moments, but the very quantities generated by the moments. We are to conceive them as the just nascent principles of finite magnitudes. Nor do we in this Lemma regard the magnitude of the moments, but their first proportion as nascent. It will be the same thing, if, instead of moments, we use either the Velocities of the increments and decrements (which may also be called the motions, mutations, and fluxions of quantities) or any finite quantities proportional to those velocities. The coefficient of any generating side is the quantity which arises by applying the Genitum to that side.

Wherefore the sense of the Lemma is, that if the moments of any quantities  $A, B, C, \&c.$  increasing or decreasing by a perpetual flux, or the velocities of the mutations which are proportional to them, be called  $a, b, c, \&c.$  the moment or mutation of the generated rectangle  $AB$  will be  $aB + bA$ ; the moment of the generated content  $ABC$  will be  $aBC + bAC + cAB$ : and the moments of the generated powers,  $A^2, A^3, A^4, A^{\frac{1}{2}}, A^{\frac{1}{3}}, A^{\frac{2}{3}}, A^{-1}, A^{-2}, A^{-\frac{1}{2}}$  will be  $2aA, 3aA^2, 4aA^3, \frac{1}{2}aA^{-\frac{1}{2}}, \frac{1}{3}aA^{\frac{1}{3}}, \frac{2}{3}aA^{-\frac{2}{3}}, \frac{2}{3}aA^{-\frac{1}{2}}, -aA^{-2}, -2aA^{-3}, -\frac{1}{2}aA^{-\frac{3}{2}}$  respectively. And in general, that the moment of any power  $A^{\frac{n}{m}}$  will be  $\frac{n}{m} aA^{\frac{n}{m}-1}$ . Also that the moment

of the generated quantity  $A^2 B$  will be  $2 a A B - \frac{1}{2} b A^2$ ; the moment of the generated quantity  $A^3 B^4 C^2$  will be  $3 a A^2 B^4 C^2 - \frac{1}{2} 4 b A^3 B^3 C^2 - \frac{1}{2} 2 c A^3 B^4 C$ ; and the moment of the generated quantity  $\frac{A^3}{B^2}$  or  $A^3 B^{-2}$  will be  $3 a A^2 B^{-2} - 2 b A^3 B^{-3}$ ; and so on. The Lemma is thus demonstrated.

CASE 1. Any rectangle as  $A B$  augmented by a perpetual flux, when, as yet, there wanted of the sides  $A$  and  $B$  half their moments  $\frac{1}{2} a$  and  $\frac{1}{2} b$ , was  $A - \frac{1}{2} a$  into  $B - \frac{1}{2} b$ , or  $A B - \frac{1}{2} a B - \frac{1}{2} b A + \frac{1}{4} a b$ ; but as soon as the sides  $A$  and  $B$  are augmented by the other half moments; the rectangle becomes  $A + \frac{1}{2} a$  into  $B + \frac{1}{2} b$  or  $A B + \frac{1}{2} a B + \frac{1}{2} b A + \frac{1}{4} a b$ . From this rectangle subduct the former rectangle, and there will remain the excess  $a B + b A$ . Therefore with the whole increments  $a$  and  $b$  of the sides, the increment  $a B + b A$  of the rectangle is generated. *Q. E. D.*

CASE 2. Suppose  $A B$  always equal to  $G$ , and then the moment of the content  $A B C$  or  $G C$  (by Case 1.) will be  $g C + c G$ , that is, (putting  $A B$  and  $a B + b A$  for  $G$  and  $g$ )  $a B C + b A C + c A B$ . And the reasoning is the same for contents under never so many sides. *Q. E. D.*

CASE 3. Suppose the sides  $A$ ,  $B$ , and  $C$ , to be always equal among themselves; and the moment  $a B + b A$ , of  $A^2$ , that is, of the rectangle  $A B$ , will be  $2 a A$ ; and the moment  $a B C + b A C + c A B$  of  $A^3$ , that is, of the content  $A B C$ , will be  $3 a A^2$ . And by the same reasoning the moment of any power  $A^n$  is  $n a A^{n-1}$ . *Q. E. D.*

CASE 4. Therefore since  $\frac{1}{A}$  into  $A$  is 1, the moment of  $\frac{1}{A}$  drawn into  $A$ , together with  $\frac{1}{A}$  drawn into  $a$ , will be the moment of 1, that is, nothing. Therefore

C 2

fore the moment of  $\frac{1}{A}$  or of  $A^{-1}$  is  $\frac{-a}{A^2}$ . And generally, since  $\frac{1}{A^n}$  into  $A^n$  is 1, the moment of  $\frac{1}{A^n}$  drawn into  $A^n$  together with  $\frac{1}{A^n}$  into  $na A^{n-1}$  will be nothing. And therefore the moment of  $\frac{1}{A^n}$  or  $A^{-n}$  will be  $-\frac{na}{A^{n+1}}$ . *Q. E. D.*

CASE 5. And since  $A^{\frac{1}{2}}$  into  $A^{\frac{1}{2}}$  is  $A$ , the moment of  $A^{\frac{1}{2}}$  drawn into  $2 A^{\frac{1}{2}}$  will be  $a$ , (by Case 3 :) and therefore the moment of  $A^{\frac{1}{2}}$  will be  $\frac{a}{2 A^{\frac{1}{2}}}$  or  $\frac{1}{2} a A^{-\frac{1}{2}}$ .

And generally, putting  $A^{\frac{m}{n}}$  equal to  $B$ , then  $A^m$  will be equal to  $B^n$ , and therefore  $ma A^{m-1}$  equal to  $nb B^{n-1}$ , and  $ma A^{-1}$  equal to  $nb B^{-1}$  or  $nb A^{-\frac{m}{n}}$ ; and therefore  $\frac{m}{n} a A^{\frac{m-n}{n}}$  is equal to  $b$ , that is, equal to the moment of  $A^{\frac{m}{n}}$ . *Q. E. D.*

CASE 6. Therefore the moment of any generated quantity  $A^m B^n$  is the moment of  $A^m$  drawn into  $B^n$ , together with the moment of  $B^n$  drawn into  $A^m$ , that is,  $ma A^{m-1} B^n + nb B^{n-1} A^m$ ; and that whether the indices  $m$  and  $n$  of the powers be whole numbers or fractions, affirmative or negative. And the reasoning is the same for contents under more powers. *Q. E. D.*

COR. 1. Hence in quantities continually proportional, if one term is given, the moments of the rest of the terms will be as the same terms multiplied by the

the number of intervals between them and the given term. Let A, B, C, D, E, F, be continually proportional; then if the term C is given, the moments of the rest of the terms will be among themselves, as — 1 A, — B, D, 2 E, 3 F.

COR. 2. And if in four proportionals the two means are given, the moments of the extremes will be as those extremes. The same is to be understood of the sides of any given rectangle.

COR. 3. And if the sum or difference of two squares is given, the moments of the sides will be reciprocally as the sides.

### SCHOLIUM.

In a letter of mine to Mr. J. Collins, dated December 10. 1672. having described a method of Tangents, which I suspected to be the same with *Slusius's* method, which at that time was not made publick; I subjoined these words; *This is one particular, or rather a corollary, of a general method, which extends itself, without any troublesome calculation, not only to the drawing of Tangents to any Curve lines, whether Geometrical or Mechanical, or any how respecting right lines or other Curves, but also to the resolving other abstruser kinds of Problems about the crookedness, areas, lengths, centres of gravity of Curves, &c. nor is it (as Hudden's method de Maximis & Minimis) limited to equations which are free from surd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite series.* So far that letter. And these last words relate to a Treatise I composed on that subject in the year 1671. The foundation of that general method is contained in the preceding Lemma.

## PROPOSITION VIII. THEOREM VI.

*If a body in an uniform medium, being uniformly acted upon by the force of gravity, ascends or descends in a right line ; and the whole space described be distinguished into equal parts, and in the beginning of each of the parts, (by adding or subducting the resisting force of the medium to or from the force of gravity, when the body ascends or descends) you collect the absolute forces ; I say that those absolute forces are in a geometrical progression.*  
Pl. 2. Fig. 1.

For let the force of gravity be expounded by the given line  $AC$  ; the force of resistance by the indefinite line  $AK$  ; the absolute force in the descent of the body, by the difference  $KC$  ; the velocity of the body by a line  $AP$ , which shall be a mean proportional between  $AK$  and  $AC$ , and therefore in a subduplicate ratio of the resistance ; the increment of the resistance made in a given particle of time by the lineola  $KL$ , and the contemporaneous increment of the velocity by the lineola  $PQ$  ; and with the centre  $C$ , and rectangular asymptotes  $CA$ ,  $CH$ , describe any Hyperbola  $BNS$ , meeting the erected perpendiculars  $AB$ ,  $KN$ ,  $LO$  in  $B$ ,  $N$ , and  $O$ . Because  $AK$  is as  $AP^2$ , the moment  $KL$  of the one will be as the moment  $2APQ$  of the other, that is, as  $AP \times KC$  ; for the increment  $PQ$  of the velocity is (by Law 2.) proportional to the generating force  $KC$ . Let the ratio of  $KL$  be compounded with the ratio of  $KN$ , and the rectangle  $KL \times KN$  will become as  $AP \times KC \times KN$  ; that is, (because the rectangle  $KC \times KN$  is given) as  $AP$ . But the ultimate ratio of the hyperbolic area  $KNOL$  to the rectangle  $KL \times KN$  becomes, when the points  $K$  and  $L$  coincide, the ratio of

of

of equality. Therefore that hyperbolic evanescent area is as  $AP$ . Therefore the whole hyperbolic area  $ABOL$  is composed of particles  $KNOL$  which are always proportional to the velocity  $AP$ ; and therefore is itself proportional to the space described with that velocity. Let that area be now divided into equal parts, as  $ABMI$ ,  $IMNK$ ,  $KNOL$ , &c. and the absolute forces  $AC$ ,  $IC$ ,  $KC$ ,  $LC$ , &c. will be in a geometrical progression. *Q.E.D.* And by a like reasoning, in the ascent of the body, taking, on the contrary side of the point  $A$ , the equal areas  $ABmi$ ,  $imnk$ ,  $knol$ , &c. it will appear that the absolute forces  $AC$ ,  $iC$ ,  $kC$ ,  $lC$ , &c. are continually proportional. Therefore if all the spaces in the ascent and descent are taken equal; all the absolute forces  $lC$ ,  $kC$ ,  $iC$ ,  $AC$ ,  $IC$ ,  $KC$ ,  $LC$ , &c. will be continually proportional. *Q.E.D.*

COR. 1. Hence if the space described be expounded by the hyperbolic area  $ABNK$ ; the force of gravity, the velocity of the body, and the resistance of the medium, may be expounded by the lines  $AC$ ,  $AP$ , and  $AK$  respectively; and *vice versa*.

COR. 2. And the greatest velocity, which the body can ever acquire in an infinite descent, will be expounded by the line  $AC$ .

COR. 3. Therefore if the resistance of the medium answering to any given velocity be known, the greatest velocity will be found, by taking it to that given velocity in a ratio subduplicate of the ratio which the force of gravity bears to that known resistance of the medium.



## PROPOSITION IX. THEOREM VII.

*Supposing what is above demonstrated, I say that if the tangents of the angles of the sector of a circle, and of an hyperbola, be taken proportional to the velocities, the radius being of a fit magnitude; all the time of the ascent to the highest place will be as the sector of the circle, and all the time of descending from the highest place as the sector of the hyperbola. Pl. 2. Fig. 2.*

To the right line  $AC$ , which expresses the force of gravity, let  $AD$  be drawn perpendicular and equal. From the centre  $D$  with the semidiameter  $AD$  describe as well the quadrant  $AtE$  of a Circle; as the rectangular Hyperbola  $AVZ$ , whose axe is  $AX$ , principal vertex  $A$ , and asymptote  $DC$ . Let  $Dp$ ,  $DP$  be drawn; and the circular sector  $AtD$  will be as all the time of the ascent to the highest place; and the hyperbolic sector  $ATD$  as all the time of descent from the highest place: If so be that the tangents  $Ap$ ,  $AP$  of those sectors be as the velocities. Fig. 2.

CASE I. Draw  $Dvq$  cutting off the moments or least particles  $tDv$  and  $qDp$ , described in the same time, of the sector  $ADt$  and of the triangle  $ADp$ . Since those particles (because of the common angle  $D$ ) are in a duplicate ratio of the sides, the particle  $tDv$  will be as  $\frac{qDp \times tD^2}{pD^2}$ , that is, (because  $tD$  is given) as  $\frac{qDp}{pD^2}$ . But  $pD^2$  is  $AD^2 + Ap^2$ , that is,  $AD^2 + AD \times Ak$ , or  $AD \times Ck$ ; and  $qDp$  is  $\frac{1}{2} AD \times pq$ . Therefore  $tDv$ , the particle of the sector, is as  $\frac{pq}{Ck}$ ; that is,

is, as the least decrement  $pq$  of the velocity directly, and the force  $Ck$ , which diminishes the velocity, inversely; and therefore as the particle of time answering to the decrement of the velocity. And, by composition, the sum of all the particles  $tDv$  in the sector  $ADt$ , will be as the sum of the particles of time answering to each of the lost particles  $pq$ , of the decreasing velocity  $Ap$ , till that velocity, being diminished into nothing, vanishes; that is, the whole sector  $ADt$  is as the whole time of ascent to the highest place. *Q.E.D.*

CASE 2. Draw  $DQV$  cutting off the least particles  $TDV$  and  $PDQ$  of the sector  $DAV$ , and of the triangle  $DAQ$ ; and these particles will be to each other as  $DT^2$  to  $DP^2$ , that is, (if  $TX$  and  $AP$  are parallel) as  $DX^2$  to  $DA^2$  or  $TX^2$  to  $AP^2$ ; and, by division, as  $DX^2 - TX^2$  to  $DA^2 - AP^2$ . But, from the nature of the hyperbola,  $DX^2 - TX^2$  is  $AD^2$ ; and, by the supposition,  $AP^2$  is  $AD \times AK$ . Therefore the particles are to each other as  $AD^2$  to  $AD^2 - AD \times AK$ ; that is, as  $AD$  to  $AD - AK$  or  $AC$  to  $CK$ : and therefore the particle  $TDV$  of the sector is  $\frac{PDQ \times AC}{CK}$ ; and therefore (because  $AC$  and  $AD$  are

given) as  $\frac{PQ}{CK}$ ; that is, as the increment of the velocity directly, and as the force generating the increment inversely; and therefore as the particle of the time answering to the increment. And, by composition, the sum of the particles of time, in which all the particles  $PQ$  of the velocity  $AP$  are generated, will be as the sum of the particles of the sector  $ATD$ ; that is, the whole time will be as the whole sector. *Q.E.D.*

COR. I. Hence if  $AB$  be equal to a fourth part of  $AC$ , the space which a body will describe by falling in any time will be to the space which the body could describe, by moving uniformly on in the same time with

with its greatest velocity  $AC$ , as the area  $ABNK$ , which expresses the space described in falling, to the area  $ATD$ , which expresses the time. For since  $AC$  is to  $AP$  as  $AP$  to  $AK$ , then (by Cor. 1. Lem. 2. of this Book)  $LK$  is to  $PQ$  as  $2AK$  to  $AP$ , that is, as  $2AP$  to  $AC$ , and thence  $LK$  is to  $\frac{1}{2}PQ$  as  $AP$  to  $\frac{1}{2}AC$  or  $AB$ ; and  $KN$  is to  $AC$  or  $AD$  as  $AB$  to  $CK$ ; and therefore, *ex aequo*,  $LKNO$  to  $DPQ$  as  $AP$  to  $CK$ . But  $DPQ$  was to  $DTV$  as  $CK$  to  $AC$ . Therefore, *ex aequo*,  $LKNO$  is to  $DTV$  as  $AP$  to  $AC$ ; that is, as the velocity of the falling body to the greatest velocity which the body by falling can acquire. Since therefore the moments  $LKNO$  and  $DTV$  of the areas  $ABNK$  and  $ATD$  are as the velocities, all the parts of those areas generated in the same time, will be as the spaces described in the same time; and therefore the whole areas  $ABNK$  and  $ADT$  generated from the beginning, will be as the whole spaces described from the beginning of the descent. *Q. E. D.*

COR. 2. The same is true also of the space described in the ascent. That is to say, that all that space is to the space described in the same time with the uniform velocity  $AC$ , as the area  $ABnk$  is to the sector  $ADt$ .

COR. 3. The velocity of the body, falling in the time  $ATD$ , is to the velocity which it would acquire in the same time in a non-resisting space, as the triangle  $APD$  to the hyperbolic sector  $ATD$ . For the velocity in a non-resisting medium would be as the time  $ATD$ , and in a resisting medium is as  $AP$ , that is, as the triangle  $APD$ . And those velocities at the beginning of the descent, are equal among themselves, as well as those areas  $ATD$ ,  $APD$ .

COR. 4. By the same argument, the velocity in the ascent is to the velocity with which the body in the same time, in a non-resisting space, would lose all

all its motion of ascent, as the triangle  $ApD$  to the circular sector  $AtD$ ; or as the right line  $Ap$  to the arc  $At$ .

COR. 5. Therefore the time in which a body by falling in a resisting medium, would acquire the velocity  $AP$ , is to the time in which it would acquire its greatest velocity  $AC$  by falling in a non-resisting space, as the sector  $ADT$  to the triangle  $ADC$ : and the time in which it would lose its velocity  $Ap$  by ascending in a resisting medium, is to the time in which it would lose the same velocity by ascending in a non-resisting space, as the arc  $At$  to its tangent  $Ap$ .

COR. 6. Hence from the given time there is given the space described in the ascent or descent. For the greatest velocity of a body descending *in infinitum* is given (by Corol. 2 and 3. Theor. 6. of this Book) and thence the time is given in which a body would acquire that velocity by falling in a non-resisting space. And taking the sector  $ADT$  or  $ADt$  to the triangle  $ADC$  in the ratio of the given time to the time just now found; there will be given both the velocity  $AP$  or  $Ap$ , and the area  $ABNK$  or  $ABnk$ , which is to the sector  $ADT$ , or  $ADt$ , as the space sought to the space which would, in the given time, be uniformly described with that greatest velocity found just before.

COR. 7. And by going backward, from the given space of ascent or descent  $ABnk$  or  $ABNK$ , there will be given the time  $ADt$  or  $ADT$ .

PRO-

## PROPOSITION X. PROBLEM III.

*Suppose the uniform force of gravity to tend directly to the plane of the horizon, and the resistance to be as the density of the medium and the square of the velocity conjunctly: it is proposed to find the density of the medium in each place, which shall make the body move in any given curve line; the velocity of the body, and the resistance of the medium in each place. Pl. 2. Fig. 3.*

Let  $PQ$  be a plane perpendicular to the plane of the scheme itself;  $PFHQ$  a curve line meeting that plane in the points  $P$  and  $Q$ ;  $G, H, I, K$  four places of the body going on in this curve from  $F$  to  $Q$ ; and  $GB, HC, ID, KE$  four parallel ordinates let fall from these points to the horizon, and standing on the horizontal line  $PQ$  at the points  $B, C, D, E$ ; and let the distances  $BC, CD, DE$ , of the ordinates be equal among themselves. From the points  $G$  and  $H$  let the right lines  $GL, HN$ , be drawn touching the curve in  $G$  and  $H$ , and meeting the ordinates  $CH, DI$ , produced upwards, in  $L$  and  $N$ ; and compleat the parallelogram  $HCDM$ . And the times, in which the body describes the arcs  $GH, HI$ , will be in a subduplicate ratio of the altitudes  $LH, NI$ , which the bodies would describe in those times, by falling from the tangents; and the velocities will be as the lengths described  $GH, HI$  directly and the times inversely. Let the times be expounded by  $T$  and  $t$ , and the velocities

by  $\frac{GH}{T}$  and  $\frac{HI}{t}$ ; and the decrement of the velocity

pro-

produced in the time  $t$  will be expounded by  $\frac{GH}{T}$  —

$\frac{HI}{t}$ . This decrement arises from the resistance which

retards the body, and from the gravity which accelerates it. Gravity, in a falling body, which in its fall describes the space  $NI$ , produces a velocity, with which it would be able to describe twice that space in the same time, as *Galileo* has demonstrated; that is, the velocity  $\frac{2NI}{t}$  : but if the body describes the arc  $HI$ ,

it augments that arc only by the length  $HI - HN$

or  $\frac{MI \times NI}{HI}$ ; and therefore generates only the velocity  $\frac{2MI \times NI}{t \times HI}$ .

Let this velocity be added to the

beforementioned decrement, and we shall have the decrement of the velocity arising from the resistance alone,

that is,  $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$ . Therefore since

in the same time, the action of gravity generates, in a

falling body, the velocity  $\frac{2NI}{t}$ ; the resistance will be

to the gravity as  $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$  to  $\frac{2NI}{t}$ ,

or as  $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$  to  $2NI$ .

Now for the abscissa's  $CB, CD, CE$  put  $o, o, o$ . For the ordinate  $CH$  put  $P$ ; and for  $MI$  put any series  $Qo + Ro^2 + So^3$  &c. And all the terms of the series after the first, that is,  $Ro^2 + So^3 +$  &c. will be  $NI$ ; and the ordinates  $DI, EK$  and  $BG$  will be  $P - Qo - Ro^2 - So^3 -$  &c.  $P - 2Qo - 4Ro^2 - 8So^3 -$  &c. and  $P + Qo - Ro^2 + So^3 -$  &c. respectively. And by squaring the dif-

differences of the ordinates  $BG - CH$  and  $CH - DI$ , and to the squares thence produced adding the squares of  $BC$  and  $CD$  themselves, you will have  $00 + QQ00 - 2QR0^3 + \&c.$  and  $00 + QQ00 + 2QR0^3 + \&c.$  the squares of the arcs  $GH, HI$ ; whose roots

$$0\sqrt{1+QQ} - \frac{QR00}{\sqrt{1+QQ}}, \text{ and } 0\sqrt{1+QQ} + \frac{QR00}{\sqrt{1+QQ}}$$

$\frac{QR00}{\sqrt{1+QQ}}$  are the arcs  $GH$  and  $HI$ . Moreover, if

from the ordinate  $CH$  there be subducted half the sum of the ordinates  $BG$  and  $DI$ , and from the ordinate  $DI$  there be subducted half the sum of the ordinates  $CH$  and  $EK$ , there will remain  $R00$  and  $R00 - 3S0^3$  the versed sines of the arcs  $GI$  and  $HK$ . And these are proportional to the lineolæ  $LH$  and  $NI$ , and therefore in the duplicate ratio of the infinitely small times

$T$  and  $t$ : and thence the ratio  $\frac{t}{T}$  is  $\sqrt{\frac{R+3S0}{R}}$  or  $\frac{R+\frac{1}{2}S0}{R}$ ; and  $\frac{t \times GH}{T} = HI + \frac{2MI \times NI}{HI}$ , by

substituting the values of  $\frac{t}{T}$ ,  $GH, HI, MI$  and  $NI$

just found, becomes  $\frac{3S00}{2R} \sqrt{1+QQ}$ . And since  $2NI$

is  $2R00$ , the resistance will be now to the gravity as  $\frac{3S00}{2R} \sqrt{1+QQ}$  to  $2R00$ , that is, as  $3S \sqrt{1+QQ}$  to  $4RR$ .

And the velocity will be such, that a body going off therewith from any place  $H$ , in the direction of the tangent  $HN$ , would describe, in vacuo, a Parabola, whose diameter is  $HC$ , and its latus rectum  $\frac{HN^2}{NI}$  or  $\frac{1+QQ}{R}$ .

And

And the resistance is as the density of the medium and the square of the velocity conjunctly; and therefore the density of the medium is as the resistance directly, and the square of the velocity inversely; that

is, as  $\frac{3 S \sqrt{1 - QQ}}{4 R R}$  directly and  $\frac{1 - QQ}{R}$  inverse-

ly; that is, as  $\frac{S}{R \sqrt{1 - QQ}}$  *Q. E. I.*

COR. 1. If the tangent *HN* be produced both ways, so as to meet any ordinate *AF* in *T*:  $\frac{HT}{AC}$  will

be equal to  $\sqrt{1 - QQ}$ , and therefore in what has gone before may be put for  $\sqrt{1 - QQ}$ . By this means the resistance will be to the gravity as  $3 S \times HT$  to

$4 R R \times AC$ ; the velocity will be as  $\frac{HT}{AC \sqrt{R}}$ , and

the density of the medium will be as  $\frac{S \times AC}{R \times HT}$ .

COR. 2. And hence, if the curve line *PFHQ* be defined by the relation between the base or abscissa *AC* and the ordinate *CH*, as is usual; and the value of the ordinate be resolved into a converging series: The problem will be expeditiously solved by the first terms of the series; as in the following examples.

EXAMPLE 1. Let the line *PFHQ* be a semi-circle described upon the diameter *PQ*; to find the density of the medium that shall make a projectile move in that line.

Bisect the diameter *PQ* in *A*; and call *AQ*, *n*; *AC*, *a*; *CH*, *e*; and *CD*, *o*: then  $DI^2$  or  $AQ^2 - AD^2 = nn - aa - 2ao - oo$ , or  $ee - 2ao - oo$ ; and the root being extracted by our method, will give

$$DI = e - \frac{ao}{e} - \frac{oo}{2e} - \frac{aao}{2e^3} - \frac{ao^3}{2e^3} - \frac{a^3 o^3}{2e^5} - \dots$$

4

&c.



&c. Here put  $nn$  for  $ee \frac{1}{a}$ , and  $DI$  will become

$$= o - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{annoo^3}{2e^5} - \&c.$$

Such series I distinguish into successive terms after this manner: I call that the first term, in which the infinitely small quantity  $o$  is not found; the second, in which that quantity is of one dimension only; the third, in which it arises to two dimensions; the fourth, in which it is of three; and so *ad infinitum*. And the first term, which here is  $e$ , will always denote the length of the ordinate  $CH$ , standing at the beginning of the indefinite quantity  $o$ . The second

term, which here is  $\frac{ao}{e}$ , will denote the difference be-

tween  $CH$  and  $DN$ ; that is, the lineola  $MN$  which is cut off by completing the parallelogram  $HCDM$ ; and therefore always determines the position of the tangent  $HN$ ; as, in this case, by taking  $MN$  to  $HM$  as

$\frac{ao}{e}$  to  $o$ , or  $a$  to  $e$ . The third term, which here is

$\frac{nnoo}{2e^3}$ , will represent the lineola  $IN$ , which lies be-

tween the tangent and the curve; and therefore determines the angle of contact  $IHN$ , or the curvature which the curve line has in  $H$ . If that lineola  $IN$  is of a finite magnitude, it will be express'd by the third term together with those that follow *in infinitum*. But if that lineola be diminished *in infinitum*, the terms following become infinitely less than the third term, and therefore may be neglected. The fourth term determines the variation of the curvature; the fifth, the variation of the variation; and so on. Whence, by the way, appears no contemptible use of these series in the solution of problems that depend upon tangents, and the curvature of curves.

Now

Now compare the series  $e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{annoo^3}{2e^5}$   
 — &c. with the series  $P - Qo - Roo - So^3 - \&c.$   
 and for  $P, Q, R$  and  $S$  put  $e, \frac{a}{e}, \frac{nn}{2e^3}$  and  $\frac{ann}{2e^5}$ , and  
 for  $\sqrt{1 + QQ}$  put  $\sqrt{1 + \frac{aa}{ee}}$  or  $\frac{n}{e}$ ; and the den-  
 sity of the medium will come out as  $\frac{a}{ne}$ , that is, (be-

cause  $n$  is given) as  $\frac{a}{e}$ , or  $\frac{AC}{CH}$ , that is, as that  
 length of the tangent  $HT$ , which is terminated at the  
 semidiameter  $AF$  standing perpendicularly on  $PQ$ ;  
 and the resistance will be to the gravity as  $3a$  to  $3n$ ,  
 that is, as  $3AC$  to the diameter  $PQ$  of the circle;  
 and the velocity will be as  $\sqrt{CH}$ . Therefore if the  
 body goes from the place  $F$ , with a due velocity, in the  
 direction of a line parallel to  $PQ$ , and the density of  
 the medium in each of the places  $H$  is as the length of  
 the tangent  $HT$ , and the resistance also in any place  $H$   
 is to the force of gravity as  $3AC$  to  $PQ$ , that body  
 will describe the quadrant  $FHQ$  of a circle. *Q. E. I.*

But if the same body should go from the place  $P$ ,  
 in the direction of a line perpendicular to  $PQ$ , and  
 should begin to move in an arc of the semi-circle  
 $PFQ$ , we must take  $AC$  or  $a$  on the contrary side of  
 the centre  $A$ ; and therefore its sign must be changed,  
 and we must put  $-a$  for  $+a$ . Then the density of  
 the medium would come out as  $-\frac{a}{e}$ . But nature does

not admit of a negative density, that is, a density  
 which accelerates the motion of bodies; and therefore it  
 cannot naturally come to pass, that a body by ascending  
 from  $P$  should describe the quadrant  $PF$  of a circle.  
 To produce such an effect, a body ought to be acce-

lerated by an impelling medium, and not impeded by a resisting one.

EXAMPLE 2. Let the line  $PFQ$  be a Parabola, having its axis  $AF$  perpendicular to the horizon  $PQ$ ; to find the density of the medium, which will make a projectile move in that line. *Fig. 4.*

From the nature of the Parabola, the rectangle  $PDQ$  is equal to the rectangle under the ordinate  $DI$  and some given right line: that is, if that right line be called  $b$ ;  $PC, a$ ;  $PQ, c$ ;  $CH, e$ ; and  $CD, o$ ; the rectangle  $a - | - o$  into  $c - a - o$  or  $ac - aa - 2ao - | - co - oo$  is equal to the rectangle  $b$  into  $DI$ , and therefore  $DI$  is equal to  $\frac{ac - aa}{b} + \frac{c - 2a}{b}o - \frac{oo}{b}$ . Now the second term  $\frac{c - 2a}{b}o$  of this series is to

be put for  $Qo$ , and the third term  $\frac{oo}{b}$  for  $Roo$ . But since there are no more terms, the coefficient  $S$  of the fourth term will vanish; and therefore the quantity  $\frac{S}{R \sqrt{1 - | - QQ}}$ , to which the density of the medium is proportional, will be nothing. Therefore, where the medium is of no density, the projectile will move in a Parabola; as *Galileo* hath heretofore demonstrated. *Q. E. I.*

EXAMPLE 3. Let the line  $AGK$  be an Hyperbola, having its asymptote  $NX$  perpendicular to the horizontal plane  $AK$ ; to find the density of the medium, that will make a projectile move in that line. *Fig. 5.*

Let  $MX$  be the other asymptote, meeting the ordinate  $DG$  produced in  $V$ ; and from the nature of the Hyperbola, the rectangle of  $XV$  into  $VG$  will be given. There is also given the ratio of  $DN$  to  $VX$ , and therefore the rectangle of  $DN$  into  $VG$  is given. Let that be  $bb$ : and, completing the parallelogram

$DNXZ,$

$DNXZ$ ; let  $BN$  be called  $a$ ;  $BD, o$ ;  $NX, c$ ; and let the given ratio of  $VZ$  to  $ZX$  or  $DN$  be  $\frac{m}{n}$ . Then

$DN$  will be equal to  $a - o$ ,  $VG$  equal to  $\frac{bb}{a - o}$ ,  $VZ$  equal to  $\frac{m}{n} \times a - o$ , and  $GD$  or  $NX - VZ - VG$

equal to  $c - \frac{m}{n} a - \frac{m}{n} o - \frac{bb}{a - o}$ . Let the term  $\frac{bb}{a - o}$

be resolved into the converging series  $\frac{bb}{a} + \frac{bb}{aa} o +$

$\frac{bb}{a^3} o^2 + \frac{bb}{a^4} o^3$  &c. and  $GD$  will become equal to  $c -$

$\frac{m}{n} a - \frac{bb}{a} - \frac{m}{n} o - \frac{bb}{aa} o - \frac{bb}{a^3} o^2 - \frac{bb}{a^4} o^3$  &c. The

second term  $\frac{m}{n} o - \frac{bb}{aa} o$  of this series is to be used for

$Qo$ , the third  $\frac{bb}{a^3} o^2$  with its sign changed for  $Ro^2$ ,

and the fourth  $\frac{bb}{a^4} o^3$  with its sign changed also for

$So^3$ , and their coefficients  $\frac{m}{n} - \frac{bb}{aa}$ ,  $\frac{bb}{a^3}$  and  $\frac{bb}{a^4}$  are

to be put for  $Q$ ,  $R$  and  $S$  in the former Rule. Which being done, the density of the medium

will come out as  $\frac{bb}{a^3} \sqrt{1 + \frac{mm}{nn} - \frac{2mbb}{naa} + \frac{b^4}{a^4}}$  or

$\sqrt[3]{aa + \frac{mm}{nn} aa - \frac{2mbb}{n} + \frac{b^4}{aa}}$ , that is, if in  $VZ$

D 2

you

you take  $VY$  equal to  $VG$ , as  $\frac{1}{XY}$ . For  $aa$  and  $\frac{m^2}{n^2}a^2$

$-\frac{2mbb}{n} + \frac{b^4}{aa}$  are the squares of  $XZ$  and  $ZY$ . But

the ratio of the resistance to gravity is found to be that of  $3XY$  to  $2YG$ ; and the velocity is that with which the body would describe a Parabola, whose vertex is  $G$ , diameter  $DG$ , latus rectum  $\frac{XY^2}{VG}$ . Suppose

therefore that the densities of the medium in each of the places  $G$  are reciprocally as the distances  $XY$ , and that the resistance in any place  $G$  is to the gravity as  $3XY$  to  $2YG$ ; and a body let go from the place  $A$ , with a due velocity, will describe that Hyperbola  $AGK$ . *Q.E.I.*

EXAMPLE 4. Suppose indefinitely, the line  $AGK$  to be an Hyperbola, described with the centre  $X$ , and the asymptotes  $MX, NX$ , so that, having constructed the rectangle  $XZDN$ , whose side  $ZD$  cuts the Hyperbola in  $G$  and its asymptote in  $V$ ,  $VG$  may be reciprocally as any power  $DN^n$  of the line  $ZX$  or  $DN$ , whose index is the number  $n$ : To find the density of the medium in which a projected body will describe this curve. *Fig. 5.*

For  $BN, BD, NX$  put  $A, O, C$  respectively, and let  $VZ$  be to  $XZ$  or  $DN$  as  $d$  to  $e$ , and  $VG$  be equal to  $\frac{bb}{DN^n}$ ; then  $DN$  will be equal to  $A - O$ ,  $VG =$

$\frac{bb}{A - O|^n}$ ,  $VZ = \frac{d}{e} \overline{A - O}$ , and  $GD$  or  $NX - VZ$

$- VG$  equal to  $C - \frac{d}{e} A + \frac{d}{e} O - \frac{bb}{A - O|^n}$ . Let

the term  $\frac{bb}{A - O|^n}$  be resolved into an infinite series  $\frac{bb}{A^n}$   
 $+$

$$+ \frac{nbb}{A^{n+1}} \times O + \frac{nn+1}{2A^{n+2}} \times bbb O^2 + \frac{n^3+3nn+2n}{6A^{n+3}}$$

$\times bbb O^3$  &c. and  $GD$  will be equal to  $C - \frac{d}{e} A -$

$$\frac{bb}{A^n} + \frac{d}{e} O - \frac{nbb}{A^{n+1}} O - \frac{1+nn+1}{2A^{n+2}} bbb O^2 -$$

$$\frac{1+n^3+3nn+2n}{6A^{n+3}} bbb O^3 \text{ \&c. The second term}$$

$\frac{d}{e} O - \frac{nbb}{A^{n+1}} O$  of this series is to be used for

$Qo$ , the third  $\frac{nn+1}{2A^{n+2}} bbb O^2$  for  $Ro$ , the fourth

$\frac{n^3+3nn+2n}{6A^{n+3}} bbb O^3$  for  $So$ . And thence the den-

sity of the medium  $\frac{S}{R \sqrt{1+QQ}}$ , in any place  $G$ ,

will be  $\frac{n+2}{3 \sqrt{A^2 + \frac{dd}{ee} A^2 - \frac{2dnbb}{eA^n} A + \frac{nnb^4}{A^{2n}}}}$ , and

therefore if in  $VZ$  you take  $VT$  equal to  $n \times VG$ , that

density is reciprocally as  $XT$ . For  $A^2$  and  $\frac{dd}{ee} A^2 -$

$\frac{2dnbb}{eA^n} A + \frac{nnb^4}{A^{2n}}$  are the squares of  $XZ$  and  $ZT$ .

But the resistance in the same place  $G$  is to the force

of gravity as  $3S \times \frac{XT}{A}$  to  $4RR$ , that is, as  $XT$  to

$\frac{2nn+2n}{n+2} VG$ . And the velocity there, is the same

wherewith the projected body would move in a Pa-

rabola, whose vertex is  $G$ , diameter  $GD$ , and latus

rectum  $\frac{1+QQ}{R}$  or  $\frac{2XT^2}{nn+1 \times VG}$ . *Q.E.I.*

SCHOL

## S C H O L I U M.

In the same manner that the density of the medium comes out to be as  $\frac{S \times AC}{R \times HT}$ , in Corol. 1. if the resistance is put as any power  $V^n$  of the velocity  $V$ , the density of the medium will come out to be as

$$\frac{S}{R^{\frac{4-n}{2}}} \times \frac{AC}{HT} \Big|^{n-1}. \quad \text{Fig. 3.}$$

And therefore if a curve can be found, such that the ratio of  $\frac{S}{R^{\frac{4-n}{2}}}$  to  $\frac{HT}{AC} \Big|^{n-1}$ , or of  $\frac{S^2}{R^{4-n}}$  to

$\frac{1-1-QQ}{1-1-QQ} \Big|^{n-1}$  may be given: the body, in an uniform medium, whose resistance is as the power  $V^n$  of the velocity  $V$ , will move in this curve. But let us return to more simple curves.

Because there can be no motion in a Parabola except in a non-resisting medium, but in the Hyperbola's here described 'tis produced by a perpetual resistance; it is evident that the line which a projectile describes in an uniformly resisting medium, approaches nearer to these Hyperbola's than to a Parabola. That line is certainly of the hyperbolic kind, but about the vertex it is more distant from the asymptotes, and in the parts remote from the vertex draws nearer to them, than these Hyperbola's here described. The difference however is not so great between the one and the other, but that these latter may be commodiously enough used in practice instead of the former. And perhaps these may prove more useful, than an Hyperbola that is more accurate, and at the same time more compounded. They may be made use of then in this manner. Fig. 5.

Coma

Compleat the parallelogram  $XTGT$ , and the right line  $GT$  will touch the hyperbola in  $G$ , and therefore the density of the medium in  $G$  is reciprocally as the tangent  $GT$ , and the velocity there, as  $\sqrt{\frac{GT^2}{GV}}$ , and the resistance is to the force of gravity as  $GT$  to  $\frac{2nn-1-2n}{n-1-2} \times GV$ .

Therefore if a body projected from the place  $A$  in the direction of the right line  $AH$ , (*Fig. 6.*) describes the Hyperbola  $AGK$ , and  $AH$  produced meets the asymptote  $NX$  in  $H$ , and  $AI$  drawn parallel to it meets the other asymptote  $MX$  in  $I$ ; the density of the medium in  $A$  will be reciprocally as  $AH$ , and the velocity of the body as  $\sqrt{\frac{AH^2}{AI}}$ , and the resistance

there to the force of gravity as  $AH$  to  $\frac{2nn-1-2n}{n-1-2} \times AI$ . Hence the following rules are deduced.

**RULE 1.** If the density of the medium at  $A$ , and the velocity with which the body is projected remain the same, and the angle  $NAH$  be changed; the lengths  $AH$ ,  $AI$ ,  $HX$  will remain. Therefore if those lengths, in any one case, are found, the Hyperbola may afterwards be easily determined from any given angle  $NAH$ .

**RULE 2.** If the angle  $NAH$ , and the density of the medium at  $A$  remain the same, and the velocity with which the body is projected be changed, the length  $AH$  will continue the same; and  $AI$  will be changed in a duplicate ratio of the velocity reciprocally.

**RULE 3.** If the angle  $NAH$ , the velocity of the body at  $A$ , and the accelerative gravity remain the same, and the proportion of the resistance at  $A$  to the motive gravity be augmented in any ratio; the proportion of  $AH$  to  $AI$  will be augmented in the same ratio,



the latus rectum of the abovementioned Parabola remaining the same, and also the length  $\frac{AH^2}{AI}$  proportional to it; and therefore  $AH$  will be diminished in the same ratio, and  $AI$  will be diminished in the duplicate of that ratio. But the proportion of the resistance to the weight is augmented, when either the specific gravity is made less, the magnitude remaining equal, or when the density of the medium is made greater, or when, by diminishing the magnitude, the resistance becomes diminished in a less ratio than the weight.

RULE 4. Because the density of the medium is greater near the vertex of the Hyperbola, than it is in the place  $A$ ; that a mean density may be preserv'd, the ratio of the least of the tangents  $GT$  to the tangent  $AH$  ought to be found, and the density in  $A$  augmented in a ratio a little greater than that of half the sum of those tangents to the least of the tangents  $GT$ .

RULE 5. If the lengths  $AH$ ,  $AI$  are given, and the figure  $AGK$  is to be described: produce  $HN$  to  $X$ , so that  $HX$  may be to  $AI$  as  $n-1$  to  $1$ ; and with the centre  $X$ , and the asymptotes  $MX$ ,  $NX$  describe an Hyperbola thro' the point  $A$ , such that  $AI$  may be to any of the lines  $VG$  as  $XV$  to  $XI$ .

RULE 6. By how much the greater the number  $n$  is, so much the more accurate are these Hyperbola's in the ascent of the body from  $A$ , and less accurate in its descent to  $K$ ; and the contrary. The Conic Hyperbola keeps a mean ratio between these, and is more simple than the rest. Therefore if the Hyperbola be of this kind, and you are to find the point  $K$ , where the projected body falls upon any right line  $AN$  passing thro' the point  $A$ : let  $AN$  produced meet the asymptotes  $MX$ ,  $NX$  in  $M$  and  $N$ , and take  $NK$  equal to  $AM$ .

RULE 7. And hence appears an expeditious method of determining this Hyperbola from the phenomena. Let

Let two similar and equal bodies be projected with the same velocity, in different angles  $HAK$ ,  $hAk$ , (*Fig. 6.*) and let them fall upon the plane of the horizon in  $K$  and  $k$ ; and note the proportion of  $AK$  to  $Ak$ . Let it be as  $d$  to  $e$ . Then erecting a perpendicular  $AI$  of any length, assume any how the length  $AH$  or  $Ah$ , and thence graphically, or by scale and compass, collect the lengths  $AK$ ,  $Ak$  (by Rule 6.) If the ratio of  $AK$  to  $Ak$  be the same with that of  $d$  to  $e$ , the length of  $AH$  was rightly assumed. If not, take on the indefinite right line  $SM$ , (*Fig. 7.*) the length  $SM$  equal to the assumed  $AH$ ; and erect a perpendicular  $MN$ , equal to the difference  $\frac{AK}{Ak} - \frac{d}{e}$  of the ratio's drawn into any given right line. By the like method, from several assumed lengths  $AH$ , you may find several points  $N$ ; and draw thro' them all a regular curve  $NNXN$ , cutting the right line  $SM$  in  $X$ . Lastly, assume  $AH$  equal to the abscissa  $SX$ , and thence find again the length  $AK$ ; and the lengths, which are to the assumed length  $AI$  and this last  $AH$ , as the length  $AK$  known by experiment, to the length  $AK$  last found, will be the true lengths  $AI$  and  $AH$ , which were to be found. But these being given, there will be given also the resisting force of the medium in the place  $A$ , it being to the force of gravity as  $AH$  to  $2AI$ . Let the density of the medium be increased by Rule 4. and if the resisting force just found be increased in the same ratio, it will become still more accurate.

**RULE 8.** The lengths  $AH$ ,  $HX$  being found; let there be now required the position of the line  $AH$ , according to which a projectile thrown with that given velocity, shall fall upon any point  $K$ . At the points  $A$  and  $K$ , (*Fig. 6.*) erect the lines  $AC$ ,  $KF$  perpendicular to the horizon; whereof let  $AC$  be drawn downwards, and be equal to  $AI$  or  $\frac{1}{2}HX$ . With the asymptotes  $AK$ ,  $KF$ , describe an Hyperbola, whose con-

conjugate shall pass thro' the point  $C$ ; and from the centre  $A$ , with the interval  $AH$ , describe a circle cutting that Hyperbola in the point  $H$ ; then the projectile thrown in the direction of the right line  $AH$  will fall upon the point  $K$ . *Q.E.I.* For the point  $H$ , because of the given length  $AH$ , must be somewhere in the circumference of the described circle. Draw  $CH$  meeting  $AK$  and  $KF$  in  $E$  and  $F$ ; and because  $CH$ ,  $MX$  are parallel, and  $AC$ ,  $AI$  equal,  $AE$  will be equal to  $AM$ , and therefore also equal to  $KN$ . But  $CE$  is to  $AE$  as  $FH$  to  $KN$ , and therefore  $CE$  and  $FH$  are equal. Therefore the point  $H$  falls upon the hyperbolic curve described with the asymptotes  $AK$ ,  $KF$ , whose conjugate passes thro' the point  $C$ ; and is therefore found in the common intersection of this hyperbolic curve and the circumference of the described circle. *Q.E.D.* It is to be observed that this operation is the same, whether the right line  $AKN$  be parallel to the horizon, or inclined thereto in any angle; and that from two intersections  $H, H$ , there arise two angles  $NAH, NAH$ ; and that in mechanical practice it is sufficient once to describe a circle, then to apply a ruler  $CH$ , of an indeterminate length, so to the point  $C$ , that its part  $FH$ , intercepted between the circle and the right line  $FK$ , may be equal to its part  $CE$  placed between the point  $C$  and the right line  $AK$ .

What has been said of Hyperbola's may be easily applied to Parabola's. For if (*Fig. 8.*) a Parabola be represented by  $XAGK$ , touched by a right line  $XV$  in the vertex  $X$ ; and the ordinates  $IA$ ,  $VG$  be as any powers  $XI^n$ ,  $XV^n$  of the abscissa's  $XI$ ,  $XV$ ; draw  $XT$ ,  $GT$ ,  $AH$ , whereof let  $XT$  be parallel to  $VG$ , and let  $GT$ ,  $AH$  touch the Parabola in  $G$  and  $A$ : and a body projected from any place  $A$ , in the direction of the right line  $AH$ , with a due velocity, will describe this Parabola, if the density of the medium in each

each of the places  $G$ , be reciprocally as the tangent  $GT$ . In that case the velocity in  $G$  will be the same as would cause a body, moving in a non-resisting space, to describe a Conic Parabola, having  $G$  for its vertex,  $VG$  produced downwards for its diameter, and

$\frac{2GT^2}{n-1 \times VG}$  for its latus rectum. And the resisting

force in  $G$  will be to the force of gravity, as  $GT$  to

$\frac{2n-2}{n-2}VG$ . Therefore if  $NAK$  represent an ho-

orizontal line, and, both the density of the medium at  $A$  and the velocity with which the body is projected, remaining the same, the angle  $NAH$  be any how alter'd; the lengths  $AH$ ,  $AI$ ,  $HX$  will remain; and thence will be given the vertex  $X$  of the Parabola, and the position of the right line  $XI$ , and by taking  $VG$  to  $IA$  as  $XV$  to  $XI$ , there will be given all the points  $G$  of the Parabola, thro' which the projectile will pass.

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## SECTION III.

*Of the Motions of Bodies which are resisted partly in the ratio of the Velocities, and partly in the duplicate of the same ratio.*

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## PROPOSITION XI. THEOREM VIII.

*If a body be resisted partly in the ratio, and partly in the duplicate ratio of its velocity, and moves in a similar medium by its innate force only; and the times be taken in arithmetical progression: then quantities reciprocally proportional to the velocities, increased by a certain given quantity, will be in geometrical progression. Pl. 3. Fig. 1.*

With the centre  $C$ , and the rectangular asymptotes  $CADd$  and  $CH$  describe an Hyperbola  $BEe$ , and let  $AB, DE, de$ , be parallel to the asymptote  $CH$ . In the asymptote  $CD$  let  $A, G$  be given points: And if the time be expounded by the hyperbolic area  $ABED$  uniformly increasing; I say that the velocity may be express'd by the length  $DF$ , whose reciprocal  $GD$  together with the given line  $CG$ , compose the length  $CD$  increasing in a geometrical progression.

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For





For let the areola  $DEed$  be the least given increment of the time, and  $Dd$  will be reciprocally as  $DE$ , and therefore directly as  $CD$ . Therefore the decrement of  $\frac{1}{GD}$ , which (by Lem. 2. Book 2.) is  $\frac{Dd}{GD^2}$

will be also as  $\frac{CD}{GD^2}$  or  $\frac{CG + GD}{GD^2}$ , that is, as

$\frac{1}{GD} + \frac{CG}{GD^2}$ . Therefore the time  $ABED$  uniformly increasing by the addition of the given particles

$EDde$ , it follows that  $\frac{1}{GD}$  decreases in the same ratio

with the velocity. For the decrement of the velocity is as the resistance, that is, (by the supposition) as the sum of two quantities, whereof one is as the velocity, and the other as the square of the velocity; and the decrement of  $\frac{1}{GD}$  is as the sum of the quantities  $\frac{1}{GD}$

and  $\frac{CG}{GD^2}$ , whereof the first is  $\frac{1}{GD}$  it self, and the

last  $\frac{CG}{GD^2}$  is as  $\frac{1}{GD^2}$ : therefore  $\frac{1}{GD}$  is as the velocity, the decrements of both being analogous. And if

the quantity  $GD$ , reciprocally proportional to  $\frac{1}{GD}$ , be

augmented by the given quantity  $CG$ ; the sum  $CD$ , the time  $ABED$  uniformly increasing, will increase in a geometrical progression. *Q. E. D.*

COR. 1. Therefore, if, having the points  $A$  and  $G$  given, the time be expounded by the hyperbolic area  $ABED$ , the velocity may be expounded by  $\frac{1}{GD}$  the reciprocal of  $GD$ .

COR. 2. And by taking  $GA$  to  $GD$  as the reciprocal of the velocity at the beginning, to the reciprocal of



of the velocity at the end of any time  $ABED$ , the point  $G$  will be found. And that point being found, the velocity may be found from any other time given.

### PROPOSITION XII. THEOREM IX.

*The same things being supposed, I say, that if the spaces described are taken in arithmetical progression, the velocities augmented by a certain given quantity will be in geometrical progression. Pl. 3. Fig. 2.*

In the asymptote  $CD$  let there be given the point  $R$ , and erecting the perpendicular  $RS$  meeting the Hyperbola in  $S$ , let the space described be expounded by the hyperbolic area  $RSED$ ; and the velocity will be as the length  $GD$ , which, together with the given line  $CG$ , composes a length  $CD$  decreasing in a geometrical progression, while the space  $RSED$  increases in an arithmetical progression.

For, because the increment  $EDde$  of the space is given, the lineola  $Dd$ , which is the decrement of  $GD$ , will be reciprocally as  $ED$ , and therefore directly as  $CD$ ; that is, as the sum of the same  $GD$  and the given length  $CG$ . But the decrement of the velocity, in a time reciprocally proportional thereto, in which the given particle of space  $DdeE$  is described, is as the resistance and the time conjunctly, that is, directly as the sum of two quantities, whereof one is as the velocity, the other as the square of the velocity, and inversely as the velocity; and therefore directly as the sum of two quantities, one of which is given, the other is as the velocity. Therefore the decrement both of the velocity and the line  $GD$ , is as a given quantity and a decreasing quantity conjunctly; and, because the decrements are analogous, the decreasing quantities will  
always

always be analogous; *viz.* the velocity, and the line  $GD$ . *Q.E.D.*

COR. 1. If the velocity be expounded by the length  $GD$ , the space described will be as the hyperbolic area  $DESR$ .

COR. 2. And if the point  $R$  be assumed any how, the point  $G$  will be found, by taking  $GR$  to  $GD$ , as the velocity at the beginning to the velocity after any space  $RSED$  is described. The point  $G$  being given, the space is given from the given velocity: and the contrary.

COR. 3. Whence since (by Prop. 11.) the velocity is given from the given time, and (by this Prop.) the space is given from the given velocity; the space will be given from the given time: and the contrary.

### PROPOSITION XIII. THEOREM X.

*Supposing that a body attracted downwards by an uniform gravity ascends or descends in a right line; and that the same is resisted, partly in the ratio of its velocity, and partly in the duplicate ratio thereof: I say that, if right lines parallel to the diameters of a Circle and an Hyperbola be drawn thro' the ends of the conjugate diameters, and the velocities be as some segments of those parallels drawn from a given point; the times will be as the sectors of the areas, cut off by right lines drawn from the centre to the ends of the segments; and the contrary. Pl. 3. Fig. 3.*

CASE 1. Suppose first that the body is ascending, and from the centre  $D$ , with any semidiameter  $DB$ , describe a quadrant  $BETF$  of a circle, and thro' the end

end  $B$  of the semidiameter  $DB$  draw the indefinite line  $BAP$ , parallel to the semidiameter  $DF$ . In that line let there be given the point  $A$ , and take the segment  $AP$  proportional to the velocity. And since one part of the resistance is as the velocity, and another part as the square of the velocity; let the whole resistance be as  $AP^2 + 2BAP$ . Join  $DA, DP$  cutting the circle in  $E$  and  $T$ , and let the gravity be expounded by  $DA^2$ , so that the gravity shall be to the resistance in  $P$ , as  $DA^2$  to  $AP^2 + 2BAP$ ; and the time of the whole ascent will be as the sector  $EDT$  of the circle.

For draw  $DVQ$ , cutting off the moment  $PQ$  of the velocity  $AP$ , and the moment  $DTV$  of the sector  $DET$  answering to a given moment of time; and that decrement  $PQ$  of the velocity will be as the sum of the forces of gravity  $DA^2$  and of resistance  $AP^2 + 2BAP$ , that is, (by 12 Prop. 2 Book Elem.) as  $DP^2$ . Then the area  $DPQ$ , which is proportional to  $PQ$ , is as  $DP^2$ , and the area  $DTV$ , which is to the area  $DPQ$  as  $DT^2$  to  $DP^2$ , is as the given quantity  $DT^2$ . Therefore the area  $EDT$  decreases uniformly according to the rate of the future time, by subduction of given particles  $DTV$ , and is therefore proportional to the time of the whole ascent. *Q. E. D.*

CASE 2. If the velocity in the ascent of the body be expounded by the length  $AP$  as before, and the resistance be made as  $AP^2 + 2BAP$ , and if the force of gravity be less than can be expressed by  $DA^2$ ; take  $BD$  (Fig. 4.) of such a length, that  $AB^2 - BD^2$  may be proportional to the gravity, and let  $DF$  be perpendicular and equal to  $DB$ , and thro' the vertex  $F$  describe the Hyperbola  $FTVE$ , whose conjugate semidiameters are  $DB$  and  $DF$ , and which cuts  $DA$  in  $E$ , and  $DP, DQ$  in  $T$  and  $V$ ; and the time of the whole ascent will be as the hyperbolic sector  $TDE$ .

For the decrement  $PQ$  of the velocity produced in a given particle of time, is as the sum of the resistance

$AP^2$

$AP^2 - 2BAP$  and of the gravity  $AB^2 - BD^2$ , that is, as  $BP^2 - BD^2$ . But the area  $DTV$  is to the area  $DPQ$  as  $DT^2$  to  $DP^2$ ; and therefore, if  $GT$  be drawn perpendicular to  $DF$ , as  $GT^2$  or  $GD^2 - DF^2$  to  $BD^2$ , and as  $GD^2$  to  $BP^2$ , and, by division, as  $DF^2$  to  $BP^2 - BD^2$ . Therefore since the area  $DPQ$  is as  $PQ$ , that is, as  $BP^2 - BD^2$ ; the area  $DTV$  will be as the given quantity  $DF^2$ . Therefore the area  $EDT$  decreases uniformly in each of the equal particles of time, by the subduction of so many given particles  $DTV$ , and therefore is proportional to the time. *Q.E.D.*

CASE 3. Let  $AP$  be the velocity in the descent of the body, and  $AP^2 - 2BAP$  the force of resistance, and  $BD^2 - AB^2$  the force of gravity, the angle  $DBA$  being a right one. And if with the centre  $D$ , and the principal vertex  $B$ , there be described a rectangular Hyperbola  $BETV$  (*Fig. 5.*) cutting  $DA$ ,  $DP$ , and  $DQ$  produced in  $E$ ,  $T$ , and  $V$ ; the sector  $DET$  of this Hyperbola will be as the whole time of descent.

For the increment  $PQ$  of the velocity, and the area  $DPQ$  proportional to it, is as the excess of the gravity above the resistance, that is, as  $BD^2 - AB^2 - 2BAP - AP^2$  or  $BD^2 - BP^2$ . And the area  $DTV$  is to the area  $DPQ$ , as  $DT^2$  to  $DP^2$ ; and therefore as  $GT^2$  or  $GD^2 - BD^2$  to  $BP^2$ , and as  $GD^2$  to  $BD^2$ , and, by division, as  $BD^2$  to  $BD^2 - BP^2$ . Therefore since the area  $DPQ$  is as  $BD^2 - BP^2$ , the area  $DTV$  will be as the given quantity  $BD^2$ . Therefore the area  $EDT$  increases uniformly in the several equal particles of time by the addition of as many given particles  $DTV$ , and therefore is proportional to the time of the descent. *Q.E.D.*

COR. If with the centre  $D$  and the semidiameter  $DA$  there be drawn thro' the vertex  $A$  an arc  $At$  similar to the arc  $ET$ , and similarly subtending the angle

VOL. II.

E

 $ADT$ :

*ADT*: the velocity *AP* will be to the velocity, which the body in the time *EDT*, in a non-resisting space, can lose in its ascent, or acquire in its descent, as the area of the triangle *DAP* to the area of the sector *DAt*; and therefore is given from the time given. For the velocity in a non-resisting medium, is proportional to the time, and therefore to this sector; in a resisting medium it is as the triangle; and in both mediums, where it is least, it approaches to the ratio of equality, as the sector and triangle do.

#### SCHOLIUM.

One may demonstrate also that case in the ascent of the body, where the force of gravity is less than can be express'd by  $DA^2$  or  $AB^2 - BD^2$ , and greater than can be express'd by  $AB^2 - DB^2$ , and must be express'd by  $AB^2$ . But I hasten to other things.

#### PROPOSITION XIV. THEOREM XI.

*The same things being supposed, I say, that the space described in the ascent or descent, is as the difference of the area by which the time is express'd, and of some other area which is augmented or diminished in an arithmetical progression; if the forces compounded of the resistance and the gravity be taken in a geometrical progression. Pl. 3. Fig. 5, 6, 7.*

Take *AC* (in the three last figures) proportional to the gravity, and *AK* to the resistance. But take them on the same side of the point *A*, if the body is descending, otherwise on the contrary. Erect *Ab*, which make to *DB* as  $DB^2$  to  $4BAC$ : and to the rectangular asymptotes *CK*, *CH*, describe the Hyperbola

bola  $bN$ , and erecting  $KN$  perpendicular to  $CK$ , the area  $AbNK$  will be augmented or diminished in an arithmetical progression, while the forces  $CK$  are taken in a geometrical progression. I say therefore that the distance of the body from its greatest altitude is as the excess of the area  $AbNK$  above the area  $DET$ .

For since  $AK$  is as the resistance, that is, as  $AP^2 + 2BAP$ ; assume any given quantity  $Z$ , and put  $AK$  equal to  $\frac{AP^2 + 2BAP}{Z}$ ; then (by Lem. 2. of this Book) the moment  $KL$  of  $AK$  will be equal to  $\frac{2APQ + 2BA \times PQ}{Z}$  or  $\frac{2BPQ}{Z}$ , and the moment  $KLON$  of the area  $AbNK$ , will be equal to  $\frac{2BPQ \times LO}{Z}$  or  $\frac{BPQ \times BD^3}{2Z \times CK \times AB}$ .

CASE 1. Now if the body ascends, and the gravity be as  $AB^2 + BD^2$ ,  $BET$ , (in Fig. 5.) being a circle, the line  $AC$ , which is proportional to the gravity, will be  $\frac{AB^2 + BD^2}{Z}$ , and  $DP^2$  or  $AP^2 + 2BAP + AB^2 + BD^2$  will be  $AK \times Z + AC \times Z$  or  $CK \times Z$ ; and therefore the area  $DTV$  will be to the area  $DPQ$  as  $DT^2$  or  $DB^2$  to  $CK \times Z$ .

CASE 2. If the body ascends, and the gravity be as  $AB^2 - BD^2$ , the line  $AC$  (in Fig. 6.) will be  $\frac{AB^2 - BD^2}{Z}$  and  $DT^2$  will be to  $DP^2$  as  $DF^2$  or  $DB^2$  to  $BP^2 - BD^2$  or  $AP^2 + 2BAP + AB^2 - BD^2$ , that is, to  $AK \times Z + AC \times Z$  or  $CK \times Z$ . And therefore the area  $DTV$  will be to the area  $DPQ$  as  $DB^2$  to  $CK \times Z$ .

CASE 3. And by the same reasoning, if the body descends, and therefore the gravity is as  $BD^2 - AB^2$ , and the line  $AC$  (in Fig. 7.) becomes equal to  $\frac{BD^2 - AB^2}{Z}$

$\frac{BD^2 - AB^2}{Z}$ ; the area  $DTV$  will be to the area  $DPQ$  as  $DB^2$  to  $CK \times Z$ : as above.

Since therefore these areas are always in this ratio; if for the area  $DTV$ , by which the moment of the time, always equal to itself, is expressed, there be put any determinate rectangle, as  $BD \times m$ , the area  $DPQ$ , that is,  $\frac{1}{2} BD \times PQ$ , will be to  $BD \times m$  as  $CK \times Z$  to  $BD^2$ . And thence  $PQ \times BD^3$  becomes equal to  $2BD \times m \times CK \times Z$ , and the moment  $KLON$  of the area  $AbNK$ , found before, becomes  $\frac{BP \times BD \times m}{AB}$ .

From the area  $DET$  subduct its moment  $DTV$  or  $BD \times m$ , and there will remain  $\frac{AP \times BD \times m}{AB}$ . There-

fore the difference of the moments, that is, the moment of the difference of the areas is equal to  $\frac{AP \times BD \times m}{AB}$ ;

and therefore (because of the given quantity  $\frac{BD \times m}{AB}$ )

as the velocity  $AP$ ; that is, as the moment of the space which the body describes in its ascent or descent. And therefore the difference of the areas, and that space, increasing or decreasing by proportional moments, and beginning together or vanishing together, are proportional. *Q. E. D.*

COR. If the length, which arises by applying the area  $DET$  to the line  $BD$ , be called  $M$ ; and another length  $V$  be taken in that ratio to the length  $M$ , which the line  $DA$  has to the line  $DE$ : the space which a body, in a resisting medium, describes in its whole ascent or descent, will be to the space, which a body, in a non-resisting medium, falling from rest can describe in the same time, as the difference of the afore-

said areas to  $\frac{BD \times V^2}{AB}$ : and therefore is given from

the

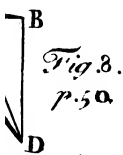
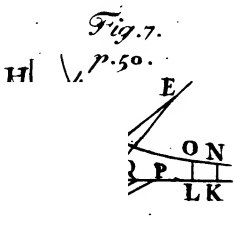
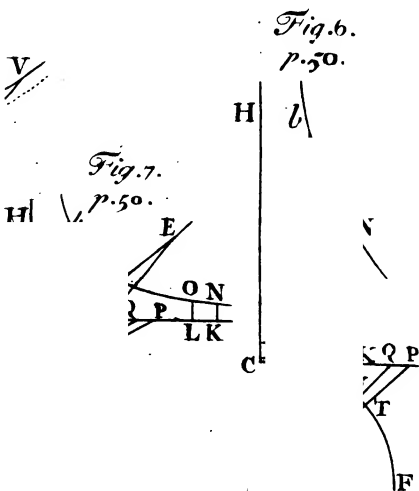
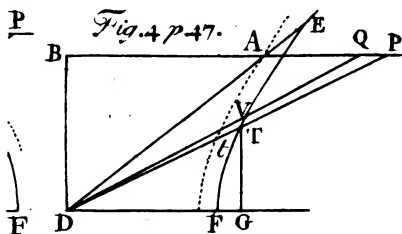
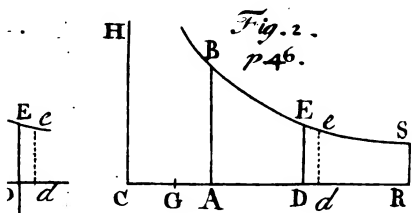
the time given. For the space in a non-resisting medium is in a duplicate ratio of the time, or as  $V^2$ ; and, because  $BD$  and  $AB$  are given, as  $\frac{BD \times V^2}{AB}$ . This area is equal to the area  $\frac{DA^2 \times BD \times M^2}{DE^2 \times AB}$ , and the moment of  $M$  is  $m$ ; and therefore the moment of this area is  $\frac{DA^2 \times BD \times 2M \times m}{DE^2 \times AB}$ . But this moment is to the moment of the difference of the aforesaid areas  $DET$  and  $AbNK$ , viz. to  $\frac{AP \times BD \times m}{AB}$ , as  $\frac{DA^2 \times BD \times M}{DE^2}$  to  $\frac{1}{2}BD \times AP$ , or as  $\frac{DA^2}{DE^2}$  into  $DET$  to  $DAP$ ; and therefore, when the areas  $DET$  and  $DAP$  are least, in the ratio of equality. Therefore the area  $\frac{BD \times V^2}{AB}$  and the difference of the areas  $DET$  and  $AbNK$ , when all these areas are least, have equal moments; and are therefore equal. Therefore since the velocities, and therefore also the spaces in both mediums described together, in the beginning of the descent, or the end of the ascent, approach to equality, and therefore are then one to another as the area  $\frac{BD \times V^2}{AB}$ , and the difference of the areas  $DET$  and  $AbNK$ ; and moreover since the space, in a non-resisting medium, is perpetually as  $\frac{BD \times V^2}{AB}$ , and the space, in a resisting medium, is perpetually as the difference of the areas  $DET$  and  $AbNK$ : it necessarily follows, that the spaces, in both mediums, described in any equal times, are one to another as that area  $\frac{BD \times V^2}{AB}$ , and the difference of the areas  $DET$  and  $AbNK$ . Q. E. D.



## SCHOLIUM.

The resistance of spherical bodies in fluids arises partly from the tenacity, partly from the attrition, and partly from the density of the medium. And that part of the resistance, which arises from the density of the fluid, is, as I said, in a duplicate ratio of the velocity, the other part, which arises from the tenacity of the fluid, is uniform, or as the moment of the time: and therefore we might now proceed to the motion of bodies, which are resisted partly by an uniform force, or in the ratio of the moments of the time, and partly in the duplicate ratio of the velocity. But it is sufficient to have cleared the way to this speculation in the 8<sup>th</sup> and 9<sup>th</sup> Prop. foregoing, and their Corollaries. For in those Propositions, instead of the uniform resistance made to an ascending body arising from its gravity, one may substitute the uniform resistance which arises from the tenacity of the medium, when the body moves by its *vis insita* alone; and when the body ascends in a right line, add this uniform resistance to the force of gravity, and subduct it when the body descends in a right line. One might also go on to the motion of bodies which are resisted in part uniformly, in part in the ratio of the velocity, and in part in the duplicate ratio of the same velocity. And I have opened a way to this in the 13<sup>th</sup> and 14<sup>th</sup> Prop. foregoing, in which the uniform resistance arising from the tenacity of the medium, may be substituted for the force of gravity, or be compounded with it as before. But I hasten to other things.

S E C.







## SECTION IV.

*Of the circular motion of bodies in  
resisting mediums.*

## LEMMA III.

Let  $PQR$  be a spiral cutting all the radii  $SP$ ,  $SQ$ ,  $SR$ , &c. in equal angles. Draw the right line  $PT$  touching the spiral in any point  $P$ , and cutting the radius  $SQ$  in  $T$ ; draw  $PO$ ,  $QO$  perpendicular to the spiral, and meeting in  $O$ , and join  $SO$ . I say, that if the points  $P$  and  $Q$  approach and coincide, the angle  $PSO$  will become a right angle, and the ultimate ratio of the rectangle  $TQ \times 2PS$  to  $PQ^2$  will be the ratio of equality. Pl. 4. Fig. 1.

For from the right angles  $OPQ$ ,  $OQR$ , subduct the equal angles  $SPQ$ ,  $SQR$ , and there will remain the equal angles  $OPS$ ,  $OQS$ . Therefore a circle which passes thro' the points  $O$ ,  $S$ ,  $P$ , will pass also thro' the point  $Q$ . Let the points  $P$  and  $Q$  coincide, and this circle will touch the spiral in the place of coincidence  $PQ$ , and will therefore cut the right line  $OP$  perpendicularly. Therefore  $OP$  will become a diameter of this circle, and the angle  $OSP$ ; being in a semicircle, becomes a right one. *Q. E. D.*

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Draw  $QD$ ,  $SE$  perpendicular to  $OP$ , and the ultimate ratio's of the lines will be as follows;  $TQ$  to  $PD$  as  $TS$  or  $PS$  to  $PE$ , or  $2PO$  to  $2PS$ ; and  $PD$  to  $PQ$  as  $PQ$  to  $2PO$ ; and, *ex æquo perturbatè*,  $TQ$  to  $PQ$  as  $PQ$  to  $2PS$ . Whence  $PQ^2$  becomes equal to  $TQ \times 2PS$ . *Q. E. D.*

### PROPOSITION XV. THEOREM XII.

*If the density of a medium in each place thereof be reciprocally as the distance of the places from an immoveable centre, and the centripetal force be in the duplicate ratio of the density: I say, that a body may revolve in a spiral which cuts all the radii drawn from that centre in a given angle. Pl. 4. Fig. 2.*

Suppose every thing to be as in the foregoing Lemma, and produce  $SQ$  to  $V$ , so that  $SV$  may be equal to  $SP$ . In any time let a body, in a resisting medium, describe the least arc  $PQ$ , and in double the time, the least arc  $PR$ ; and the decrements of those arcs arising from the resistance, or their differences from the arcs which would be described in a non-resisting medium in the same times, will be to each other, as the squares of the times in which they are generated: Therefore the decrement of the arc  $PQ$  is the fourth part of the decrement of the arc  $PR$ . Whence also if the area  $QSV$  be taken equal to the area  $PSQ$ , the decrement of the arc  $PQ$  will be equal to half the lineola  $Rr$ ; and therefore the force of resistance and the centripetal force are to each other as the lineola's  $\frac{1}{2}Rr$  and  $TQ$  which they generate in the same time. Because the centripetal force with which the body is urged in  $P$ , is reciprocally as  $SP^2$ , and (by Lem. 10. Book 1.) the lineola  $TQ$ , which is generated by that force, is in a ra-

tio compounded of the ratio of this force and the duplicate ratio of the time in which the arc  $PQ$  is described, (for in this case I neglect the resistance, as being infinitely less than the centripetal force,) it follows, that  $TQ \times SP^2$ , that is, (by the last Lemma)  $\frac{1}{2}PQ^2 \times SP$ , will be in a duplicate ratio of the time, and therefore the time is as  $PQ \times \sqrt{SP}$ ; and the velocity of the body, with which the arc  $PQ$  is described in

that time, as  $\frac{PQ}{PQ \times \sqrt{SP}}$  or  $\frac{1}{\sqrt{SP}}$ , that is, in the subduplicate ratio of  $SP$  reciprocally. And by a like reasoning, the velocity with which the arc  $QR$  is described, is in the subduplicate ratio of  $SQ$  reciprocally. Now those arcs  $PQ$  and  $QR$  are as the describing velocities to each other; that is, in the subduplicate ratio of  $SQ$  to  $SP$ , or as  $SQ$  to  $\sqrt{SP \times SQ}$ ; and, because of the equal angles  $SPQ$ ,  $SQR$ , and the equal areas  $PSQ$ ,  $QSR$ , the arc  $PQ$  is to the arc  $QR$  as  $SQ$  to  $SP$ . Take the differences of the proportional consequents, and the arc  $PQ$  will be to the arc  $Rr$  as  $SQ$  to  $SP - \sqrt{SP \times SQ}$ , or  $\frac{1}{2}VQ$ . For the points  $P$  and  $Q$  coinciding, the ultimate ratio of  $SP - \sqrt{SP \times SQ}$  to  $\frac{1}{2}VQ$  is the ratio of equality. Because the decrement of the arc  $PQ$  arising from the resistance, or its double  $Rr$ , is as the resistance and the square of the

time conjunctly; the resistance will be as  $\frac{Rr}{PQ^2 \times SP}$ .

But  $PQ$  was to  $Rr$ , as  $SQ$  to  $\frac{1}{2}VQ$ , and thence

$\frac{Rr}{PQ^2 \times SP}$  becomes as  $\frac{\frac{1}{2}VQ}{PQ \times SP \times SQ}$  or as  $\frac{\frac{1}{2}OS}{OP \times SP^2}$ .

For the points  $P$  and  $Q$  coinciding,  $SP$  and  $SQ$  coincide also, and the angle  $PVQ$  becomes a right one; and, because of the similar triangles  $PVQ$ ,  $PSO$ ,  $PQ$  becomes to  $\frac{1}{2}VQ$  as  $OP$  to  $\frac{1}{2}OS$ . Therefore

$\frac{OS}{OP \times SP^2}$  is as the resistance, that is, in the ratio of the

the density of the medium in  $P$  and the duplicate ratio of the velocity conjunctly. Subduct the duplicate ratio of the velocity, namely the ratio  $\frac{1}{SP}$ , and there

will remain the density of the medium in  $P$  as  $\frac{OS}{OP \times SP}$ .

Let the spiral be given, and, because of the given ratio of  $OS$  to  $OP$ , the density of the medium in  $P$  will be as  $\frac{1}{SP}$ . Therefore in a medium whose density is

reciprocally as  $SP$  the distance from the centre, a body will revolve in this spiral. *Q. E. D.*

COR. 1. The velocity in any place  $P$ , is always the same wherewith a body in a non-resisting medium with the same centripetal force would revolve in a circle, at the same distance  $SP$  from the centre.

COR. 2. The density of the medium, if the distance  $SP$  be given, is as  $\frac{OS}{OP}$ , but if that distance is not

given, as  $\frac{OS}{OP \times SP}$ . And thence a spiral may be fitted to any density of the medium.

COR. 3. The force of the resistance in any place  $P$ , is to the centripetal force in the same place as  $\frac{1}{2} OS$  to  $OP$ . For those forces are to each other as  $\frac{1}{2} Rr$  and  $TQ$  or as  $\frac{\frac{1}{2} VQ \times PQ}{SQ}$  and  $\frac{\frac{1}{2} PQ^2}{SP}$ , that is, as  $\frac{1}{2} VQ$  and  $PQ$ , or  $\frac{1}{2} OS$  and  $OP$ . The spiral therefore being given, there is given the proportion of the resistance to the centripetal force; and *vice versa*, from that proportion given the spiral is given.

COR. 4. Therefore the body can't revolve in this spiral, except where the force of resistance is less than half the centripetal force. Let the resistance be made equal to half the centripetal force, and the spiral will coincide with the right line  $PS$ , and in that right line the

the body will descend to the centre with a velocity, that is to the velocity, with which it was proved before in the case of the Parabola, (Theor. 10. Book I.) the descent would be made in a non-resisting medium, in the subduplicate ratio of unity to the number two. And the times of the descent will be here reciprocally as the velocities, and therefore given.

COR. 5. And because at equal distances from the centre, the velocity is the same in the spiral  $PQR$  as it is in the right line  $SP$ , and the length of the spiral is to the length of the right line  $PS$ , in a given ratio, namely in the ratio of  $OP$  to  $OS$ ; the time of the descent in the spiral will be to the time of the descent in the right line  $SP$  in the same given ratio, and therefore given.

COR. 6. If from the centre  $S$  with any two given intervals, two circles are described; and these circles remaining, the angle which the spiral makes with the radius  $PS$  be any how changed; the number of revolutions which the body can complete in the space between the circumferences of those circles, going round in the spiral from one circumference to another, will be as  $\frac{PS}{OS}$ , or as the tangent of the angle which the spiral makes with the radius  $PS$ ; and the time of the same revolutions will be as  $\frac{OP}{OS}$ , that is, as the secant of the same angle, or reciprocally as the density of the medium.

COR. 7. If a body, in a medium whose density is reciprocally as the distances of places from the centre, revolves in any curve  $AEB$  (Fig. 3.) about that centre, and cuts the first radius  $AS$  in the same angle in  $B$  as it did before in  $A$ , and that with a velocity, that shall be to its first velocity in  $A$  reciprocally in a subduplicate ratio of the distances from the centre (that is, as  $AS$  to a mean proportional between  $AS$  and  $BS$ ) that



that body will continue to describe innumerable similar revolutions  $BFC$ ,  $CGD$ , &c. and by its intersections will distinguish the radius  $AS$  into parts  $AS$ ,  $BS$ ,  $CS$ ,  $DS$ , &c. that are continually proportional. But the times of the revolutions will be as the perimeters of the orbits  $AEB$ ,  $BFC$ ,  $CGD$ , &c. directly, and the velocities at the beginnings  $A, B, C$  of those orbits, inversely; that is, as  $AS^{\frac{1}{2}}$ ,  $BS^{\frac{1}{2}}$ ,  $CS^{\frac{1}{2}}$ . And the whole time in which the body will arrive at the centre, will be to the time of the first revolution, as the sum of all the continued proportionals  $AS^{\frac{1}{2}}$ ,  $BS^{\frac{1}{2}}$ ,  $CS^{\frac{1}{2}}$ , going on *ad infinitum*, to the first term  $AS^{\frac{1}{2}}$ ; that is, as the first term  $AS^{\frac{1}{2}}$  to the difference of the two first  $AS^{\frac{1}{2}} - BS^{\frac{1}{2}}$ , or as  $\frac{2}{3} AS$  to  $AB$  very nearly. Whence the whole time may be easily found.

COR. 8. From hence also may be deduced, near enough, the motions of bodies in mediums whose density is either uniform or observes any other assigned law. From the centre  $S$ , with intervals  $SA$ ,  $SB$ ,  $SC$ , &c. continually proportional, describe as many circles; and suppose the time of the revolutions between the perimeters of any two of those circles, in the medium whereof we treated, to be to the time of the revolutions between the same in the medium proposed, as the mean density of the proposed medium between those circles, to the mean density of the medium whereof we treated, between the same circles, nearly: And that the secant of the angle in which the spiral above determined, in the medium whereof we treated, cuts the radius  $AS$ , is in the same ratio to the secant of the angle in which the new spiral, in the proposed medium, cuts the same radius: And also that the number of all the revolutions between the same two circles is nearly as the tangents of those angles. If this be done every where between every two circles, the motion will

will be continued thro' all the circles. And by this means one may without difficulty conceive at what rate and in what time bodies ought to revolve in any regular medium.

COR. 9. And altho these motions becoming excentric should be performed in spirals approaching to an oval figure; yet conceiving the several revolutions of those spirals to be at the same distances from each other, and to approach to the centre by the same degrees as the spiral above described, we may also understand how the motions of bodies may be performed in spirals of that kind.

# PROPOSITION XVI. THEOREM XIII.

*If the density of the medium in each of the places be reciprocally as the distance of the places from the immoveable centre, and the centripetal force be reciprocally as any power of the same distance, I say, that the body may revolve in a spiral intersecting all the radii drawn from that centre in a given angle.*

Pl. 4. Fig. 2.

This is demonstrated in the same manner as the foregoing proposition. For if the centripetal force in  $P$  be reciprocally as any power  $SP^{n+1}$  of the distance  $SP$  whose index is  $n+1$ : it will be collected as above, that the time in which the body describes any arc  $PQ$ , will be as  $PQ \times PS^{\frac{1}{2}n}$ ; and the resistance in  $P$  as

$$\frac{Rr}{PQ^2 \times SP^n}, \text{ or as } \frac{1 - \frac{1}{2}n \times VQ}{PQ \times SP^n \times SQ}, \text{ and therefore}$$

$$\text{as } \frac{1 - \frac{1}{2}n \times OS}{OP \times SP^{n+1}}, \text{ that is, (because } \frac{1 - \frac{1}{2}n \times OS}{OP} \text{ is a}$$

given

given quantity) reciprocally as  $SP^{n+1}$ . And therefore, since the velocity is reciprocally as  $SP^{\frac{1}{2}n}$ , the density in  $P$  will be reciprocally as  $SP$ .

COR. 1. The resistance is to the centripetal force as  $1 - \frac{1}{2}n \times OS$  to  $OP$ .

COR. 2. If the centripetal force be reciprocally as  $SP^3$ ,  $1 - \frac{1}{2}n$  will be  $= 0$ ; and therefore the resistance and density of the medium will be nothing, as in Prop. 9. Book 1.

COR. 3. If the centripetal force be reciprocally as any power of the radius  $SP$ , whose index is greater than the number 3, the affirmative resistance will be changed into a negative.

#### SCHOLIUM.

This Proposition and the former which relate to mediums of unequal density, are to be understood of the motion of bodies that are so small, that the greater density of the medium on one side of the body, above that on the other, is not to be consider'd. I suppose also the resistance, *ceteris paribus*, to be proportional to its density. Whence in mediums whose force of resistance is not as the density, the density must be so much augmented or diminished, that either the excess of the resistance may be taken away, or the defect supplied.

#### PROPOSITION XVII. PROBLEM IV.

*To find the centripetal force and the resisting force of the medium, by which a body, the law of the velocity being given, shall revolve in a given spiral. Pl. 4. Fig. 4.*

Let that spiral be  $PQR$ . From the velocity, with which the body goes over the very small arc  $PQ$ , the  
time

time will be given ; and from the altitude  $TQ$ , which is as the centripetal force, and the square of the time, that force will be given. Then from the difference  $RSr$ , of the areas  $PSQ$  and  $QSR$  described in equal particles of time, the retardation of the body will be given ; and from the retardation will be found the resisting force and density of the medium.

### PROPOSITION XVIII. PROBLEM V.

*The law of centripetal force being given, to find the density of the medium in each of the places thereof, by which a body may describe a given spiral.*

From the centripetal force the velocity in each place must be found ; then from the retardation of the velocity, the density of the medium is found, as in the foregoing Proposition.

But I have explain'd the method of managing these Problems in the tenth Proposition and second Lemma of this Book ; and will no longer detain the reader in these perplex'd disquisitions. I shall now add some things relating to the forces of progressive bodies, and to the density and resistance of those mediums in which the motions hitherto treated of, and those akin to them, are performed.



S E C-



## SECTION V.

*Of the density and compression of fluids; and of Hydrostatics.*

The Definition of a Fluid.

*A fluid is any body whose parts yield to any force impressed on it, and, by yielding, are easily moved among themselves.*

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## PROPOSITION XIX. THEOREM XIV.

*All the parts of a homogeneous and unmoved fluid included in any unmoved vessel, and compressed on every side, (setting aside the consideration of condensation, gravity, and all centripetal forces) will be equally pressed on every side, and remain in their places without any motion arising from that pressure.*  
Pl. 4. Fig. 5.

CASE I. Let a fluid be included in the sphærical vessel *ABC* and uniformly compressed on every side: I say, that no part of it will be moved by that pressure. For if any part, as *D*, be moved, all such parts at the same distance from the centre on every side, must necessarily be moved at the same time by a like motion; because the pressure of them all is similar and equal; and all other motion is excluded that does not arise from that

that pressure. But if these parts come all of them nearer to the centre, the fluid must be condensed towards the centre, contrary to the supposition. If they recede from it, the fluid must be condensed towards the circumference; which is also contrary to the supposition. Neither can they move in any one direction retaining their distance from the centre, because for the same reason they may move in a contrary direction; but the same part cannot be moved contrary ways at the same time. Therefore no part of the fluid will be moved from its place. *Q. E. D.*

CASE 2. I say now, that all the spherical parts of this fluid are equally pressed on every side. For let *EF* be a spherical part of the fluid; if this be not pressed equally on every side, augment the lesser pressure till it be pressed equally on every side; and its parts (by Case 1.) will remain in their places. But before the increase of the pressure, they would remain in their places, (by Case 1.) and by the addition of a new pressure, they will be moved, by the definition of a fluid, from those places. Now these two conclusions contradict each other. Therefore it was false to say, that the sphere *EF* was not pressed equally on every side. *Q. E. D.*

CASE 3. I say besides, that different spherical parts have equal pressures. For the contiguous spherical parts press each other mutually and equally in the point of contact, (by Law 3.) But (by Case 2.) they are pressed on every side with the same force. Therefore any two spherical parts not contiguous, since an intermediate spherical part can touch both, will be pressed with the same force. *Q. E. D.*

CASE 4. I say now, that all the parts of the fluid are every where pressed equally. For any two parts may be touched by spherical parts in any points whatever; and there they will equally press those spherical

parts, (by Case 3.) and are, reciprocally, equally pressed by them, (by Law 3.) *Q. E. D.*

CASE 5. Since therefore any part  $GHI$  of the fluid is inclosed by the rest of the fluid as in a vessel, and is equally pressed on every side; and also its parts equally press one another, and are at rest among themselves; it is manifest that all the parts of any fluid as  $GHI$ , which is pressed equally on every side, do press each other mutually and equally, and are at rest among themselves. *Q. E. D.*

CASE 6. Therefore if that fluid be included in a vessel of a yielding substance, or that is not rigid, and be not equally pressed on every side; the same will give way to a stronger pressure, by the definition of fluidity.

CASE 7. And therefore in an inflexible or rigid vessel, a fluid will not sustain a stronger pressure on one side than on the other, but will give way to it, and that in a moment of time; because the rigid side of the vessel does not follow the yielding liquor. But the fluid, by thus yielding, will press against the opposite side, and so the pressure will tend on every side to equality. And because the fluid, as soon as it endeavours to recede from the part that is most pressed, is withstood by the resistance of the vessel on the opposite side; the pressure will on every side be reduced to equality, in a moment of time, without any local motion: and from thence the parts of the fluid, (by Case 5.) will press each other mutually and equally, and be at rest among themselves. *Q. E. D.*

COR. Whence neither will a motion of the parts of the fluid among themselves, be changed by a pressure communicated to the external superficies, except so far as either the figure of the superficies may be somewhere alter'd, or that all the parts of the fluid, by pressing one another more intensely or remissly, may slide with more or less difficulty among themselves.

PRO-

## PROPOSITION XX. THEOREM XV.

*If all the parts of a spherical fluid, homogeneous at equal distances from the centre, lying on a spherical concentric bottom, gravitate towards the centre of the whole; the bottom will sustain the weight of a cylinder, whose base is equal to the superficies of the bottom, and whose altitude is the same with that of the incumbent fluid. Pl. 4. Fig. 6.*

Let  $DHM$  be the superficies of the bottom, and  $AEI$  the upper superficies of the fluid. Let the fluid be distinguished into concentric orbs of equal thickness, by the innumerable spherical superficies  $BFK$ ,  $CGL$ ; and conceive the force of gravity to act only in the upper superficies of every orb, and the actions to be equal on the equal parts of all the superficies. Therefore the upper superficies  $AE$  is pressed by the single force of its own gravity, by which all the parts of the upper orb, and the second superficies  $BFK$  will, (by Prop. 19.) according to its measure, be equally pressed. The second superficies  $BFK$  is pressed likewise by the force of its own gravity, which added to the former force, makes the pressure double. The third superficies  $CGL$  is, according to its measure, acted on by this pressure and the force of its own gravity besides, which makes its pressure triple. And in like manner the fourth superficies receives a quadruple pressure, the fifth superficies a quintuple, and so on. Therefore the pressure acting on every superficies, is not as the solid quantity of the incumbent fluid, but as the number of the orbs reaching to the upper surface of the fluid; and is equal to the gravity of the lowest orb multiplied by the number of orbs: that is, to the gravity of a solid

F 2

whose



whose ultimate ratio to the cylinder abovementioned (when the number of the orbs is increased and their thickness diminished *ad infinitum*, so that the action of gravity from the lowest superficies to the uppermost may become continued) is the ratio of equality. Therefore the lowest superficies sustains the weight of the cylinder above-determined. *Q.E.D.* And by a like reasoning the Proposition will be evident, where the gravity of the fluid decreases in any assigned ratio of the distance from the centre, and also where the fluid is more rare above and denser below. *Q.E.D.*

COR. 1. Therefore the bottom is not pressed by the whole weight of the incumbent fluid, but only sustains that part of it which is described in the Proposition; the rest of the weight being sustained archwise by the spherical figure of the fluid.

COR. 2. The quantity of the pressure is the same always at equal distances from the centre, whether the superficies pressed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid, continued upwards from the compressed superficies, rises perpendicularly in a rectilinear direction, or creeps obliquely thro' crooked cavities and canals, whether those passages be regular or irregular, wide or narrow. That the pressure is not alter'd by any of these circumstances, may be collected by applying the demonstration of this Theorem to the several cases of fluids.

COR. 3. From the same demonstration it may also be collected, (by Prop. 19.) that the parts of an heavy fluid acquire no motion among themselves, by the pressure of the incumbent weight; except that motion which arises from condensation.

COR. 4. And therefore if another body of the same specific gravity, incapable of condensation, be immersed in this fluid, it will acquire no motion by the pressure of the incumbent weight: it will neither descend, nor ascend, nor change its figure. If it be  
spherical,

Fig. 1.

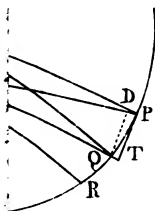


Fig. 2.  
p. 56. 61.

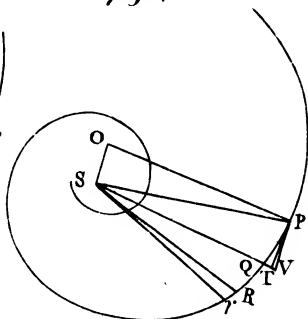


Fig. 3.

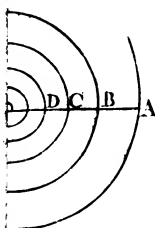


Fig. 4.  
p. 62.

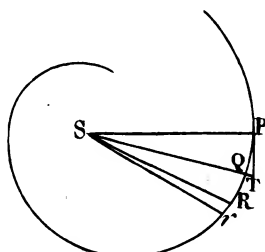


Fig. 5.

p. 64.

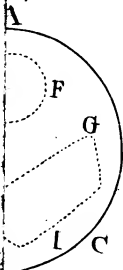
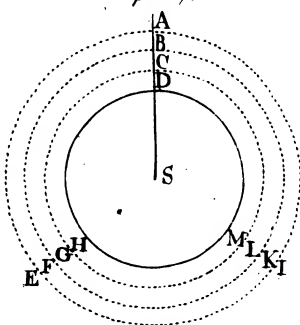


Fig. 6.  
p. 67.





spherical, it will remain so notwithstanding the pressure; if it be square, it will remain square: and that whether it be soft, or fluid; whether it swims freely in the fluid, or lies at the bottom. For any internal part of a fluid is in the same state with the submersed body; and the case of all submersed bodies that have the same magnitude, figure, and specific gravity, is alike. If a submersed body retaining its weight, should dissolve and put on the form of a fluid, this body, if before it would have ascended, descended, or from any pressure assume a new figure, would now likewise ascend, descend, or put on a new figure; and that because its gravity and the other causes of its motion remain. But (by Case 5. Prop. 19.) it would now be at rest and retain its figure. Therefore also in the former case.

COR. 5. Therefore a body that is specifically heavier than a fluid contiguous to it, will sink, and that which is specifically lighter will ascend, and attain so much motion and change of figure, as that excess or defect of gravity is able to produce. For that excess or defect is the same thing as an impulse, by which a body, otherwise *in equilibrio* with the parts of the fluid, is acted on; and may be compared with the excess or defect of a weight in one of the scales of a balance.

COR. 6. Therefore bodies placed in fluids have a twofold gravity; the one true and absolute, the other apparent, vulgar and comparative. Absolute gravity is the whole force with which the body tends downwards: relative and vulgar gravity is the excess of gravity with which the body tends downwards more than the ambient fluid. By the first kind of gravity, the parts of all fluids and bodies gravitate in their proper places; and therefore their weights taken together, compose the weight of the whole. For the whole taken together is heavy, as may be experienced in vessels full of liquor; and the weight of the whole is equal to the weights of all the parts, and is therefore composed of

them. By the other kind of gravity bodies do not gravitate in their places, that is, compared with one another, they do not preponderate, but hindering one another's endeavours to descend, remain in their proper places, as if they were not heavy. Those things which are in the air and do not preponderate, are commonly looked on as not heavy. Those which do preponderate are commonly reckoned heavy, in as much as they are not sustained by the weight of the air. The common weights are nothing else but the excess of the true weights above the weight of the air. Hence also vulgarly those things are called light, which are less heavy; and by yielding to the preponderating air, mount upwards. But these are only comparatively light, and not truly so, because they descend *in vacuo*. Thus in water, bodies which, by their greater or less gravity, descend or ascend, are comparatively and apparently heavy or light, and their comparative and apparent gravity or levity is the excess or defect by which their true gravity either exceeds the gravity of the water or is exceeded by it. But those things which neither by preponderating descend, nor, by yielding to the preponderating fluid, ascend, altho' by their true weight they do increase the weight of the whole, yet comparatively, and in the sense of the vulgar, they do not gravitate in the water. For these cases are alike demonstrated.

COR. 7. These things which have been demonstrated concerning gravity, take place in any other centripetal forces.

COR. 8. Therefore if the medium in which any body moves be acted on either by its own gravity, or by any other centripetal force, and the body be urged more powerfully by the same force; the difference of the forces is that very motive force, which in the foregoing Propositions I have consider'd as a centripetal force. But if the body be more lightly urg'd by that

that force, the difference of the forces becomes a centrifugal force, and is to be consider'd as such.

COR. 9. But since fluids by pressing the included bodies do not change their external figures, it appears also, (by Cor. Prop. 19.) that they will not change the situation of their internal parts in relation to one another; and therefore if animals were immersed therein, and that all sensation did arise from the motion of their parts; the fluid will neither hurt the immersed bodies, nor excite any sensation, unless so far as those bodies may be condensed by the compression. And the case is the same of any system of bodies encompassed with a compressing fluid. All the parts of the system will be agitated with the same motions, as if they were placed in a vacuum, and would only retain their comparative gravity; unless so far as the fluid may somewhat resist their motions, or be requisite to conglutinate them by compression.

# PROPOSITION XXI. THEOREM XVI.

*Let the density of any fluid be proportional to the compression, and its parts be attracted downwards by a centripetal force reciprocally proportional to the distances from the centre: I say, that, if those distances be taken continually proportional, the densities of the fluid at the same distances will be also continually proportional.* Pl. 5. Fig. 1.

Let  $ATV$  denote the sphaerical bottom of the fluid,  $S$  the centre,  $SA, SB, SC, SD, SE, SF, \&c.$  distances continually proportional. Erect the perpendiculars  $AH, BI, CK, DL, EM, FN, \&c.$  which shall be as the densities of the medium in the places  $A, B, C, D, E, F$ ; and the specific gravities in those places will be

F 4

as

as  $\frac{AH}{AS}, \frac{BI}{BS}, \frac{CK}{CS},$  &c. or, which is all one, as

$\frac{AH}{AB}, \frac{BI}{BC}, \frac{CK}{CD},$  &c. Suppose first these gravities to

be uniformly continued from  $A$  to  $B$ , from  $B$  to  $C$ , from  $C$  to  $D$ , &c. the decrements in the points  $B, C, D$ , &c. being taken by steps. And these gravities drawn into the altitudes  $AB, BC, CD$ , &c. will give the pressures  $AH, BI, CK$ , &c. by which the bottom  $ATV$  is acted on, (by Theor. 15.) Therefore the particle  $A$  sustains all the pressures  $AH, BI, CK, DL$ , &c. proceeding in *infinitum*; and the particle  $B$  sustains the pressures of all but the first  $AH$ ; and the particle  $C$  all but the two first  $AH, BI$ ; and so on: and therefore the density  $AH$  of the first particle  $A$  is to the density  $BI$  of the second particle  $B$  as the sum of all  $AH - BI - CK - DL$ , in *infinitum*, to the sum of all  $BI - CK - DL$ , &c. And  $BI$  the density of the second particle  $B$  is to  $CK$  the density of the third  $C$ , as the sum of all  $BI - CK - DL$ , &c. to the sum of all  $CK - DL$ , &c. Therefore these sums are proportional to their differences  $AH, BI, CK$ , &c. and therefore continually proportional, (by Lem. 1. of this Book) and therefore the differences  $AH, BI, CK$ , &c. proportional to the sums, are also continually proportional. Wherefore since the densities in the places  $A, B, C$ , &c. are as  $AH, BI, CK$ , &c. they will also be continually proportional. Proceed intermissively, and, *ex æquo*, at the distances  $SA, SC, SE$  continually proportional, the densities  $AH, CK, EM$  will be continually proportional. And by the same reasoning, at any distances  $SA, SD, SG$  continually proportional, the densities  $AH, DL, GO$  will be continually proportional. Let now the points  $A, B, C, D, E$ , &c. coincide, so that the progression of the specific gravities from the bottom  $A$  to the top of the fluid may be made continual; and at any distances  $SA, SD,$   
 $SG$

$SG$  continually proportional, the densities  $AH, DL, GQ$  being all along continually proportional, will still remain continually proportional. *Q. E. D.*

COR Hence if the density of the fluid in two places as  $A$  and  $E$  be given, its density in any other place  $Q$  may be collected. With the centre  $S$ , and the rectangular asymptotes  $SQ, SX$  describe (*Fig. 2.*) an Hyperbola cutting the perpendiculars  $AH, EM, QT$  in  $a, e$ , and  $q$ , as also the perpendiculars  $HX, MY, TZ$  let fall upon the asymptote  $SX$  in  $b, m$ , and  $t$ . Make the area  $TmtZ$  to the given area  $YmbX$  as the given area  $EeqQ$  to the given area  $EeaA$ ; and the line  $Zt$  produced will cut off the line  $QT$  proportional to the density. For if the lines  $SA, SE, SQ$  are continually proportional, the areas  $EeqQ, EeaA$  will be equal, and thence the areas  $TmtZ, XbmY$  proportional to them will be also equal, and the lines  $SX, SY, SZ$ , that is,  $AH, EM, QT$  continually proportional as they ought to be. And if the lines  $SA, SE, SQ$  obtain any other order in the series of continued proportionals, the lines  $AH, EM, QT$ , because of the proportional hyperbolic areas, will obtain the same order in another series of quantities continually proportional.

## PROPOSITION XXII. THEOREM XVII.

*Let the density of any fluid be proportional to the compression, and its parts be attracted downwards by a gravitation reciprocally proportional to the squares of the distances from the centre: I say, that, if the distances be taken in harmonic progression, the densities of the fluid at those distances will be in a geometrical progression.* Pl. 5. Fig. 3.

Let  $S$  denote the centre, and  $SA, SB, SC, SD, SE$ , the distances in Geometrical progression. Erect the  
the



the perpendiculars  $AH, BI, CK$ , &c. which shall be as the densities of the fluid in the places  $A, B, C, D, E$ , &c. and the specific gravities thereof in those places will be as  $\frac{AH}{SA^2}, \frac{BI}{SB^2}, \frac{CK}{SC^2}$ , &c. Suppose these gravities to be uniformly continued, the first from  $A$  to  $B$ , the second from  $B$  to  $C$ , the third from  $C$  to  $D$ , &c. And these drawn into the altitudes  $AB, BC, CD, DE$ , &c. or, which is the same thing, into the distances  $SA, SB, SC$ , &c. proportional to those altitudes, will give  $\frac{AH}{SA}, \frac{BI}{SB}, \frac{CK}{SC}$ , &c. the exponents of the pressures. Therefore since the densities are as the sums of those pressures, the differences  $AH - BI, BI - CK$ , &c. of the densities will be as the differences of those sums  $\frac{AH}{SA}, \frac{BI}{SB}, \frac{CK}{SC}$ , &c. With the centre  $S$ , and the asymptotes  $SA, Sx$ , describe any Hyperbola, cutting the perpendiculars  $AH, BI, CK$ , &c. in  $a, b, c$ , &c. and the perpendiculars  $Ht, In, Kw$  let fall upon the asymptote  $Sx$ , in  $h, i, k$ ; and the differences of the densities  $in, nw$ , &c. will be as  $\frac{AH}{SA}, \frac{BI}{SB}$ , &c. And the rectangles  $in \times tb, nw \times ui$ , &c. or  $tp, uq$ , &c. as  $\frac{AH \times tb}{SA}, \frac{BI \times ui}{SB}$ , &c. that is, as  $Aa, Bb$ , &c. For, by the nature of the Hyperbola,  $SA$  is to  $AH$  or  $St$ , as  $tb$  to  $Aa$ , and therefore  $\frac{AH \times tb}{SA}$  is equal to  $Aa$ . And, by a like reasoning,  $\frac{BI \times ui}{SB}$  is equal to  $Bb$ , &c. But  $Aa, Bb, Cc$ , &c. are continually proportional, and therefore proportional to their differences  $Aa - Bb, Bb - Cc$ , &c. and therefore the rectangles  $tp, uq$ , &c. are proportional

to those differences ; as also the sums of the rectangles  $tp + nq$  or  $tp + nq + wr$  to the sums of the differences  $Aa - Cc$  or  $Aa - Dd$ . Suppose several of these terms, and the sum of all the differences, as  $Aa - Ff$ , will be proportional to the sum of all the rectangles, as  $ztbn$ . Increase the number of terms, and diminish the distances of the points  $A, B, C$ , &c. in *infinitum*, and those rectangles will become equal to the hyperbolic area  $ztbn$ , and therefore the difference  $Aa - Ff$  is proportional to this area. Take now any distances as  $SA, SD, SF$  in harmonic progression, and the differences  $Aa - Dd, Dd - Ff$  will be equal ; and therefore the areas  $tblx, xlnz$  proportional to those differences will be equal among themselves, and the densities  $St, Sx, Sz$ , that is,  $AH, DL, FN$  continually proportional. *Q. E. D.*

COR. 2. Hence if any two densities of the fluid, as  $AH$  and  $BI$  be given, the area  $tbiz$ , answering to their difference  $tn$  will be given ; and thence the density  $FN$  will be found at any height  $SF$ , by taking the area  $tbiz$  to that given area  $tbiz$  as the difference  $Aa - Ff$  to the difference  $Aa - Bb$ .

# SCHOLIUM.

By a like reasoning, it may be proved, that if the gravity of the particles of a fluid be diminished in a triplicate ratio of the distances from the centre ; and the reciprocals of the squares of the distances  $SA, SB, SC$ , &c. (namely  $\frac{SA^3}{SA^2}, \frac{SA^3}{SB^2}, \frac{SA^3}{SC^2}$ ) be taken in an Arithmetical progression, the densities  $AH, BI, CK$ , &c. will be in a Geometrical progression. And if the gravity be diminished in a quadruplicate ratio of the distances, and the reciprocals of the cubes of the distances (as  $\frac{SA^4}{SA^3}, \frac{SA^4}{SB^3}, \frac{SA^4}{SC^3}$  &c.) be taken in

Arith-

Arithmetical progression, the densities  $AH$ ,  $BI$ ,  $CK$ , &c. will be in Geometrical progression. And so *in infinitum*. Again, if the gravity of the particles of the fluid be the same at all distances, and the distances be in Arithmetical progression, the densities will be in a Geometrical progression, as Dr. Halley has found. If the gravity be as the distance, and the squares of the distances be in Arithmetical progression, the densities will be in Geometrical progression. And so *in infinitum*. These things will be so, when the density of the fluid condensed by compression is as the force of compression, or, which is the same thing, when the space possessed by the fluid is reciprocally as this force. Other laws of condensation may be supposed, as that the cube of the compressing force may be as the biquadrate of the density; or the triplicate ratio of the force the same with the quadruplicate ratio of the density: In which case, if the gravity be reciprocally as the square of the distance from the centre, the density will be reciprocally as the cube of the distance. Suppose that the cube of the compressing force be as the quadrato-cube of the density; and if the gravity be reciprocally as the square of the distance, the density will be reciprocally in a sesquiplicate ratio of the distance. Suppose the compressing force to be in a duplicate ratio of the density, and the gravity reciprocally in a duplicate ratio of the distance, and the density will be reciprocally as the distance. To run over all the cases that might be offer'd, would be tedious. But as to our own air, this is certain from experiment, that its density is either accurately or very nearly at least as the compressing force; and therefore the density of the air in the atmosphere of the earth is as the weight of the whole incumbent air, that is, as the height of the mercury in the barometer.

PRO-

## PROPOSITION XXIII. THEOREM XVIII.

*If a fluid be composed of particles mutually flying each other, and the density be as the compression, the centrifugal forces of the particles will be reciprocally proportional to the distances of their centres. And vice versa, particles flying each other with forces that are reciprocally proportional to the distances of their centres, compose an elastic fluid, whose density is as the compression. Pl. 5. Fig. 4.*

Let the fluid be supposed to be included in a cubic space  $ACE$ , and then to be reduced by compression into a lesser cubic space  $ace$ ; and the distances of the particles retaining a like situation with respect to each other in both the spaces, will be as the sides  $AB$ ,  $ab$  of the cubes; and the densities of the mediums will be reciprocally as the containing spaces  $AB^3$ ,  $ab^3$ . In the plane side of the greater cube  $ABCD$  take the square  $DP$  equal to the plane side  $db$  of the lesser cube: and, by the supposition, the pressure with which the square  $DP$  urges the inclosed fluid, will be to the pressure with which that square  $db$  urges the inclosed fluid, as the densities of the mediums are to each other, that is, as  $ab^3$  to  $AB^3$ . But the pressure with which the square  $DB$  urges the included fluid, is to the pressure with which the square  $DP$  urges the same fluid, as the square  $DB$  to the square  $DP$ , that is, as  $AB^2$  to  $ab^2$ . Therefore, *ex aqno*, the pressure with which the square  $DB$  urges the fluid is to the pressure with which the square  $db$  urges the fluid; as  $ab$  to  $AB$ . Let the planes  $FGH$ ,  $fgb$ , be drawn thro' the middles of the two cubes, and divide the fluid into two parts. These parts will press

press each other mutually with the same forces with which they are themselves pressed by the planes  $AC$ ,  $ac$ , that is, in the proportion of  $ab$  to  $AB$ ; and therefore the centrifugal forces by which these pressures are sustained, are in the same ratio. The number of the particles being equal, and the situation alike, in both cubes, the forces which all the particles exert, according to the planes  $FGH$ ,  $fgb$ , upon all, are as the forces which each exerts on each. Therefore the forces which each exerts on each according to the plane  $FGH$  in the greater cube, are to the forces which each exerts on each according to the plane  $fgb$  in the lesser cube, as  $ab$  to  $AB$ , that is, reciprocally as the distances of the particles from each other. *Q. E. D.*

And, *vice versa*, if the forces of the single particles are reciprocally as the distances, that is, reciprocally as the sides of the cubes  $AB$ ,  $ab$ ; the sums of the forces will be in the same ratio, and the pressures of the sides  $DB$ ,  $db$  as the sums of the forces; and the pressure of the square  $DP$  to the pressure of the side  $DB$  as  $ab^2$  to  $AB^2$ . And, *ex aequo*, the pressure of the square  $DP$  to the pressure of the side  $db$  as  $ab^3$  to  $AB^3$ , that is, the force of compression in the one to the force of compression in the other, as the density in the former to the density in the latter. *Q. E. D.*

#### SCHOLIUM.

By a like reasoning, if the centrifugal forces of the particles are reciprocally in the duplicate ratio of the distances between the centres, the cubes of the compressing forces will be as the biquadrates of the densities. If the centrifugal forces be reciprocally in the triplicate or quadruplicate ratio of the distances, the cubes of the compressing forces will be as the quadrato-cubes, or cubo-cubes of the densities. And universally, if  $D$  be put for the distance, and  $E$  for the density

sity of the compressed fluid, and the centrifugal forces be reciprocally as any power  $D^n$  of the distance, whose index is the number  $n$ ; the compressing forces will be as the cube roots of the power  $E^{n+2}$ , whose index is the number  $n+2$ : and the contrary. All these things are to be understood of particles whose centrifugal forces terminate in those particles that are next them, or are diffused not much further. We have an example of this in magnetical bodies. Their attractive virtue is terminated nearly in bodies of their own kind that are next them. The virtue of the magnet is contracted by the interposition of an iron plate; and is almost terminated at it. For bodies further off are not attracted by the magnet so much as by the iron plate. If in this manner particles repel others of their own kind that lie next them, but do not exert their virtue on the more remote, particles of this kind will compose such fluids as are treated of in this proposition. If the virtue of any particle diffuse itself every way *in infinitum*, there will be required a greater force to produce an equal condensation of a greater quantity of the fluid. But whether elastic fluids do really consist of particles so repelling each other, is a physical question. We have here demonstrated mathematically the property of fluids consisting of particles of this kind, that hence philosophers may take occasion to discuss that question.





## SECTION VI.

*Of the motion and resistance of funependulous bodies.*

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## PROPOSITION XXIV. THEOREM XIX.

*The quantities of matter in funependulous bodies, whose centres of oscillation are equally distant from the centre of suspension, are in a ratio compounded of the ratio of the weights and the duplicate ratio of the times of the oscillations in vacuo.*

For the velocity, which a given force can generate in a given matter in a given time, is as the force and the time directly, and the matter inversely. The greater the force or the time is, or the less the matter, the greater velocity will be generated. This is manifest from the second law of motion. Now if pendulums are of the same length, the motive forces in places equally distant from the perpendicular are as the weights: and therefore if two bodies by oscillating describe equal arcs, and those arcs are divided into equal parts; since the times in which the bodies describe each of the correspondent parts of the arcs are as the times of the whole oscillations, the velocities in the correspondent parts of the

the

the oscillations will be to each other, as the motive forces and the whole times of the oscillations directly, and the quantities of matter reciprocally : and therefore the quantities of matter are as the forces and the times of the oscillations directly and the velocities reciprocally. But the velocities reciprocally are as the times, and therefore the times directly and the velocities reciprocally are as the squares of the times ; and therefore the quantities of matter are as the motive forces and the squares of the times, that is, as the weights and the squares of the times. *Q. E. D.*

COR. 1. Therefore if the times are equal, the quantities of matter in each of the bodies are as the weights.

COR. 2. If the weights are equal, the quantities of matter will be as the squares of the times.

COR. 3. If the quantities of matter are equal, the weights will be reciprocally as the squares of the times.

COR. 4. Whence since the squares of the times, *ceteris paribus*, are as the lengths of the pendulums ; therefore if both the times and quantities of matter are equal, the weights will be as the lengths of the pendulums.

COR. 5. And universally, the quantity of matter in the pendulous body is as the weight and the square of the time directly, and the length of the pendulum inversely.

COR. 6. But in a non-resisting medium, the quantity of matter in the pendulous body is as the comparative weight and the square of the time directly, and the length of the pendulum inversely. For the comparative weight is the motive force of the body in any heavy medium, as was shewn above ; and therefore does the same thing in such a non-resisting medium, as the absolute weight does in a vacuum.

COR. 7. And hence appears a method both of comparing bodies one among another, as to the quantity of



matter in each ; and of comparing the weights of the same body in different places, to know the variation of its gravity. And by experiments made with the greatest accuracy, I have always found the quantity of matter in bodies to be proportional to their weight.

### PROPOSITION XXV. THEOREM XX.

*Funipendulous bodies that are, in any medium, resisted in the ratio of the moments of time, and funipendulous bodies that move in a non-resisting medium of the same specific gravity, perform their oscillations in a cycloid in the same time, and describe proportional parts of arcs together. Pl. 5. Fig. 5.*

Let  $AB$  be an arc of a cycloid, which a body  $D$ , by vibrating in a non-resisting medium shall describe in any time. Bisect that arc in  $C$ , so that  $C$  may be the lowest point thereof ; and the accelerative force with which the body is urged in any place  $D$  or  $d$  or  $E$  will be as the length of the arc  $CD$  or  $Cd$  or  $CE$ . Let that force be expressed by that same arc ; and since the resistance is as the moment of the time, and therefore given, let it be express'd by the given part  $CO$  of the cycloidal arc, and take the arc  $Od$  in the same ratio to the arc  $CD$  that the arc  $OB$  has to the arc  $CB$  : and the force with which the body in  $d$  is urged in a resisting medium, being the excess of the force  $Cd$  above the resistance  $CO$ , will be expressed by the arc  $Od$ , and will therefore be to the force with which the body  $D$  is urged in a non-resisting medium in the place  $D$ , as the arc  $Od$  to the arc  $CD$  ; and therefore also in the place  $B$ , as the arc  $OB$  to the arc  $CB$ . Therefore if two bodies  $D, d$  go from the place  $B$ , and are urged by these forces ; since the forces at the beginning are as the  
2
arcs

arcs  $CB$  and  $OB$ , the first velocities and arcs first described will be in the same ratio. Let those arcs be  $BD$  and  $Bd$ , and the remaining arcs  $CD$ ,  $Od$ , will be in the same ratio. Therefore the forces, being proportional to those arcs  $CD$ ,  $Od$ , will remain in the same ratio as at the beginning, and therefore the bodies will continue describing together arcs in the same ratio. Therefore the forces and velocities and the remaining arcs  $CD$ ,  $Od$ , will be always as the whole arcs  $CB$ ,  $OB$ , and therefore those remaining arcs will be described together. Therefore the two bodies  $D$  and  $d$  will arrive together at the places  $C$  and  $O$ ; that which moves in the non-resisting medium, at the place  $C$ , and the other, in the resisting medium, at the place  $O$ . Now since the velocities in  $C$  and  $O$  are as the arcs  $CB$ ,  $OB$ , the arcs which the bodies describe when they go farther, will be in the same ratio. Let those arcs be  $CE$  and  $Oe$ . The force with which the body  $D$  in a non-resisting medium is retarded in  $E$  is as  $CE$ , and the force with which the body  $d$  in the resisting medium is retarded in  $e$ , is as the sum of the force  $Ce$  and the resistance  $CO$ , that is, as  $Oe$ ; and therefore the forces with which the bodies are retarded, are as the arcs  $CB$ ,  $OB$ , proportional to the arcs  $CE$ ,  $Oe$ ; and therefore the velocities, retarded in that given ratio, remain in the same given ratio. Therefore the velocities and the arcs described with those velocities, are always to each other in that given ratio of the arcs  $CB$  and  $OB$ ; and therefore if the entire arcs  $AB$ ,  $aB$  are taken in the same ratio, the bodies  $D$  and  $d$  will describe those arcs together, and in the places  $A$  and  $a$  will lose all their motion together. Therefore the whole oscillations are isochronal, or are performed in equal times; and any parts of the arcs, as  $BD$ ,  $Bd$ , or  $BE$ ,  $Be$ , that are described together, are proportional to the whole arcs  $BA$ ,  $Ba$ . *Q. E. D.*

COR. Therefore the swiftest motion in a resisting medium does not fall upon the lowest point  $C$ , but is found in that point  $O$ , in which the whole arc described  $Ba$  is bisected. And the body proceeding from thence to  $a$ , is retarded at the same rate with which it was accelerated before in its descent from  $B$  to  $O$ .

# PROPOSITION XXVI. THEOREM XXI.

*Funipendulous bodies, that are resisted in the ratio of the velocity, have their oscillations in a cycloid isochronal.*

For if two bodies, equally distant from their centres of suspension, describe, in oscillating, unequal arcs, and the velocities in the correspondent parts of the arcs be to each other as the whole arcs; the resistances, proportional to the velocities, will be also to each other as the same arcs. Therefore if these resistances be subtracted from or added to the motive forces arising from gravity which are as the same arcs, the differences or sums will be to each other in the same ratio of the arcs: and since the increments and decrements of the velocities are as these differences or sums, the velocities will be always as the whole arcs: Therefore if the velocities are in any one case as the whole arcs, they will remain always in the same ratio. But at the beginning of the motion, when the bodies begin to descend and describe those arcs, the forces, which at that time are proportional to the arcs, will generate velocities proportional to the arcs. Therefore the velocities will be always as the whole arcs to be described, and therefore those arcs will be described in the same time. *Q. E. D.*

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## PROPOSITION XXVII. THEOREM XXII.

*If funipendulous bodies are resisted in the duplicate ratio of their velocities, the differences between the times of the oscillations in a resisting medium, and the times of the oscillations in a non-resisting medium of the same specific gravity, will be proportional to the arcs described in oscillating nearly.*

For let equal pendulums in a resisting medium describe the unequal arcs A, B ; and the resistance of the body in the arc A will be to the resistance of the body in the correspondent part of the arc B in the duplicate ratio of the velocities, that is, as A A to B B nearly. If the resistance in the arc B were to the resistance in the arc A as A B to A A ; the times in the arcs A and B would be equal (by the last Prop.) Therefore the resistance A A in the arc A, or A B in the arc B, causes the excess of the time in the arc A above the time in a non-resisting medium ; and the resistance B B causes the excess of the time in the arc B above the time in a non-resisting medium. But those excesses are as the efficient forces A B and B B nearly, that is, as the arcs A and B. *Q.E.D.*

**COR. 1.** Hence from the times of the oscillations in unequal arcs in a resisting medium, may be known the times of the oscillations in a non-resisting medium of the same specific gravity. For the difference of the times will be to the excess of the time in the lesser arc above the time in a non-resisting medium, as the difference of the arcs to the lesser arc.

**COR. 2.** The shorter oscillations are more isochronal, and very short ones are performed nearly in the same

same times as in a non-resisting medium. But the times of those which are performed in greater arcs are a little greater, because the resistance in the descent of the body, by which the time is prolonged, is greater, in proportion to the length described in the descent, than the resistance in the subsequent ascent, by which the time is contracted. But the time of the oscillations, both short and long, seems to be prolonged in some measure by the motion of the medium. For retarded bodies are resisted somewhat less, in proportion to the velocity, and accelerated bodies somewhat more, than those that proceed uniformly forwards; because the medium, by the motion it has received from the bodies, going forwards the same way with them, is more agitated in the former case, and less in the latter; and so conspires more or less with the bodies moved. Therefore it resists the pendulums in their descent more, and in their ascent less, than in proportion to the velocity; and these two causes concurring prolong the time.

### PROPOSITION XXVIII. THEOREM XXIII.

*If a funipendulous body, oscillating in a cycloid, be resisted in the ratio of the moments of the time, its resistance will be to the force of gravity as the excess of the arc described in the whole descent above the arc described in the subsequent ascent, to twice the length of the pendulum. Pl. 5. Fig. 5.*

Let  $BC$  represent the arc described in the descent,  $Ca$  the arc described in the ascent, and  $Aa$  the difference of the arcs: and things remaining as they were constructed and demonstrated in Prop. 25. the force with which the  
oscillating

oscillating body is urged in any place  $D$ , will be to the force of resistance as the arc  $CD$  to the arc  $CO$ , which is half of that difference  $Aa$ . Therefore the force with which the oscillating body is urged at the beginning or the highest point of the cycloid, that is, the force of gravity, will be to the resistance as the arc of the cycloid, between that highest point and lowest point  $C$ , is to the arc  $CO$ ; that is, (doubling those arcs) as the whole cycloidal arc, or twice the length of the pendulum, to the arc  $Aa$ . *Q.E.D.*

### PROPOSITION XXIX. PROBLEM VI.

*Supposing that a body oscillating in a cycloid is resisted in a duplicate ratio of the velocity: to find the resistance in each place. Pl. 5. Fig. 6.*

Let  $Ba$  be an arc described in one entire oscillation,  $C$  the lowest point of the cycloid, and  $CZ$  half the whole cycloidal arc, equal to the length of the pendulum; and let it be required to find the resistance of the body in any place  $D$ . Cut the indefinite right line  $OQ$  in the points  $O, S, P, Q$ , so that (erecting the perpendiculars  $OK, ST, PI, QE$ , and with the centre  $O$ , and the asymptotes  $OK, OQ$  describing the hyperbola  $TIGE$  cutting the perpendiculars  $ST, PI, QE$  in  $T, I$  and  $E$ , and thro' the point  $I$  drawing  $KF$ , parallel to the asymptote  $OQ$ , meeting the asymptote  $OK$  in  $K$ , and the perpendiculars  $ST$  and  $QE$  in  $L$  and  $F$ ) the hyperbolic area  $PIEQ$  may be to the hyperbolic area  $PITS$  as the arc  $BC$ , described in the descent of the body, to the arc  $Ca$  described in the ascent; and that the area  $IEF$  may be to the area  $ILT$  as  $OQ$  to  $OS$ . Then with the perpendicular  $MN$  cut off the hyperbolic area  $PINM$ , and let that area be to the hyperbolic area  $PIEQ$  as the arc  $CZ$  to the arc

G 4

BC

$BC$  described in the descent. And if the perpendicular  $RG$  cut off the hyperbolic area  $PIGR$ , which shall be to the area  $PIEQ$  as any arc  $CD$  to the arc  $BC$  described in the whole descent; the resistance in any place  $D$  will be to the force of gravity, as the area  $\frac{OR}{OQ} IEF - IGH$  to the area  $PINM$ .

For since the forces arising from gravity with which the body is urged in the places  $Z, B, D, a$ , are as the arcs  $CZ, CB, CD, Ca$ , and those arcs are as the areas  $PINM, PIEQ, PIGR, PITS$ ; let those areas be the exponents both of the arcs and of the forces respectively. Let  $Dd$  be a very small space described by the body in its descent; and let it be expressed by the very small area  $RGgr$  comprehended between the parallels  $RG, rg$ ; and produce  $rg$  to  $h$ , so that  $GHhg$ , and  $RGgr$  may be the contemporaneous decrements of the areas  $IGH, PIGR$ . And the increment  $GHhg - \frac{Rr}{OQ} IEF$ , or  $Rr \times HG - \frac{Rr}{OQ} IEF$ , of the area  $\frac{OR}{OQ} IEF - IGH$  will be to the decrement  $RGgr$ , or  $Rr \times RG$ , of the area  $PIGR$ , as  $HG - \frac{IEF}{OQ}$  to  $RG$ ; and therefore as  $OR \times HG - \frac{OR}{OQ} IEF$  to  $OR \times GR$  or  $OP \times PI$ , that is (because of the equal quantities  $OR \times HG, OR \times HR - OR \times GR, ORHK - OPIK, PIHR$  and  $PIGR - IGH$ ) as  $PIGR - IGH - \frac{OR}{OQ} IEF$  to  $OPIK$ . Therefore if the area  $\frac{OR}{OQ} IEF - IGH$  be called  $Y$ , and  $RGgr$  the decrement of the area  $PIGR$  be given, the increment of the area  $Y$  will be as  $PIGR - Y$ .

Then

Then if  $V$  represent the force arising from the gravity, proportional to the arc  $CD$  to be described, by which the body is acted upon in  $D$ , and  $R$  be put for the resistance;  $V - R$  will be the whole force with which the body is urged in  $D$ . Therefore the increment of the velocity is as  $V - R$  and the particle of time in which it is generated conjunctly. But the velocity itself is as the contemporaneous increment of the space described directly and the same particle of time inversely. Therefore, since the resistance is, by the supposition, as the square of the velocity, the increment of the resistance will (by Lem. 2.) be as the velocity and the increment of the velocity conjunctly, that is, as the moment of the space and  $V - R$  conjunctly; and therefore, if the moment of the space be given, as  $V - R$ ; that is, if for the force  $V$  we put its exponent  $PIGR$ , and the resistance  $R$  be expressed by any other area  $Z$ , as  $PIGR - Z$ .

Therefore the area  $PIGR$  uniformly decreasing by the subduction of given moments, the area  $Y$  increases in proportion of  $PIGR - Y$ , and the area  $Z$  in proportion of  $PIGR - Z$ . And therefore if the areas  $Y$  and  $Z$  begin together, and at the beginning are equal, these, by the addition of equal moments, will continue to be equal; and in like manner decreasing by equal moments will vanish together. And, *vice versa*, if they together begin and vanish, they will have equal moments and be always equal: and that, because if the resistance  $Z$  be augmented, the velocity together with the arc  $Ca$ , described in the ascent of the body, will be diminished; and the point in which all the motion together with the resistance ceases, coming nearer to the point  $C$ , the resistance vanishes sooner than the area  $Y$ . And the contrary will happen when the resistance is diminished.

Now the area  $Z$  begins and ends where the resistance is nothing, that is, at the beginning of the motion where the arc  $CD$  is equal to the arc  $CB$ , and the right  
line



line  $RG$  falls upon the right line  $QE$ ; and at the end of the motion where the arc  $CD$  is equal to the arc  $Ca$ , and  $RG$  falls upon the right line  $ST$ . And the area  $Y$  or  $\frac{OR}{OQ} IEF - IGH$  begins and ends also where the resistance is nothing, and therefore where  $\frac{OR}{OQ} IEF$  and  $IGH$  are equal; that is, (by the construction) where the right line  $RG$  falls successively upon the right lines  $QE$  and  $ST$ . Therefore those areas begin and vanish together, and are therefore always equal. Therefore the area  $\frac{OR}{OQ} IEF - IGH$  is equal to the area  $Z$ , by which the resistance is expressed, and therefore is to the area  $PINM$  by which the gravity is expressed as the resistance to the gravity. *Q.E.D.*

COR. 1. Therefore the resistance in the lowest place  $C$  is to the force of gravity, as the area  $\frac{OP}{OQ} IEF$  to the area  $PINM$ .

COR. 2. But it becomes greatest, where the area  $PIHR$  is to the area  $IEF$  as  $OR$  to  $OQ$ . For in that case its moment (that is,  $PIGR - Y$ ) becomes nothing.

COR. 3. Hence also may be known the velocity in each place: as being in the subduplicate ratio of the resistance, and at the beginning of the motion equal to the velocity of the body oscillating in the same cycloid without any resistance.

However, by reason of the difficulty of the calculation by which the resistance and the velocity are found by this Proposition, we have thought fit to subjoin the Proposition following.

PRO-





## PROPOSITION XXX. THEOREM XXIV.

*If a right line  $aB$  (Pl. 6. Fig. 1.) be equal to the arc of a cycloid which an oscillating body describes, and at each of its points  $D$  the perpendiculars  $DK$  be erected, which shall be to the length of the pendulum as the resistance of the body in the corresponding points of the arc to the force of gravity: I say, that the difference between the arc described in the whole descent and the arc described in the whole subsequent ascent drawn into half the sum of the same arcs, will be equal to the area  $BK a$  which all those perpendiculars take up.*

Let the arc of the cycloid, described in one entire oscillation, be expressed by the right line  $aB$ , equal to it, and the arc which would have been described *in vacuo*, by the length  $AB$ . Bisect  $AB$  in  $C$ , and the point  $C$  will represent the lowest point of the cycloid, and  $CD$  will be as the force arising from gravity, with which the body in  $D$  is urged in the direction of the tangent of the cycloid, and will have the same ratio to the length of the pendulum as the force in  $D$  has to the force of gravity. Let that force therefore be expressed by that length  $CD$ , and the force of gravity by the length of the pendulum, and if in  $DE$  you take  $DK$  in the same ratio to the length of the pendulum as the resistance has to the gravity,  $DK$  will be the exponent of the resistance. From the centre  $C$  with the interval  $CA$  or  $CB$  describe a semi-circle  $BEeA$ . Let the body describe, in the least time, the space  $Dd$ , and erecting the perpendiculars  $DE$ ,  $de$ , meeting the circumference in  $E$  and  $e$ , they will be as the velocities which

which the body descending *in vacuo* from the point *B* would acquire in the places *D* and *d*. This appears by Prop. 52. Book I. Let therefore these velocities be expressed by those perpendiculars *DE*, *de*; and let *DF* be the velocity which it acquires in *D* by falling from *B* in the resisting medium. And if from the centre *C* with the interval *CF* we describe the circle *FfM* meeting the right lines *de* and *AB* in *f* and *M*, then *M* will be the place to which it would thenceforward, without farther resistance, ascend, and *df* the velocity it would acquire in *d*. Whence also if *Fg* represent the moment of the velocity which the body *D*, in describing the least space *Dd*, loses by the resistance of the medium; and *CN* be taken equal to *Cg*: then will *N* be the place to which the body, if it met no farther resistance, would thenceforward ascend, and *MN* will be the decrement of the ascent arising from the loss of that velocity. Draw *Fm* perpendicular to *df*, and the decrement *Fg* of the velocity *DF* generated by the resistance *DK* will be to the increment *fm* of the same velocity generated by the force *CD*, as the generating force *DK* to the generating force *CD*. But because of the similar triangles *Fmf*, *Fhg*, *FDC*, *fm* is to *Fm* or *Dd* as *CD* to *DF*; and, *ex aequo*, *Fg* to *Dd* as *DK* to *DF*. Also *Fh* is to *Fg* as *DF* to *CF*; and, *ex aequo perturbatè*, *Fh* or *MN* to *Dd* as *DK* to *CF* or *CM*; and therefore the sum of all the *MN*  $\times$  *CM* will be equal to the sum of all the *Dd*  $\times$  *DK*. At the moveable point *M* suppose always a rectangular ordinate erected equal to the indeterminate *CM*, which by a continual motion is drawn into the whole length *Aa*; and the trapezium described by that motion, or its equal, the rectangle *Aa*  $\times$   $\frac{1}{2}$  *aB*, will be equal to the sum of all the *MN*  $\times$  *CM*, and therefore to the sum of all the *Dd*  $\times$  *DK*, that is, to the area *BKVTa*. Q. E. D.

Con

COR. Hence from the law of resistance and the difference  $Aa$  of the arcs  $Ca, CB$  may be collected the proportion of the resistance to the gravity nearly.

For if the resistance  $DK$  be uniform, the figure  $BKTa$  will be a rectangle under  $Ba$  and  $DK$ ; and thence the rectangle under  $\frac{1}{2}Ba$  and  $Aa$  will be equal to the rectangle under  $Ba$  and  $DK$ , and  $DK$  will be equal to  $\frac{1}{2}Aa$ . Wherefore since  $DK$  is the exponent of the resistance, and the length of the pendulum the exponent of the gravity, the resistance will be to the gravity as  $\frac{1}{2}Aa$  to the length of the pendulum; altogether as in Prop. 28. is demonstrated.

If the resistance be as the velocity, the figure  $BKTa$  will be nearly an ellipsis. For if a body, in a non-resisting medium, by one entire oscillation, should describe the length  $BA$ , the velocity in any place  $D$  would be as the ordinate  $DE$  of the circle described on the diameter  $AB$ . Therefore since  $Ba$  in the resisting medium, and  $BA$  in the non-resisting one, are described nearly in the same times; and therefore the velocities in each of the points of  $Ba$ , are to the velocities in the correspondent points of the length  $BA$  nearly as  $Ba$  is to  $BA$ ; the velocity in the point  $D$  in the resisting medium will be as the ordinate of the circle or ellipsis described upon the diameter  $Ba$ ; and therefore the figure  $BKVTa$  will be nearly an ellipsis. Since the resistance is supposed proportional to the velocity, let  $OV$  be the exponent of the resistance in the middle point  $O$ ; and an ellipsis  $BRVSt$  described with the centre  $O$ , and the semiaxes  $OB, OV$  will be nearly equal to the figure  $BKVTa$ , and to its equal the rectangle  $Aa \times BO$ . Therefore  $Aa \times BO$  is to  $OV \times BO$  as the area of this ellipsis to  $OV \times BO$ ; that is,  $Aa$  is to  $OV$  as the area of the semicircle to the square of the radius, or as 11 to 7 nearly; and therefore  $\frac{7}{11}Aa$  is to the length of the pendulum, as the resistance of the oscillating body in  $O$  to its gravity.

Now

Now if the resistance  $DK$  be in the duplicate ratio of the velocity, the figure  $BKVTa$  will be almost a Parabola having  $V$  for its vertex and  $OV$  for its axis, and therefore will be nearly equal to the rectangle under  $\frac{1}{2}Ba$  and  $OV$ . Therefore the rectangle under  $\frac{1}{2}Ba$  and  $Aa$  is equal to the rectangle  $\frac{1}{2}Ba \times OV$ , and therefore  $OV$  is equal to  $\frac{1}{2}Aa$ : and therefore the resistance in  $O$  made to the oscillating body is to its gravity as  $\frac{1}{2}Aa$  to the length of the pendulum.

And I take these conclusions to be accurate enough for practical uses. For since an Ellipsis or Parabola  $BRV Sa$  falls in with the figure  $BKVTa$  in the middle point  $V$ , that figure, if greater towards the part  $BRV$  or  $V Sa$  than the other, is less towards the contrary part, and is therefore nearly equal to it.

### PROPOSITION XXXI. THEOREM XXV.

*If the resistance made to an oscillating body in each of the proportional parts of the arcs described be augmented or diminished in a given ratio; the difference between the arc described in the descent and the arc described in the subsequent ascent, will be augmented or diminished in the same ratio.*

For that difference arises from the retardation of the pendulum by the resistance of the medium, and therefore is as the whole retardation, and the retarding resistance proportional thereto. In the foregoing Proposition the rectangle under the right line  $\frac{1}{2}aB$  and the difference  $Aa$  of the arcs  $CB$ ,  $Ca$  was equal to the area  $BK Ta$ . And that area, if the length  $aB$  remains, is augmented or diminished in the ratio of the ordinates  $DK$ ; that is, in the ratio of the resistance, and is therefore as the length  $aB$  and the resistance conjunctly.

conjunctly. And therefore the rectangle under  $Aa$  and  $\frac{2}{3}aB$  is as  $aB$  and the resistance conjunctly, and therefore  $Aa$  is as the resistance. *Q. E. D.*

COR. 1. Hence if the resistance be as the velocity, the difference of the arcs in the same medium will be as the whole arc described : and the contrary.

COR. 2. If the resistance be in the duplicate ratio of the velocity, that difference will be in the duplicate ratio of the whole arc : and the contrary.

COR. 3. And universally, if the resistance be in the triplicate or any other ratio of the velocity, the difference will be in the same ratio of the whole arc : and the contrary.

COR. 4. If the resistance be partly in the simple ratio of the velocity, and partly in the duplicate ratio of the same, the difference will be partly in the ratio of the whole arc, and partly in the duplicate ratio of it : and the contrary. So that the law and ratio of the resistance will be the same for the velocity, as the law and ratio of that difference for the length of the arc.

COR. 5. And therefore if a pendulum describe successively unequal arcs, and we can find the ratio of the increment or decrement of this difference for the length of the arc described ; there will be had also the ratio of the increment or decrement of the resistance for a greater or less velocity.

#### GENERAL SCHOLIUM.

From these Propositions, we may find the resistance of mediums by pendulums oscillating therein. I found the resistance of the air by the following experiments. I suspended a wooden globe or ball weighing  $57\frac{2}{3}$  ounces Averdupois, its diameter  $6\frac{1}{8}$  London inches, by a fine thread on a firm hook, so that the distance between the hook and the centre of oscillation of the globe was  $10\frac{1}{2}$  foot. I marked on the thread a point 10 foot and



1 inch distant from the centre of suspension ; and even with that point I placed a ruler divided into inches, by the help whereof I observed the lengths of the arcs described by the pendulum. Then I number'd the oscillations, in which the globe would lose  $\frac{1}{8}$  part of its motion. If the pendulum was drawn aside from the perpendicular to the distance of 2 inches, and thence let go, so that in its whole descent it described an arc of two inches, and in the first whole oscillation, compounded of the descent and subsequent ascent, an arc of almost four inches: the same in 164 oscillations lost  $\frac{1}{8}$  part of its motion, so as in its last ascent to describe an arc of  $1\frac{1}{4}$  inches. If in the first descent it described an arc of 4 inches ; it lost  $\frac{1}{8}$  part of its motion in 121 oscillations, so as in its last ascent to describe an arc of  $3\frac{1}{2}$  inches. If in the first descent it described an arc of 8, 16, 32, or 64 inches ; it lost  $\frac{1}{8}$  part of its motion in 69,  $35\frac{1}{2}$ ,  $18\frac{1}{2}$ ,  $9\frac{3}{4}$  oscillations, respectively. Therefore the difference between the arcs described in the first descent and the last ascent, was in the 1<sup>st</sup>, 2<sup>d</sup>, 3<sup>d</sup>, 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup> case,  $\frac{1}{4}$ ,  $\frac{1}{2}$ , 1, 2, 4, 8 inches, respectively. Divide those differences by the number of oscillations in each case, and in one mean oscillation, wherein an arc of  $3\frac{1}{4}$ ,  $7\frac{1}{2}$ , 15, 30, 60, 120 inches was described, the difference of the arcs described in the descent and subsequent ascent will be  $\frac{1}{656}$ ,  $\frac{1}{242}$ ,  $\frac{1}{69}$ ,  $\frac{4}{71}$ ,  $\frac{8}{37}$ ,  $\frac{24}{29}$  parts of an inch, respectively.

But these differences in the greater oscillations are in the duplicate ratio of the arcs described nearly, but in lesser oscillations something greater than in that ratio ; and therefore (by Cor. 2. Prop. 31. of this Book) the resistance of the globe, when it moves very swift, is in the duplicate ratio of the velocity, nearly ; and when it moves slowly, somewhat greater than in that ratio.

Let

Now let  $V$  represent the greatest velocity in any oscillation, and let  $A$ ,  $B$ , and  $C$  be given quantities, and let us suppose the difference of the arcs to be  $AV + BV^{\frac{1}{2}} + CV^2$ . Since the greatest velocities are in the cycloid as  $\frac{1}{2}$  the arcs described in oscillating, and in the circle as  $\frac{1}{2}$  the chords of those arcs; and therefore in equal arcs are greater in the cycloid than in the circle, in the ratio of  $\frac{1}{2}$  the arcs to their chords; but the times in the circle are greater than in the cycloid, in a reciprocal ratio of the velocity; it is plain that the differences of the arcs (which are as the resistance and the square of the time conjunctly) are nearly the same, in both curves: for in the cycloid those differences must be on the one hand augmented, with the resistance, in about the duplicate ratio of the arc to the chord, because of the velocity augmented in the simple ratio of the same; and on the other hand diminished, with the square of the time, in the same duplicate ratio. Therefore to reduce these observations to the cycloid, we must take the same differences of the arcs as were observed in the circle, and suppose the greatest velocities analogous to the half, or the whole arcs, that is, to the numbers  $\frac{1}{2}$ , 1, 2, 4, 8, 16. Therefore in the 2<sup>d</sup>, 4<sup>th</sup>, and 6<sup>th</sup> case, put 1, 4 and 16 for  $V$ ; and the difference of the arcs in the 2<sup>d</sup> case will become  $\frac{\frac{1}{2}}{121} = A + B$

$+ C$ ; in the 4<sup>th</sup> case  $\frac{2}{35^{\frac{1}{2}}} = 4A + 8B + 16C$ ; in

the 6<sup>th</sup> case  $\frac{8}{9^{\frac{2}{3}}} = 16A + 64B + 256C$ . These equations reduced give  $A = 0,0000916$ ,  $B = 0,0010847$ , and  $C = 0,0029558$ . Therefore the difference of

the arcs is as  $0,0000916V + 0,0010847V^{\frac{1}{2}} + 0,0029558V^2$ : and therefore since (by Cor. Prop. 30. applied to this case) the resistance of the globe in the

middle of the arc described in oscillating, where the velocity is  $V$ , is to its weight as  $\frac{1}{11}AV + \frac{1}{10}BV^{\frac{1}{2}} + \frac{1}{4}CV^2$  to the length of the pendulum; if for  $A$ ,  $B$ , and  $C$  you put the numbers found, the resistance of the globe will be to its weight, as  $0,0000583V + 0,0007593V^{\frac{1}{2}} + 0,0022169V^2$  to the length of the pendulum between the centre of suspension and the ruler, that is, to 121 inches. Therefore since  $V$  in the 2<sup>d</sup> case represents 1, in the 4<sup>th</sup> case 4, and in the 6<sup>th</sup> case 16: the resistance will be to the weight of the globe, in the 2<sup>d</sup> case as 0,0030345 to 121, in the 4<sup>th</sup> as 0,041748 to 121, in the 6<sup>th</sup> as 0,61705 to 121.

The arc which the point marked in the thread described in the 6<sup>th</sup> case, was of  $120 - \frac{8}{9^{\frac{1}{2}}}$  or  $119\frac{1}{2}$ ,

inches. And therefore since the radius was 121 inches, and the length of the pendulum between the point of suspension and the centre of the globe was 126 inches, the arc which the centre of the globe described was  $124\frac{1}{3}$  inches. Because the greatest velocity of the oscillating body, by reason of the resistance of the air, does not fall on the lowest point of the arc described, but near the middle place of the whole arc: this velocity will be nearly the same as if the globe in its whole descent in a non-resisting medium should describe  $62\frac{1}{2}$  inches the half of that arc, and that in a cycloid, to which we have above reduced the motion of the pendulum: and therefore that velocity will be equal to that which the globe would acquire by falling perpendicularly from a height equal to the versed sine of that arc. But that versed sine in the cycloid is to that arc  $62\frac{1}{2}$  as the same arc to twice the length of the pendulum 252, and therefore equal to 15,278 inches. Therefore the velocity of the pendulum is the same which a body would acquire by falling, and in its fall de-

describing a space of 15,278 inches. Therefore with such a velocity the globe meets with a resistance, which is to its weight as 0,61705 to 121, or (if we take that part only of the resistance which is in the duplicate ratio of the velocity) as 0,56752 to 121.

I found by an hydrostatical experiment, that the weight of this wooden globe was to the weight of a globe of water of the same magnitude as 55 to 97: and therefore since 121 is to 213,4 in the same ratio, the resistance made to this globe of water moving forwards with the abovementioned velocity, will be to its weight as 0,56752 to 213,4, that is, as 1 to  $376\frac{1}{5}$ . Whence since the weight of a globe of water, in the time in which the globe with a velocity uniformly continued describes a length of 30,556 inches, will generate all that velocity in the falling globe; it is manifest that the force of resistance uniformly continued in the same time will take away a velocity, which will be less than the other in the ratio of 1 to  $376\frac{1}{5}$ , that is, the

$\frac{1}{376\frac{1}{5}}$  part of the whole velocity. And therefore in the time that the globe, with the same velocity uniformly continued, would describe the length of its semi-diameter, or  $3\frac{1}{6}$  inches, it would lose the  $\frac{1}{314}$  part of its motion.

I also counted the oscillations in which the pendulum lost  $\frac{1}{4}$  part of its motion. In the following table the upper numbers denote the length of the arc described in the first descent, expressed in inches and parts of an inch; the middle numbers denote the length of the arc described in the last ascent; and in the lowest place are the numbers of the oscillations. I give an account of this experiment, as being more accurate than that in which only  $\frac{1}{8}$  part of the motion was lost. I leave the calculation to such as are disposed to make it.

H 2

First

<i>First descent</i>	2	4	8	16	32	64
<i>Last ascent</i>	$1\frac{1}{2}$	3	6	12	24	48
<i>Numb. of oscill.</i>	374	272	$162\frac{1}{2}$	$83\frac{1}{3}$	$41\frac{2}{3}$	$22\frac{2}{3}$

I afterwards suspended a leaden globe of 2 inches in diameter, weighing  $26\frac{1}{4}$  ounces Averdupois by the same thread, so that between the centre of the globe and the point of suspension there was an interval of  $10\frac{1}{2}$  feet, and I counted the oscillations in which a given part of the motion was lost. The first of the following tables exhibits the number of oscillations in which  $\frac{1}{8}$  part of the whole motion was lost; the second the number of oscillations in which there was lost  $\frac{1}{4}$  part of the same.

<i>First descent</i>	1	2	4	8	16	32	64
<i>Last ascent</i>	$\frac{2}{3}$	$\frac{2}{4}$	$3\frac{1}{2}$	7	14	28	56
<i>Numb. of oscill.</i>	226	228	193	140	$90\frac{1}{2}$	53	30

<i>First descent</i>	1	2	4	8	16	32	64
<i>Last ascent</i>	$\frac{3}{4}$	$1\frac{1}{2}$	3	6	12	24	48
<i>Numb. of oscill.</i>	510	518	420	318	204	121	70

Selecting in the first table the 3<sup>d</sup>, 5<sup>th</sup>, and 7<sup>th</sup> observation, and expressing the greatest velocities in these observations particularly by the numbers 1, 4, 16 respectively, and generally by the quantity V as above:

there will come out in the 3<sup>d</sup> observation  $\frac{\frac{1}{2}}{193} = A +$

$B + C$ , in the 5<sup>th</sup> observation  $\frac{2}{90\frac{1}{2}} = 4A + 8B +$

$16C$ , in the 7<sup>th</sup> observation  $\frac{8}{30} = 16A + 64B +$

$256C$ . These equations reduced give  $A = 0,001414$ ,  $B = 0,000297$ ,  $C = 0,000879$ . And thence the resistance of the globe moving with the velocity V will be to its weight  $26\frac{1}{4}$  ounces, in the same ratio as

$0,9009V + 0,000208V^{\frac{1}{2}} + 0,000659V^2$  to 121 inches

inches the length of the pendulum. And if we regard that part only of the resistance which is in the duplicate ratio of the velocity, it will be to the weight of the globe as  $0,000659V^2$  to 121 inches. But this part of the resistance in the 1<sup>st</sup> experiment was to the weight of the wooden globe of  $57\frac{2}{3}$  ounces as  $0,002217V^2$  to 121; and thence the resistance of the wooden globe is to the resistance of the leaden one (their velocities being equal) as  $57\frac{2}{3}$  into  $0,002217$  to  $26\frac{1}{4}$  into  $0,000659$ , that is, as  $7\frac{1}{3}$  to 1. The diameters of the two globes were  $6\frac{7}{8}$  and 2 inches, and the squares of these are to each other as  $47\frac{1}{4}$  and 4, or  $11\frac{1}{2}$  and 1, nearly. Therefore the resistances of these equally swift globes were in less than a duplicate ratio of the diameters. But we have not yet consider'd the resistance of the thread, which was certainly very considerable, and ought to be subducted from the resistance of the pendulums here found. I could not determine this accurately, but I found it greater than a third part of the whole resistance of the lesser pendulum; and thence I gathered that the resistances of the globes, when the resistance of the thread is subducted, are nearly in the duplicate ratio of their diameters. For the ratio of  $7\frac{1}{3} - \frac{1}{3}$  to  $1 - \frac{1}{3}$ , or  $10\frac{1}{2}$  to 1 is not very different from the duplicate ratio of the diameters,  $11\frac{1}{2}$  to 1.

Since the resistance of the thread is of less moment in greater globes, I tried the experiment also with a globe whose diameter was  $18\frac{1}{4}$  inches. The length of the pendulum between the point of suspension and the centre of oscillation was  $122\frac{1}{2}$  inches, and between the point of suspension and the knot in the thread  $109\frac{1}{2}$  inches. The arc described by the knot at the first descent of the pendulum was 32 inches. The arc described by the same knot in the last ascent after five oscillations was 28 inches. The sum of the arcs or the whole arc described in one mean oscillation was 60 inches.

H 3

inches. The difference of the arcs 4 inches. The  $\frac{1}{10}$  part of this, or the difference between the descent and ascent in one mean oscillation is  $\frac{2}{5}$  of an inch. Then as the radius  $109\frac{1}{2}$  to the radius  $122\frac{1}{2}$  so is the whole arc of 60 inches described by the knot in one mean oscillation to the whole arc of  $67\frac{1}{8}$  inches described by the centre of the globe in one mean oscillation; and so is the difference  $\frac{2}{5}$  to a new difference 0,4475. If the length of the arc described were to remain, and the length of the pendulum should be augmented in the ratio of 126 to  $122\frac{1}{2}$ ; the time of the oscillation would be augmented, and the velocity of the pendulum would be diminished in the subduplicate of that ratio; so that the difference 0,4475 of the arcs described in the descent and subsequent ascent would remain. Then if the arc described be augmented in the ratio of  $124\frac{3}{4}$  to  $67\frac{1}{8}$ , that difference 0,4475 would be augmented in the duplicate of that ratio, and so would become 1,5295. These things would be so upon the supposition, that the resistance of the pendulum were in the duplicate ratio of the velocity. Therefore if the pendulum describe the whole arc of  $124\frac{3}{4}$  inches, and its length between the point of suspension and the centre of oscillation be 126 inches, the difference of the arcs described in the descent and subsequent ascent would be 1,5295 inches. And this difference multiplied into the weight of the pendulous globe, which was 208 ounces, produces 318,136. Again in the pendulum above-mentioned, made of a wooden globe, when its centre of oscillation, being 126 inches from the point of suspension, described the whole arc of  $124\frac{3}{4}$  inches, the difference of the arcs described in the descent and ascent was  $\frac{126}{121}$  into  $\frac{8}{9\frac{1}{2}}$ . This multiplied into the weight of the globe, which was  $57\frac{1}{2}$  ounces, produces 49,396. But I multiply these differences into the weights of the globes, in order to find their resistances. For the disse-

differences arise from the resistances, and are as the resistances directly and the weights inversely. Therefore the resistances are as the numbers 318,136 and 49,396. But that part of the resistance of the lesser globe, which is in the duplicate ratio of the velocity, was to the whole resistance as 0,56752 to 0,61675, that is, as 45,453 to 49,396; whereas that part of the resistance of the greater globe is almost equal to its whole resistance; and so those parts are nearly as 318,136 and 45,453, that is, as 7 and 1. But the diameters of the globes are  $18\frac{1}{4}$  and  $6\frac{7}{8}$ ; and their squares  $351\frac{1}{2}$  and  $47\frac{1}{4}$  are as 7,438 and 1, that is, as the resistances of the globes 7 and 1, nearly. The difference of these ratio's is scarce greater than may arise from the resistance of the thread. Therefore those parts of the resistances which are, when the globes are equal, as the squares of the velocities; are also, when the velocities are equal, as the squares of the diameters of the globes.

But the greatest of the globes, I used in these experiments, was not perfectly spherical, and therefore in this calculation I have, for brevity's sake, neglected some little niceties; being not very solicitous for an accurate calculus, in an experiment that was not very accurate. So that I could wish, that these experiments were tried again with other globes, of a larger size, more in number, and more accurately formed; since the demonstration of a vacuum depends thereon. If the globes be taken in a geometrical proportion, as suppose whose diameters are 4, 8, 16, 32 inches; one may collect from the progression observed in the experiments what would happen if the globes were still larger.

In order to compare the resistances of different fluids with each other, I made the following trials. I procured a wooden vessel 4 feet long, 1 foot broad, and 1 foot high. This vessel, being uncover'd, I fill'd with spring-water, and having immersed pendulums therein, I made them oscillate in the water. And I

H 4

found



found that a leaden globe weighing  $166\frac{1}{8}$  ounces, and in diameter  $3\frac{1}{8}$  inches, moved therein as it is set down in the following table; the length of the pendulum from the point of suspension to a certain point marked in the thread being 126 inches, and to the centre of oscillation  $134\frac{1}{8}$  inches.

The arc described in the first descent by a point marked in the thread, was inches	64 . 32 . 16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$ . $\frac{1}{4}$
The arc described in the last ascent, was inches	48 . 24 . 12 . 6 . 3 . $1\frac{1}{2}$ . $\frac{3}{4}$ . $\frac{3}{8}$ . $\frac{3}{16}$
The difference of the arcs proportional to the motion lost, was inches	16 . 8 . 4 . 2 . 1 . $\frac{1}{2}$ . $\frac{1}{4}$ . $\frac{3}{8}$ . $\frac{1}{16}$
The number of the oscillations in water	$\frac{22}{60}$ . $1\frac{1}{3}$ . 3 . 7 . $11\frac{1}{4}$ . $12\frac{2}{3}$ . $13\frac{5}{6}$
The number of the oscillations in air.	$85\frac{1}{2}$ . 287 . 535

In the experiments of the 4<sup>th</sup> column, there were equal motions lost in 535 oscillations made in the air, and  $1\frac{1}{3}$  in water. The oscillations in the air were indeed a little swifter than those in the water. But if the oscillations in the water were accelerated in such a ratio that the motions of the pendulums might be equally swift in both mediums, there would be still the same number  $1\frac{1}{3}$  of oscillations in the water, and by these the same quantity of motion would be lost as before; because the resistance is increased and the square of the time diminished in the same duplicate ratio. The pendulums therefore being of equal velocities, there were equal

equal motions lost in 535 oscillations in the air, and  $1\frac{1}{2}$  in the water; and therefore the resistance of the pendulum in the water is to its resistance in the air as 535 to  $1\frac{1}{2}$ . This is the proportion of the whole resistances in the case of the 4<sup>th</sup> column.

Now let  $AV - CV^2$  represent the difference of the arcs described in the descent and subsequent ascent by the globe moving in air with the greatest velocity  $V$ ; and since the greatest velocity is in the case of the 4<sup>th</sup> column to the greatest velocity in the case of the 1<sup>st</sup> column as 1 to 8; and that difference of the arcs in the case of the 4<sup>th</sup> column to the difference in the case of

the 1<sup>st</sup> column, as  $\frac{2}{535}$  to  $\frac{16}{85\frac{1}{2}}$ , or as  $85\frac{1}{2}$  to 4280;

put in these cases 1 and 8 for the velocities, and  $85\frac{1}{2}$  and 4280 for the differences of the arcs, and  $A - C$  will be  $= 85\frac{1}{2}$ , and  $8A - 64C = 4280$  or  $A - 8C = 535$ ; and then, by reducing these equations, there will come out  $7C = 449\frac{1}{2}$  and  $C = 64\frac{1}{4}$  and  $A = 21\frac{3}{4}$ ; and therefore the resistance, which is as  $\frac{1}{11} AV - \frac{1}{4} CV^2$ , will become as  $13\frac{5}{11} V - 48\frac{2}{3} V^2$ . Therefore in the case of the 4<sup>th</sup> column, where the velocity was 1, the whole resistance is to its part proportional to the square of the velocity, as  $13\frac{5}{11} - 48\frac{2}{3}$  or  $61\frac{1}{7}$  to  $48\frac{2}{3}$ ; and therefore the resistance of the pendulum in water is to that part of the resistance in air, which is proportional to the square of the velocity, and which in swift motions is the only part that deserves consideration, as  $61\frac{1}{7}$  to  $48\frac{2}{3}$  and 535 to  $1\frac{1}{2}$  conjunctly, that is, as 571 to 1. If the whole thread of the pendulum oscillating in the water had been immersed, its resistance would have been still greater; so that the resistance of the pendulum oscillating in the water, that is, that part which is proportional to the square of the velocity, and which only needs to be consider'd in swift bodies, is to the resistance of the same whole pendulum, oscillating in air with

with the same velocity, as about 850 to 1, that is, as the density of water to the density of air, nearly.

In this calculation, we ought also to have taken in that part of the resistance of the pendulum in the water, which was as the square of the velocity, but I found (which will perhaps seem strange) that the resistance in the water was augmented in more than a duplicate ratio of the velocity. In searching after the cause, I thought upon this, that the vessel was too narrow for the magnitude of the pendulous globe, and by its narrowness obstructed the motion of the water as it yielded to the oscillating globe. For when I immersed a pendulous globe, whose diameter was one inch only; the resistance was augmented nearly in a duplicate ratio of the velocity. I tried this by making a pendulum of two globes, of which the lesser and lower oscillated in the water, and the greater and higher was fastened to the thread just above the water, and by oscillating in the air, assisted the motion of the pendulum, and continued it longer. The experiments made by this contrivance proved according to the following table.

<i>Arc descr. in first descent</i>	16	.	8	.	4	.	2	.	1	.	$\frac{1}{2}$	.	$\frac{1}{4}$
<i>Arc descr. in last ascent</i>	12	.	6	.	3	.	$1\frac{1}{2}$	.	$\frac{3}{4}$	.	$\frac{1}{2}$	.	$1\frac{1}{6}$
<i>Diff. of arcs, proport. to mos. lost</i>	} 4 . 2 . 1 . $\frac{1}{2}$ . $\frac{1}{4}$ . $\frac{1}{8}$ . $1\frac{1}{6}$												
<i>Number of oscillations</i>	3 $\frac{1}{8}$ . 6 $\frac{1}{2}$ . 12 $\frac{1}{2}$ . 21 $\frac{1}{3}$ . 34 . 53 . 62 $\frac{1}{3}$												

In comparing the resistances of the mediums with each other, I also caused iron pendulums to oscillate in quicksilver. The length of the iron wire was about 3 feet, and the diameter of the pendulous globe about  $\frac{1}{3}$  of an inch. To the wire, just above the quicksilver, there was fixed another leaden globe of a bigness sufficient to continue the motion of the pendulum for some time. Then a vessel, that would hold about 3 pounds of quicksilver, was filled by turns with quicksilver and common

common water, that by making the pendulum oscillate successively in these two different fluids, I might find the proportion of their resistances: and the resistance of the quicksilver proved to be to the resistance of water as about 13 or 14 to 1; that is, as the density of quicksilver to the density of water. When I made use of a pendulous globe something bigger, as of one whose diameter was about  $\frac{1}{2}$  or  $\frac{2}{3}$  of an inch, the resistance of the quicksilver proved to be to the resistance of the water as about 12 or 10 to 1. But the former experiment is more to be relied on, because in the latter the vessel was too narrow in proportion to the magnitude of the immersed globe: For the vessel ought to have been enlarged together with the globe. I intended to have repeated these experiments with larger vessels, and in melted metals, and other liquors both cold and hot: but I had not leisure to try all; and besides, from what is already described, it appears sufficiently that the resistance of bodies moving swiftly is nearly proportional to the densities of the fluids in which they move. I don't say accurately. For more tenacious fluids, of equal density, will undoubtedly resist more than those that are more liquid, as cold oil more than warm, warm oil more than rain-water, and water more than spirit of wine. But in liquors, which are sensibly fluid enough, as in air, in salt and fresh water, in spirit of wine, of turpentine and salts, in oil cleared of its fæces by distillation and warmed, in oil of vitriol and in mercury, and melted metals, and any other such like, that are fluid enough to retain for some time the motion impressed upon them by the agitation of the vessel, and which being poured out are easily resolv'd into drops: I doubt not but the rule already laid down may be accurate enough, especially if the experiments be made with larger pendulous bodies, and more swiftly moved.

Lastly, since it is the opinion of some, that there is a certain æthereal medium extremely rare and subtile, which

which freely pervades the pores of all bodies; and from such a medium so pervading the pores of bodies, some resistance must needs arise: in order to try whether the resistance, which we experience in bodies in motion, be made upon their outward superficies only, or whether their internal parts meet with any considerable resistance upon their superficies; I thought of the following experiment. I suspended a round deal box by a thread 11 feet long, on a steel hook by means of a ring of the same metal, so as to make a pendulum of the aforesaid length. The hook had a sharp hollow edge on its upper part, so that the upper arc of the ring pressing on the edge might move the more freely: and the thread was fastened to the lower arc of the ring. The pendulum being thus prepared, I drew it aside from the perpendicular to the distance of about 6 feet, and that in a plane perpendicular to the edge of the hook, lest the ring, while the pendulum oscillated, should slide to and fro on the edge of the hook: For the point of suspension, in which the ring touches the hook, ought to remain immovable. I therefore accurately noted the place, to which the pendulum was brought, and letting it go, I marked three other places, to which it returned at the end of the 1<sup>st</sup>, 2<sup>d</sup>, and 3<sup>d</sup> oscillation. This I often repeated, that I might find those places as accurately as possible. Then I filled the box with lead and other heavy metals, that were near at hand. But first I weighed the box when empty, and that part of the thread that went round it, and half the remaining part extended between the hook and the suspended box. For the thread so extended always acts upon the pendulum, when drawn aside from the perpendicular, with half its weight. To this weight I added the weight of the air contained in the box. And this whole weight was about  $\frac{1}{8}$  of the weight of the box when filled with the metals. Then because the box when full of the metals, by extending the thread with its weight, in-

creased the length of the pendulum, I shortened the thread so as to make the length of the pendulum, when oscillating, the same as before. Then drawing aside the pendulum to the place first marked, and letting it go, I reckoned about 77 oscillations, before the box returned to the second mark, and as many afterwards before it came to the third mark, and as many after that, before it came to the fourth mark. From whence I conclude that the whole resistance of the box, when full, had not a greater proportion to the resistance of the box, when empty, than 78 to 77. For if their resistances were equal, the box, when full, by reason of its *vis insita*, which was 78 times greater than the *vis insita* of the same when empty, ought to have continued its oscillating motion so much the longer, and therefore to have returned to those marks at the end of 78 oscillations. But it returned to them at the end of 77 oscillations.

Let therefore A represent the resistance of the box upon its external superficies, and B the resistance of the empty box on its internal superficies; and if the resistances to the internal parts of bodies equally swift be as the matter, or the number of particles that are resisted: then  $78B$  will be the resistance made to the internal parts of the box, when full; and therefore the whole resistance  $A + B$  of the empty box will be to the whole resistance  $A + 78B$  of the full box as 77 to 78, and, by division,  $A + B$  to  $77B$ , as 77 to 1, and thence  $A + B$  to  $B$  as  $77 \times 77$  to 1, and, by division again,  $A$  to  $B$  as 5928 to 1. Therefore the resistance of the empty box in its internal parts will be above 5000 times less than the resistance on its external superficies. This reasoning depends upon the supposition that the greater resistance of the full box arises, not from any other latent cause, but only from the action of some subtile fluid upon the included metal.

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This experiment is related by memory, the paper being lost, in which I had described it; so that I have been obliged to omit some fractional parts, which are flipt out of my memory. And I have no leisure to try it again. The first time I made it, the hook being weak, the full box was retarded sooner. The cause I found to be, that the hook was not strong enough to bear the weight of the box; so that as it oscillated to and fro, the hook was bent sometimes this and sometimes that way. I therefore procured a hook of sufficient strength, so that the point of suspension might remain unmoved, and then all things happened as is above described.

S E C.



## SECTION VII.

*Of the motion of fluids and the resistance made to projected bodies.*

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## PROPOSITION XXXII. THEOREM XXVI.

*Suppose two similar systems of bodies consisting of an equal number of particles, and let the correspondent particles be similar and proportional, each in one system to each in the other, and have a like situation among themselves, and the same given ratio of density to each other; and let them begin to move among themselves in proportional times, and with like motions, (that is, those in one system among one another, and those in the other among one another.) And if the particles that are in the same system do not touch one another, except in the moments of reflexion; nor attract, nor repel each other, except with accelerative forces that are as the diameters of the correspondent particles inversely, and the squares of the velocities directly: I say, that the particles of those systems will continue to move among themselves with like motions and in proportional times.*

Like bodies in like situations are said to be moved among themselves with like motions and in proportional times,



times, when their situations at the end of those times are always found alike in respect of each other: as suppose we compare the particles in one system with the correspondent particles in the other. Hence the times will be proportional, in which similar and proportional parts of similar figures will be described by correspondent particles. Therefore if we suppose two systems of this kind, the correspondent particles, by reason of the similitude of the motions at their beginning, will continue to be moved with like motions, so long as they move without meeting one another. For if they are acted on by no forces, they will go on uniformly in right lines by the 1<sup>st</sup> law. But if they do agitate one another, with some certain forces, and those forces are as the diameters of the correspondent particles inversely and the squares of the velocities directly; then because the particles are in like situations, and their forces are proportional, the whole forces with which correspondent particles are agitated, and which are compounded of each of the agitating forces, (by Corol. 2. of the Laws) will have like directions, and have the same effect as if they respected centres placed alike among the particles; and those whole forces will be to each other as the several forces which compose them, that is, as the diameters of the correspondent particles inversely, and the squares of the velocities directly: and therefore will cause correspondent particles to continue to describe like figures. These things will be so (by Cor. 1 and 8. Prop. 4. Book 1.) if those centres are at rest. But if they are moved, yet by reason of the similitude of the translations, their situations among the particles of the system will remain similar; so that the changes introduced into the figures described by the particles will still be similar. So that the motions of correspondent and similar particles will continue similar till their first meeting with each other; and thence will arise similar collisions, and similar reflexions; which will again beget  
similar

similar motions of the particles among themselves (by what was just now shewn) till they mutually fall upon one another again, and so on *ad infinitum*.

COR. 1. Hence if any two bodies, which are similar and in like situations to the correspondent particles of the systems, begin to move amongst them in like manner and in proportional times, and their magnitudes and densities be to each other as the magnitudes and densities of the corresponding particles: these bodies will continue to be moved in like manner and in proportional times. For the case of the greater parts of both systems and of the particles is the very same.

COR. 2. And if all the similar and similarly situated parts of both systems be at rest among themselves: and two of them, which are greater than the rest, and mutually correspondent in both systems, begin to move in lines alike posited, with any similar motion whatsoever; they will excite similar motions in the rest of the parts of the systems, and will continue to move among those parts in like manner and in proportional times; and will therefore describe spaces proportional to their diameters.

# PROPOSITION XXXIII. THEOREM XXVII.

*The same things being supposed, I say that the greater parts of the systems are resisted in a ratio compounded of the duplicate ratio of their velocities, and the duplicate ratio of their diameters, and the simple ratio of the density of the parts of the systems.*

For the resistance arises partly from the centripetal or centrifugal forces with which the particles of the system mutually act on each other, partly from the collisions and reflexions of the particles and the greater parts.

The resistances of the first kind are to each other as the whole motive forces from which they arise, that is, as the whole accelerative forces and the quantities of matter in corresponding parts; that is, (by the supposition) as the squares of the velocities directly, and the distances of the corresponding particles inversely, and the quantities of matter in the correspondent parts directly: and therefore since the distances of the particles in one system are to the correspondent distances of the particles of the other, as the diameter of one particle or part in the former system to the diameter of the correspondent particle or part in the other, and since the quantities of matter are as the densities of the parts and the cubes of the diameters; the resistances are to each other as the squares of the velocities and the squares of the diameters and the densities of the parts of the systems. *Q. E. D.* The resistances of the latter sort are as the number of correspondent reflexions and the forces of those reflexions conjunctly. But the number of the reflexions are to each other as the velocities of the corresponding parts directly and the spaces between their reflexions inversely. And the forces of the reflexions are as the velocities and the magnitudes and the densities of the corresponding parts conjunctly; that is, as the velocities and the cubes of the diameters and the densities of the parts. And joining all these ratios, the resistances of the corresponding parts are to each other as the squares of the velocities and the squares of the diameters and the densities of the parts conjunctly.

*Q. E. D.*

**COR. I.** Therefore if those systems are two elastic fluids, like our air, and their parts are at rest among themselves; and two similar bodies proportional in magnitude and density to the parts of the fluids and similarly situated among those parts, be any how projected in the direction of lines similarly posited; and the accelerative forces with which the particles of the fluids

mutually

mutually act upon each other, are as the diameters of the bodies projected inversely and the squares of their velocities directly: those bodies will excite similar motions in the fluids in proportional times, and will describe similar spaces and proportional to their diameters.

COR. 2. Therefore in the same fluid a projected body that moves swiftly meets with a resistance that is in the duplicate ratio of its velocity, nearly. For if the forces, with which distant particles act mutually upon one another, should be augmented in the duplicate ratio of the velocity, the projected body would be resisted in the same duplicate ratio accurately; and therefore in a medium, whose parts when at a distance do not act mutually with any force on one another, the resistance is in the duplicate ratio of the velocity accurately. Let there be therefore three mediums *A, B, C*, consisting of similar and equal parts regularly disposed at equal distances. Let the parts of the mediums *A* and *B* recede from each other with forces that are among themselves as *T* and *V*; and let the parts of the medium *C* be entirely destitute of any such forces. And if four equal bodies *D, E, F, G* move in these mediums, the two first *D* and *E* in the two first *A* and *B*, and the other two *F* and *G* in the third *C*; and if the velocity of the body *D* be to the velocity of the body *E*, and the velocity of the body *F* to the velocity of the body *G* in the subduplicate ratio of the force *T* to the force *V*: the resistance of the body *D* to the resistance of the body *E*, and the resistance of the body *F* to the resistance of the body *G* will be in the duplicate ratio of the velocities; and therefore the resistance of the body *D* will be to the resistance of the body *F*, as the resistance of the body *E* to the resistance of the body *G*. Let the bodies *D* and *F* be equally swift, as also the bodies *E* and *G*; and augmenting the velocities of the bodies *D* and *F* in any ratio, and diminishing the forces of the particles of the medium *B* in the duplicate of the same

I 2

ratio,

ratio, the medium *B* will approach to the form and condition of the medium *C* at pleasure; and therefore the resistances of the equal and equally swift bodies *E* and *G* in these mediums will perpetually approach to equality, so that their difference will at last become less than any given. Therefore since the resistances of the bodies *D* and *F* are to each other as the resistances of the bodies *E* and *G*, those will also in like manner approach to the ratio of equality. Therefore the bodies *D* and *F*, when they move with very great swiftness, meet with resistances very nearly equal; and therefore since the resistance of the body *F* is in a duplicate ratio of the velocity, the resistance of the body *D* will be nearly in the same ratio.

COR. 3. The resistance of a body moving very swift in an elastic fluid is almost the same as if the parts of the fluid were destitute of their centrifugal forces, and did not fly from each other: if so be that the elasticity of the fluid arise from the centrifugal forces of the particles, and the velocity be so great as not to allow the particles time enough to act.

COR. 4. Therefore since the resistances of similar and equally swift bodies, in a medium whose distant parts do not fly from each other, are as the squares of the diameters; the resistances made to bodies moving with very great and equal velocities in an elastic fluid, will be as the squares of the diameters, nearly.

COR. 5. And since similar, equal, and equally swift bodies, moving thro' mediums of the same density, whose particles do not fly from each other mutually, will strike against an equal quantity of matter in equal times, whether the particles of which the medium consists be more and smaller, or fewer and greater, and therefore impress on that matter an equal quantity of motion, and in return (by the 3<sup>d</sup> law of motion) suffer an equal re-action from the same, that is, are equally resisted: it is manifest also, that in elastic fluids of  
the

the same density, when the bodies move with extreme swiftness, their resistances are nearly equal; whether the fluids consist of gross parts, or of parts never so subtile. For the resistance of projectiles moving with exceeding great celerities, is not much diminished by the subtilty of the medium.

COR. 6. All these things are so in fluids, whose elastic force takes its rise from the centrifugal forces of the particles. But if that force arise from some other cause, as from the expansion of the particles after the manner of wool, or the boughs of trees, or any other cause, by which the particles are hindered from moving freely among themselves; the resistance, by reason of the lesser fluidity of the medium, will be greater than in the corollaries above.

# PROPOSITION XXXIV. THEOREM XXVIII.

*If in a rare medium, consisting of equal particles freely disposed at equal distances from each other, a globe and a cylinder described on equal diameters move with equal velocities, in the direction of the axis of the cylinder: the resistance of the globe will be but half so great as that of the cylinder.*

For since the action of the medium upon the body is the same (by Cor. 5. of the laws) whether the body move in a quiescent medium, or whether the particles of the medium impinge with the same velocity upon the quiescent body: let us consider the body as if it were quiescent, and see with what force it would be impelled by the moving medium. Let therefore *ABKI* (*Pl. 6. Fig. 2.*) represent a spherical body described from the centre *C* with the semidiameter *CA*, and let the particles of the medium impinge with a gi-

ven velocity upon that sphaerical body, in the directions of right lines parallel to  $AC$ ; and let  $FB$  be one of those right lines. In  $FB$  take  $LB$  equal to the semidiameter  $CB$ , and draw  $BD$  touching the sphere in  $B$ . Upon  $KC$  and  $BD$  let fall the perpendiculars  $BE$ ,  $LD$ , and the force with which a particle of the medium, impinging on the globe obliquely in the direction  $FB$ , would strike the globe in  $B$ , will be to the force with which the same particle, meeting the cylinder  $ONGQ$  described about the globe with the axis  $ACI$ , would strike it perpendicularly in  $b$ , as  $LD$  to  $LB$  or  $BE$  to  $BC$ . Again, the efficacy of this force to move the globe according to the direction of its incidence  $FB$  or  $AC$ , is to the efficacy of the same to move the globe according to the direction of its determination, that is, in the direction of the right line  $BC$  in which it impels the globe directly, as  $BE$  to  $BC$ . And joining these ratio's the efficacy of a particle, falling upon the globe obliquely in the direction of the right line  $FB$ , to move the globe in the direction of its incidence, is to the efficacy of the same particle falling in the same line perpendicularly on the cylinder, to move it in the same direction, as  $BE^2$  to  $BC^2$ . Therefore if in  $bE$ , which is perpendicular to the circular base of the cylinder  $NAO$ , and equal to the radius  $AC$ , we take  $bH$  equal to  $\frac{BE^2}{CB}$ : then  $bH$  will be to

$bE$  as the effect of the particle upon the globe to the effect of the particle upon the cylinder. And therefore the solid which is formed by all the right lines  $bH$  will be to the solid formed by all the right lines  $bE$  as the effect of all the particles upon the globe to the effect of all the particles upon the cylinder. But the former of these solids is a paraboloid whose vertex is  $C$ , its axis  $CA$  and latus rectum  $CA$ ; and the latter solid is a cylinder circumscribing the paraboloid :  
and

and it is known that a paraboloid is half its circumscribed cylinder. Therefore the whole force of the medium upon the globe is half of the entire force of the same upon the cylinder. And therefore if the particles of the medium are at rest, and the cylinder and globe move with equal velocities, the resistance of the globe will be half the resistance of the cylinder. *Q. E. D.*

### SCHOLIUM.

By the same method other figures may be compared together as to their resistance; and those may be found which are most apt to continue their motions in resisting mediums. As if upon the circular base *CEBH* (*Pl. 6. Fig. 3.*) from the centre *O*, with the radius *OC*, and the altitude *OD*, one would construct a frustum *CBGF* of a cone, which should meet with less resistance than any other frustum constructed with the same base and altitude, and going forwards towards *D* in the direction of its axis: bisect the altitude *OD* in *Q*, and produce *OQ* to *S*, so that *QS* may be equal to *QC*, and *S* will be the vertex of the cone whose frustum is sought.

Whence by the bye, since the angle *CSB* is always acute, it follows, that if the solid *ADBE* (*Pl. 6. Fig. 4.*) be generated by the convolution of an elliptical or oval figure *ADBE* about its axe *AB*, and the generating figure be touched by three right lines *FG*, *GH*, *HI* in the points *F*, *B*, and *I*, so that *GH* shall be perpendicular to the axe in the point of contact *B*, and *FG*, *HI* may be inclined to *GH* in the angles *FGB*, *BHI* of 135 degrees; the solid arising from the convolution of the figure *ADFGHIE* about the same axe *AB*, will be less resisted than the former solid; if so be that both move forward in the direction of their axe *AB*, and that the extremity *B* of each go foremost. Which proposition I conceive may be of use in the building of ships.



If the figure  $DNFG$  be such a curve, that if from any point thereof as  $N$  the perpendicular  $NM$  be let fall on the axe  $AB$ , and from the given point  $G$  there be drawn the right line  $GR$  parallel to a right line touching the figure in  $N$ , and cutting the axe produced in  $R$ ,  $MN$  becomes to  $GR$  as  $GR^3$  to  $4BR \times GB^2$ ; the solid described by the revolution of this figure about its axe  $AB$ , moving in the beforementioned rare medium from  $A$  towards  $B$ , will be less resisted than any other circular solid whatsoever, described of the same length and breadth.

*The demonstration of these curious Theorems being omitted by the author, the analysis thereof, communicated by a friend, is added at the end of this volume.*

### PROPOSITION XXXV. PROBLEM VII.

*If a rare medium consist of very small quiescent particles of equal magnitudes and freely disposed at equal distances from one another: to find the resistance of a globe moving uniformly forwards in this medium.*

CASE I. Let a cylinder described with the same diameter and altitude be conceived to go forward with the same velocity in the direction of its axis, thro' the same medium. And let us suppose that the particles of the medium, on which the globe or cylinder falls, fly back with as great a force of reflexion as possible. Then since the resistance of the globe (by the last Proposition) is but half the resistance of the cylinder, and since the globe is to the cylinder as 2 to 3, and since the cylinder by falling perpendicularly on the particles, and reflecting them with the utmost force communicates to them a velocity double to its own: it follows that the cylinder, in moving forward uniformly half the length of its axis, will communicate a motion to the particles, which is to the whole motion of the cylinder  
as





as the density of the medium to the density of the cylinder; and that the globe, in the time it describes one length of its diameter in moving uniformly forwards, will communicate the same motion to the particles; and in the time that it describes two thirds of its diameter, will communicate a motion to the particles, which is to the whole motion of the globe as the density of the medium to the density of the globe. And therefore the globe meets with a resistance, which is to the force by which its whole motion may be either taken away or generated in the time in which it describes two thirds of its diameter moving uniformly forwards, as the density of the medium to the density of the globe.

CASE 2. Let us suppose that the particles of the medium incident on the globe or cylinder are not reflected; and then the cylinder falling perpendicularly on the particles will communicate its own simple velocity to them, and therefore meets a resistance but half so great as in the former case, and the globe also meets with a resistance but half so great.

CASE 3. Let us suppose the particles of the medium to fly back from the globe with a force which is neither the greatest nor yet none at all, but with a certain mean force; then the resistance of the globe will be in the same mean ratio between the resistance in the first case and the resistance in the second. *Q.E.I.*

COR. 1. Hence if the globe and the particles are infinitely hard, and destitute of all elastic force, and therefore of all force of reflexion; the resistance of the globe will be to the force by which its whole motion may be destroyed or generated, in the time that the globe describes four third parts of its diameter, as the density of the medium to the density of the globe.

COR. 2. The resistance of the globe, *ceteris paribus*, is in the duplicate ratio of the velocity.

COR. 3. The resistance of the globe, *ceteris paribus*, is in the duplicate ratio of the diameter.

COR.

COR. 4. The resistance of the globe is, *ceteris paribus*, as the density of the medium.

COR. 5. The resistance of the globe is in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the density of the medium.

COR. 6. The motion of the globe and its resistance may be thus expounded. Let  $AB$  (Pl. 7. Fig. 1.) be the time in which the globe may, by its resistance uniformly continued, lose its whole motion. Erect  $AD$ ,  $BC$  perpendicular to  $AB$ . Let  $BC$  be that whole motion, and thro' the point  $C$ , the asymptotes being  $AD$ ,  $AB$ , describe the hyperbola  $CF$ . Produce  $AB$  to any point  $E$ . Erect the perpendicular  $EF$  meeting the hyperbola in  $F$ . Compleat the parallelogram  $CBE G$ , and draw  $AF$  meeting  $BC$  in  $H$ . Then if the globe in any time  $BE$ , with its first motion  $BC$  uniformly continued, describes in a non-resisting medium the space  $CBE G$  expounded by the area of the parallelogram, the same in a resisting medium will describe the space  $CBE F$  expounded by the area of the hyperbola; and its motion at the end of that time will be expounded by  $EF$  the ordinate of the hyperbola; there being lost of its motion the part  $FG$ . And its resistance at the end of the same time will be expounded by the length  $BH$ ; there being lost of its resistance the part  $CH$ . All these things appear by Cor. 1 and 3. Prop. 5. Book 2.

COR. 7. Hence if the globe in the time  $T$  by the resistance  $R$  uniformly continued, lose its whole motion  $M$ : the same globe in the time  $t$  in a resisting medium, wherein the resistance  $R$  decreases in a duplicate ratio of the velocity, will lose out of its motion  $M$  the part  $\frac{tM}{T+t}$ , the part  $\frac{TM}{T+t}$  remaining; and will describe a space which is to the space described in the same

same time  $t$  with the uniform motion  $M$ , as the logarithm of the number  $\frac{T-t}{T}$  multiplied by the number

2,302585092994 is to the number  $\frac{t}{T}$ , because the hyperbolic area  $BCFE$  is to the rectangle  $BCGE$  in that proportion.

SCHOLIUM.

I have exhibited in this Proposition the resistance and retardation of spherical projectiles in mediums that are not continued, and shewn that this resistance is to the force by which the whole motion of the globe may be destroyed or produced in the time in which the globe can describe two thirds of its diameter, with a velocity uniformly continued, as the density of the medium to the density of the globe, if so be the globe and the particles of the medium be perfectly elastic, and are indued with the utmost force of reflexion: and that this force, where the globe and particles of the medium are infinitely hard and void of any reflecting force, is diminished one half. But in continued mediums, as water, hot oil, and quicksilver, the globe as it passes thro them does not immediately strike against all the particles of the fluid that generate the resistance made to it, but presses only the particles that lie next to it, which press the particles beyond, which press other particles, and so on; and in these mediums the resistance is diminished one other half. A globe in these extremely fluid mediums meets with a resistance that is to the force by which its whole motion may be destroyed or generated in the time wherein it can describe, with that motion uniformly continued, eight third parts of its diameter, as the density of the medium to the density of the globe. This I shall endeavour to shew in what follows.

PRO-

## PROPOSITION XXXVI. PROBLEM VIII.

*To define the motion of water running out of a cylindrical vessel thro' a hole made at the bottom.*

Let  $ACDB$  (Pl. 7. Fig. 2.) be a cylindrical vessel,  $AB$  the mouth of it,  $CD$  the bottom parallel to the horizon,  $EF$  a circular hole in the middle of the bottom,  $G$  the centre of the hole, and  $GH$  the axis of the cylinder perpendicular to the horizon. And suppose a cylinder of ice  $APQB$  to be of the same breadth with the cavity of the vessel, and to have the same axis, and to descend perpetually with an uniform motion, and that its parts as soon as they touch the superficies  $AB$  dissolve into water, and flow down by their weight into the vessel, and in their fall compose the cataract or column of water  $ABNFEM$ , passing thro' the hole  $EF$ , and filling up the same exactly. Let the uniform velocity of the descending ice and of the contiguous water in the circle  $AB$  be that which the water would acquire by falling thro' the space  $IH$ ; and let  $IH$  and  $HG$  lie in the same right line, and thro' the point  $I$  let there be drawn the right line  $KL$  parallel to the horizon, and meeting the ice on both the sides thereof in  $K$  and  $L$ . Then the velocity of the water running out at the hole  $EF$  will be the same that it would acquire by falling from  $I$  thro' the space  $IG$ . Therefore, by Galileo's Theorems,  $IG$  will be to  $IH$  in the duplicate ratio of the velocity of the water that runs out at the hole to the velocity of the water in the circle  $AB$ , that is, in the duplicate ratio of the circle  $AB$  to the circle  $EF$ ; those circles being reciprocally as the velocities of the water which in the same time and in equal quantities passes severally thro' each

each of them, and compleatly fills them both. We are now considering the velocity with which the water tends to the plane of the horizon. But the motion parallel to the same by which the parts of the falling water approach to each other, is not here taken notice of; since it is neither produced by gravity, nor at all changes the motion perpendicular to the horizon which the gravity produces. We suppose indeed that the parts of the water cohere a little, that by their cohesion they may in falling approach to each other with motions parallel to the horizon, in order to form one single cataract, and to prevent their being divided into several: but the motion parallel to the horizon arising from this cohesion does not come under our present consideration.

CASE I. Conceive now the whole cavity in the vessel, which encompasses the falling water *ABNFE M*, to be full of ice, so that the water may pass thro' the ice as thro' a funnel. Then if the water pass very near to the ice only, without touching it; or, which is the same thing, if, by reason of the perfect smoothness of the surface of the ice, the water, tho' touching it, glides over it with the utmost freedom, and without the least resistance; the water will run thro' the hole *EF* with the same velocity as before, and the whole weight of the column of water *ABNFE M* will be all taken up as before in forcing out the water, and the bottom of the vessel will sustain the weight of the ice encompassing that column.

Let now the ice in the vessel dissolve into water; yet will the efflux of the water remain, as to its velocity, the same as before. It will not be less, because the ice now dissolved will endeavour to descend; it will not be greater, because the ice now become water cannot descend without hindering the descent of other water equal to its own descent. The same force ought always to generate the same velocity in the effluent water.

But



But the hole at the bottom of the vessel, by reason of the oblique motions of the particles of the effluent water, must be a little greater than before. For now the particles of the water do not all of them pass thro' the hole perpendicularly; but flowing down on all parts from the sides of the vessel, and converging towards the hole, pass thro' it with oblique motions; and in tending downwards meet in a stream whose diameter is a little smaller below the hole than at the hole itself, its diameter being to the diameter of the hole as 5 to 6, or as  $5\frac{1}{2}$  to  $6\frac{1}{2}$ , very nearly, if I took the measures of those diameters right. I procured a very thin flat plate having a hole pierced in the middle, the diameter of the circular hole being  $\frac{1}{8}$  parts of an inch. And that the stream of running water might not be accelerated in falling, and by that acceleration become narrower, I fixed this plate, not to the bottom, but to the side of the vessel, so as to make the water go out in the direction of a line parallel to the horizon. Then when the vessel was full of water, I opened the hole to let it run out; and the diameter of the stream, measured with great accuracy at the distance of about half an inch from the hole, was  $\frac{2}{5}$  of an inch. Therefore the diameter of this circular hole was to the diameter of the stream very nearly as 25 to 21. So that the water in passing thro' the hole, converges on all sides, and after it has run out of the vessel, becomes smaller by converging in that manner, and by becoming smaller is accelerated till it comes to the distance of half an inch from the hole, and at that distance flows in a smaller stream and with greater celerity than in the hole itself, and this in the ratio of  $25 \times 25$  to  $21 \times 21$  or 17 to 12 very nearly, that is, in about the subduplicate ratio of 2 to 1. Now it is certain from experiments, that the quantity of water, running out in a given time thro' a circular hole made in the bottom of a vessel is equal to the quantity, which, flowing with the aforesaid velocity,

city, would run out in the same time, thro' another circular hole, whose diameter is to the diameter of the former as 21 to 25. And therefore that running water in passing thro' the hole itself has a velocity downwards equal to that which a heavy body would acquire in falling thro' half the height of the stagnant water in the vessel, nearly. But then after it has run out, it is still accelerated by converging, till it arrives at a distance from the hole that is nearly equal to its diameter, and acquires a velocity greater than the other in about the subduplicate ratio of 2 to 1; which velocity a heavy body would nearly acquire, by falling thro' the whole height of the stagnant water in the vessel.

Therefore in what follows let the diameter of the stream be represented by that lesser hole which we called *EF*. And imagine another plane *VW* above the hole *EF*, (*Pl. 7. Fig. 3.*) and parallel to the plane thereof, to be placed at a distance equal to the diameter of the same hole, and to be pierced thro' with a greater hole *ST*, of such a magnitude that a stream which will exactly fill the lower hole *EF* may pass thro' it; the diameter of which hole will therefore be to the diameter of the lower hole as 25 to 21, nearly. By this means the water will run perpendicularly out at the lower hole; and the quantity of the water running out will be, according to the magnitude of this last hole, the same, very nearly, which the solution of the problem requires. The space included between the two planes and the falling stream may be consider'd as the bottom of the vessel. But to make the solution more simple and mathematical, it is better to take the lower plane alone for the bottom of the vessel, and to suppose that the water which flowed thro' the ice as thro' a funnel, and ran out of the vessel thro' the hole *EF* made in the lower plane, preserves its motion continually, and that the ice continues at rest. Therefore in what follows let *ST* be the diameter of a circular hole described from the centre *Z*, and let the stream run out  
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of the vessel thro' that hole when the water in the vessel is all fluid. And let  $EF$  be the diameter of the hole which the stream, in falling thro', exactly fills up, whether the water runs out of the vessel by that upper hole  $ST$ , or flows thro' the middle of the ice in the vessel, as thro' a funnel. And let the diameter of the upper hole  $ST$  be to the diameter of the lower  $EF$  as about 25 to 21, and let the perpendicular distance between the planes of the holes be equal to the diameter of the lesser hole  $EF$ . Then the velocity of the water downwards in running out of the vessel thro' the hole  $ST$ , will be in that hole the same that a body may acquire by falling from half the height  $IZ$ : and the velocity of both the falling streams will be, in the hole  $EF$ , the same which a body would acquire by falling from the whole height  $IG$ .

CASE 2. If the hole  $EF$  be not in the middle of the bottom of the vessel, but in some other part thereof, the water will still run out with the same velocity as before, if the magnitude of the hole be the same. For tho' an heavy body takes a longer time in descending to the same depth, by an oblique line, than by a perpendicular line; yet in both cases it acquires in its descent the same velocity, as *Galileo* has demonstrated.

CASE 3. The velocity of the water is the same when it runs out thro' a hole in the side of the vessel. For if the hole be small, so that the interval between the superficies  $AB$  and  $KL$  may vanish as to sense, and the stream of water horizontally issuing out may form a parabolic figure: from the latus rectum of this parabola may be collected, that the velocity of the effluent water is that which a body may acquire by falling the height  $IG$  or  $HG$  of the stagnant water in the vessel. For by making an experiment, I found that if the height of the stagnant water above the hole were 20 inches, and the height

height of the hole above a plane parallel to the horizon were also 20 inches, a stream of water springing out from thence would fall upon the plane, at the distance of 37 inches, very nearly, from a perpendicular let fall upon that plane from the hole. For without resistance the stream would have fallen upon the plane at the distance of 40 inches, the latus rectum of the parabolic stream being 80 inches.

CASE 4. If the effluent water tend upwards, it will still issue forth with the same velocity. For the small stream of water springing upwards, ascends with a perpendicular motion to  $GH$  or  $GI$  the height of the stagnant water in the vessel; excepting in so far as its ascent is hindered a little by the resistance of the air; and therefore it springs out with the same velocity that it would acquire in falling from that height. Every particle of the stagnant water is equally pressed on all sides, (by Prop. 19. Book 2.) and yielding to the pressure, tends all ways with an equal force, whether it descends thro' the hole in the bottom of the vessel, or gushes out in an horizontal direction thro' an hole in the side, or passes into a canal, and springs up from thence thro' a little hole made in the upper part of the canal. And it may not only be collected from reasoning, but is manifest also from the well-known experiments just mentioned, that the velocity with which the water runs out is the very same that is assigned in this Proposition.

CASE 5. The velocity of the effluent water is the same, whether the figure of the hole be circular, or square, or triangular, or any other figure equal to the circular. For the velocity of the effluent water does not depend upon the figure of the hole, but arises from its depth below the plane  $KL$ .

CASE 6. If the lower part of the vessel  $ABDC$  be immersed into stagnant water, and the height of the stagnant water above the bottom of the vessel be  $GR$ ; the velocity with which the water that is in the vessel

and  $EF$  to the sum of the same circles, (by Cor. 4.) and the weight of the whole water in the vessel is to the weight of the whole water perpendicularly incumbent on the bottom as the circle  $AB$  to the difference of the circles  $AB$  and  $EF$ . Therefore, *ex æquo perturbatè*, that part of the weight which presses upon the bottom is to the weight of the whole water perpendicularly incumbent thereon as the circle  $AB$  to the sums of the circles  $AB$  and  $EF$ , or the excess of twice the circle  $AB$  above the bottom.

COR. 7. If in the middle of the hole  $EF$  there be placed the little circle  $PQ$  described about the centre  $G$ , and parallel to the horizon; the weight of water which that little circle sustains is greater than the weight of a third part of a cylinder of water whose base is that little circle and its height  $GH$ . For let  $ABNFEM$  (Pl. 7. Fig. 4.) be the cataract or column of falling water whose axis is  $GH$  as above, and let all the water, whose fluidity is not requisite for the ready and quick descent of the water, be supposed to be congealed; as well round about the cataract, as above the little circle. And let  $PHQ$  be the column of water, congealed above the little circle, whose vertex is  $H$ , and its altitude  $GH$ . And suppose this cataract to fall with its whole weight downwards, and not in the least to lie against or to press  $PHQ$ , but to glide freely by it without any friction, unless perhaps just at the very vertex of the ice where the cataract at the beginning of its fall may tends to a concave figure. And as the congealed water  $AMEC$ ,  $BNFD$  lying round the cataract, is convex in its internal superficies  $AME$ ,  $BNF$  towards the falling cataract, so this column  $PHQ$  will be convex towards the cataract also, and will therefore be greater than a cone whose base is that little circle  $PQ$  and its altitude  $GH$ , that is, greater than a third part of a cylinder described with the same base and altitude. Now that little circle sustains the weight of this column,

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that

that is, a weight greater than the weight of the cone or a third part of the cylinder.

COR. 8. The weight of water which the circle  $PQ$ , when very small, sustains, seems to be less than the weight of two thirds of a cylinder of water whose base is that little circle, and its altitude  $HG$ . For, things standing as above supposed, imagine the half of a sphæroid described whose base is that little circle, and its semi-axis or altitude  $HG$ . This figure will be equal to two thirds of that cylinder, and will comprehend within it the column of congealed water  $PHQ$ , the weight of which is sustained by that little circle. For tho' the motion of the water tends directly downwards, the external superficies of that column must yet meet the base  $PQ$  in an angle somewhat acute, because the water in its fall is perpetually accelerated, and by reason of that acceleration becomes narrower. Therefore, since that angle is less than a right one, this column in the lower parts thereof will lie within the hemi-sphæroid. In the upper parts also it will be acute or pointed; because, to make it otherwise, the horizontal motion of the water must be at the vertex infinitely more swift than its motion towards the horizon. And the less this circle  $PQ$  is, the more acute will the vertex of this column be; and the circle being diminished in infinitum, the angle  $PHQ$  will be diminished in infinitum, and therefore the column will lie within the hemi-sphæroid. Therefore that column is less than that hemi-sphæroid, or than two third parts of the cylinder whose base is that little circle, and its altitude  $GH$ . Now the little circle sustains a force of water equal to the weight of this column, the weight of the ambient water being employed in causing its efflux out at the hole.

COR. 9. The weight of water which the little circle  $PQ$  sustains when it is very small, is very nearly equal to the weight of a cylinder of water whose base is that little circle, and its altitude  $\frac{1}{2}GH$ . For this weight is

an arithmetical mean between the weights of the cone and the hemi-sphæroid abovementioned. But if that little circle be not very small, but on the contrary increased till it be equal to the hole  $EF$ ; it will sustain the weight of all the water lying perpendicularly above it, that is, the weight of a cylinder of water whose base is that little circle and its altitude  $GH$ .

COR. 10. And (as far as I can judge) the weight which this little circle sustains is always to the weight of a cylinder of water whose base is that little circle and its altitude  $\frac{1}{2}GH$ , as  $EF^2$  to  $EF^2 - \frac{1}{2}PQ^2$ , or as the circle  $EF$  to the excess of this circle above half the little circle  $PQ$ , very nearly.

#### LEMMA IV.

*If a cylinder move uniformly forwards in the direction of its length, the resistance made thereto is not at all changed by augmenting or diminishing that length; and is therefore the same with the resistance of a circle, described with the same diameter, and moving forwards with the same velocity in the direction of a right line perpendicular to its plane.*

For the sides are not at all opposed to the motion; and a cylinder becomes a circle when its length is diminished in infinitum.

PRO-

## PROPOSITION XXXVII. THEOREM XXIX.

*If a cylinder move uniformly forwards in a compressed, infinite, and non-elastic fluid, in the direction of its length; the resistance arising from the magnitude of its transverse section, is to the force by which its whole motion may be destroyed or generated, in the time that it moves four times its length, as the density of the medium to the density of the cylinder, nearly.*

For let the vessel  $ABDC$  (Pl. 7. Fig. 5.) touch the surface of stagnant water with its bottom  $CD$ , and let the water run out of this vessel into the stagnant water thro' the cylindric canal  $EFTS$  perpendicular to the horizon; and let the little circle  $PQ$  be placed parallel to the horizon any where in the middle of the canal; and produce  $CA$  to  $K$ , so that  $AK$  may be to  $CK$  in the duplicate of the ratio, which the excess of the orifice of the canal  $EF$  above the little circle  $PQ$ , bears to the circle  $AB$ . Then 'tis manifest (by Case 5. Case 6. and Cor. 1. Prop. 36.) that the velocity of the water passing thro' the annular space between the little circle and the sides of the vessel, will be the very same which the water would acquire by falling, and in its fall describing the altitude  $KC$  or  $IG$ .

And (by Cor. 10. Prop. 36.) if the breadth of the vessel be infinite, so that the lineola  $HI$  may vanish, and the altitudes  $IG$ ,  $HG$  become equal; the force of the water that flows down, and presses upon the circle will be to the weight of a cylinder whose base is that little circle and the altitude  $\frac{1}{2}IG$ , as  $EF^2$  to  $EF^2 - \frac{1}{2}PQ^2$  very nearly. For the force of the water flowing downwards uniformly thro' the whole

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canal



canal will be the same upon the little circle  $PQ$  in whatsoever part of the canal it be placed.

Let now the orifices of the canal  $EF$ ,  $ST$  be closed, and let the little circle ascend in the fluid compressed on every side, and by its ascent let it oblige the water that lies above it to descend thro' the annular space between the little circle and the sides of the canal. Then will the velocity of the ascending little circle be to the velocity of the descending water as the difference of the circles  $EF$  and  $PQ$  is to the circle  $PQ$ ; and the velocity of the ascending little circle will be to the sum of the velocities, that is, to the relative velocity of the descending water with which it passes by the little circle in its ascent, as the difference of the circles  $EF$  and  $PQ$  to the circle  $EF$ , or as  $EF^2 - PQ^2$  to  $EF^2$ . Let that relative velocity be equal to the velocity with which it was shewn above that the water would pass thro' the annular space if the circle were to remain unmoved, that is, to the velocity which the water would acquire by falling, and in its fall describing the altitude  $IG$ ; and the force of the water upon the ascending circle will be the same as before, (by cor. 5. of the laws of motion) that is, the resistance of the ascending little circle will be to the weight of a cylinder of water whose base is that little circle and its altitude  $\frac{1}{2}IG$ , as  $EF^2$  to  $EF^2 - \frac{1}{2}PQ^2$  nearly. But the velocity of the little circle will be to the velocity which the water acquires by falling, and in its fall describing the altitude  $IG$ , as  $EF^2 - PQ^2$  to  $EF^2$ .

Let the breadth of the canal be increased in infinitum; and the ratio's between  $EF^2 - PQ^2$  and  $EF^2$ , and between  $EF^2$  and  $EF^2 - \frac{1}{2}PQ^2$  will become at last ratio's of equality. And therefore the velocity of the little circle will now be the same which the water would acquire in falling, and in its fall describing the altitude  $IG$ ; and the resistance will become equal to the weight of a cylinder whose base is that little circle, and

and its altitude half the altitude  $IG$ , from which the cylinder must fall to acquire the velocity of the ascending circle. And with this velocity the cylinder in the time of its fall will describe four times its length. But the resistance of the cylinder moving forwards with this velocity in the direction of its length, is the same with the resistance of the little circle, (by Lem. 4.) and is therefore nearly equal to the force by which its motion may be generated while it describes four times its length.

If the length of the cylinder be augmented or diminished, its motion, and the time in which it describes four times its length, will be augmented or diminished in the same ratio; and therefore the force by which the motion, so increased or diminished, may be destroyed or generated, will continue the same; because the time is increased or diminished in the same proportion; and therefore that force remains still equal to the resistance of the cylinder, because (by Lem. 4.) that resistance will also remain the same.

If the density of the cylinder be augmented or diminished, its motion, and the force by which its motion may be generated or destroyed in the same time, will be augmented or diminished in the same ratio. Therefore the resistance of any cylinder whatsoever will be to the force by which its whole motion may be generated or destroyed in the time during which it moves four times its length, as the density of the medium to the density of the cylinder, nearly. *Q. E. D.*

A fluid must be compressed to become continued; it must be continued and non-elastic, that all the pressure arising from its compression may be propagated in an instant; and so acting equally upon all parts of the body moved, may produce no change of the resistance. The pressure arising from the motion of the body is spent in generating a motion in the parts of the fluid, and this creates the resistance. But the pressure arising  
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from the compression of the fluid, be it never so forcible, if it be propagated in an instant, generates no motion in the parts of a continued fluid, produces no change at all of motion therein; and therefore neither augments nor lessens the resistance. This is certain, that the action of the fluid arising from the compression cannot be stronger on the hinder parts of the body moved than on its fore parts, and therefore cannot lessen the resistance described in this Proposition. And if its propagation be infinitely swifter than the motion of the body pressed, it will not be stronger on the fore parts than on the hinder parts. But that action will be infinitely swifter and propagated in an instant, if the fluid be continued and non-elastic.

COR. 1. The resistances made to cylinders going uniformly forwards in the direction of their lengths thro' continued infinite mediums, are in a ratio compounded of the duplicate ratio of the velocities and the duplicate ratio of the diameters, and the ratio of the density of the mediums.

COR. 2. If the breadth of the canal be not infinitely increased, but the cylinder go forwards in the direction of its length through an included quiescent medium, its axis all the while coinciding with the axis of the canal; its resistance will be to the force by which its whole motion in the time in which it describes four times its length, may be generated or destroyed, in a ratio compounded of the ratio of  $EF^2$  to  $EF^2 - \frac{1}{2}PQ^2$  once, and the ratio of  $EF^2$  to  $EF^2 - PQ^2$  twice, and the ratio of the density of the medium to the density of the cylinder.

COR. 3. The same things supposed, and that a length  $L$  is to the quadruple of the length of the cylinder in a ratio compounded of the ratio  $EF^2 - \frac{1}{2}PQ^2$  to  $EF^2$  once, and the ratio of  $EF^2 - PQ^2$  to  $EF^2$  twice; the resistance of the cylinder will be to the force by which its whole motion, in the time during which it describes the

the length  $L$ , may be destroyed or generated, as the density of the medium to the density of the cylinder.

# SCHOLIUM.

In this proposition we have investigated that resistance alone which arises from the magnitude of the transverse section of the cylinder, neglecting that part of the same which may arise from the obliquity of the motions. For as in Case 1. of Prop. 36. the obliquity of the motions with which the parts of the water in the vessel converged on every side to the hole  $EF$ , hindered the efflux of the water thro' the hole; so in this proposition, the obliquity of the motions, with which the parts of the water, pressed by the antecedent extremity of the cylinder, yield to the pressure and diverge on all sides, retards their passage, thro' the places that lie round that antecedent extremity, towards the hinder parts of the cylinder, and causes the fluid to be moved to a greater distance; which increases the resistance, and that in the same ratio almost in which it diminished the efflux of the water out of the vessel, that is, in the duplicate ratio of 25 to 21, nearly. And as in Case 1. of that Proposition, we made the parts of the water pass thro' the hole  $EF$  perpendicularly and in the greatest plenty, by supposing all the water in the vessel lying round the cataract to be frozen, and that part of the water whose motion was oblique and useless to remain without motion; so in this proposition, that the obliquity of the motions may be taken away, and the parts of the water may give the freest passage to the cylinder, by yielding to it with the most direct and quick motion possible, so that only so much resistance may remain as arises from the magnitude of the transverse section, and which is incapable of diminution, unless by diminishing the diameter of the cylinder; we must conceive those parts of the fluid whose motions  
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are oblique and useleſs, and produce reſiſtance, to be at reſt among themſelves at both extremities of the cylinder, and there to cohere, and be joined to the cylinder. Let  $ABCD$  (Pl. 7. Fig. 6.) be a rectangle, and let  $AE$  and  $BE$  be two parabolic arcs, deſcribed with the axis  $AB$ , and with a latus reſtūm that is to the ſpace  $HG$ , which muſt be deſcribed by the cylinder in falling in order to acquire the velocity with which it moves, as  $HG$  to  $\frac{1}{2}AB$ . Let  $CF$  and  $DF$  be two other parabolic arcs deſcribed with the axis  $CD$ , and a latus reſtūm quadruple of the former; and by the convolution of the figure about the axis  $EF$  let there be generated a ſolid, whoſe middle part  $ABDC$  is the cylinder we are here ſpeaking of, and the extreme parts  $ABE$  and  $CDF$  contain the parts of the fluid, at reſt among themſelves, and concreted into two hard bodies, adhering to the cylinder at each end like a head and tail. Then if this ſolid  $EACFDB$  move in the direction of the length of its axis  $FE$  towards the parts beyond  $E$ , the reſiſtance will be the ſame which we have here determined in this propoſition, nearly; that is, it will have the ſame ratio to the force with which the whole motion of the cylinder may be deſtroyed or generated in the time that it is deſcribing the length  $4AC$  with that motion uniformly continued, as the density of the fluid has to the density of the cylinder, nearly. And (by Cor. 7. Prop. 36.) the reſiſtance muſt be to this force in the ratio of 2 to 3, at the leaſt.

LEMMA





## LEMMA V.

*If a cylinder, a sphere, and a sphæroid, of equal breadths be placed successively in the middle of a cylindric canal, so that their axes may coincide with the axis of the canal; these bodies will equally hinder the passage of the water thro' the canal.*

For the spaces, lying between the sides of the canal, and the cylinder, sphere, and sphæroid, thro' which the water passes, are equal; and the water will pass equally thro' equal spaces.

This is true upon the supposition that all the water above the cylinder, sphere, or sphæroid, whose fluidity is not necessary to make the passage of the water the quickest possible, is congealed, as was explained above in Cor. 7. Prop. 36.

## LEMMA VI.

*The same supposition remaining, the forementioned bodies are equally acted on by the water flowing thro' the canal.*

This appears by Lem. 5. and the third law. For the water and the bodies act upon each other mutually, and equally.

## LEMMA VII.

*If the water be at rest in the canal, and these bodies move with equal velocity and the contrary way thro' the canal, their resistances will be equal among themselves.*

This appears from the last Lemma, for the relative motions remain the same among themselves.

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## SCHOLIUM.

The case is the same of all convex and round bodies, whose axes coincide with the axis of the canal. Some difference may arise from a greater or less friction; but in these *lemmata* we suppose the bodies to be perfectly smooth, and the medium to be void of all tenacity and friction; and that those parts of the fluid which by their oblique and superfluous motions may disturb, hinder, and retard the flux of the water thro' the canal, are at rest amongst themselves; being fixed like water by frost, and adhering to the fore and hinder parts of the bodies in the manner explained in the Scholium of the last Proposition. For in what follows, we consider the very least resistance that round bodies described with the greatest given transverse sections can possibly meet with.

Bodies swimming upon fluids, when they move straight forwards, cause the fluid to ascend at their fore parts and subside at their hinder parts, especially if they are of an obtuse figure; and thence they meet with a little more resistance than if they were acute at the head and tail. And bodies moving in elastic fluids, if they are obtuse behind and before, condense the fluid a little more at their fore parts, and relax the same at their hinder parts; and therefore meet also with a little more resistance than if they were acute at the head and tail. But in these lemma's and propositions we are not treating of elastic, but non-elastic fluids; not of bodies floating on the surface of the fluid, but deeply immersed therein. And when the resistance of bodies in non-elastic fluids is once known, we may then augment this resistance a little in elastic fluids, as our air; and in the surfaces of stagnating fluids, as lakes and seas.

PROP-

## PROPOSITION XXXVIII. THEOREM XXX.

*If a globe move uniformly forward in a compressed, infinite, and non-elastic fluid, its resistance is to the force by which its whole motion may be destroyed or generated in the time that it describes eight third parts of its diameter, as the density of the fluid to the density of the globe, very nearly.*

For the globe is to its circumscribed cylinder as two to three; and therefore the force which can destroy all the motion of the cylinder while the same cylinder is describing the length of four of its diameters, will destroy all the motion of the globe while the globe is describing two thirds of this length, that is, eight third parts of its own diameter. Now the resistance of the cylinder is to this force very nearly as the density of the fluid to the density of the cylinder or globe (by Prop. 37.) and the resistance of the globe is equal to the resistance of the cylinder (by Lem. 5, 6, 7.)

*Q. E. D.*

COR. 1. The resistances of globes in infinite compressed mediums are in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the density of the mediums.

COR. 2. The greatest velocity with which a globe can descend by its comparative weight thro' a resisting fluid, is the same which it may acquire by falling with the same weight, and without any resistance, and in its fall describing a space that is to four third parts of its diameter, as the density of the globe to the density of the fluid. For the globe in the time of its fall, moving with the velocity acquired in falling, will describe a space

space that will be to eight third parts of its diameter as the density of the globe to the density of the fluid; and the force of its weight which generates this motion, will be to the force that can generate the same motion in the time that the globe describes eight third parts of its diameter, with the same velocity as the density of the fluid to the density of the globe; and therefore (by this Proposition) the force of weight will be equal to the force of resistance, and therefore cannot accelerate the globe.

COR. 3. If there be given both the density of the globe and its velocity at the beginning of the motion, and the density of the compressed quiescent fluid in which the globe moves; there is given at any time both the velocity of the globe and its resistance, and the space described by it, (by Cor. 7. Prop. 35.)

COR. 4. A globe moving in a compressed quiescent fluid of the same density with itself, will lose half its motion before it can describe the length of two of its diameters, (by the same Cor. 7.)

### PROPOSITION XXXIX. THEOREM XXXI.

*If a globe move uniformly forward thro' a fluid inclosed and compressed in a cylindric canal, its resistance is to the force by which its whole motion may be generated or destroyed in the time in which it describes eight third parts of its diameter, in a ratio compounded of the ratio of the orifice of the canal, to the excess of that orifice above half the greatest circle of the globe; and the duplicate ratio of the orifice of the canal, to the excess of that orifice above the greatest circle of the globe; and the ratio of the density of the fluid to the density of the globe, nearly.*

But

This appears by cor. 2. prop. 37. and the demonstration proceeds in the same manner as in the foregoing proposition.

# SCHOLIUM.

In the two last propositions we suppose (as was done before in lem. 5.) that all the water which precedes the globe, and whose fluidity increases the resistance of the same, is congealed. Now if that water becomes fluid, it will somewhat increase the resistance. But in these propositions that increase is so small, that it may be neglected, because the convex superficies of the globe produces the very same effect almost as the congelation of the water.

## PROPOSITION XL. PROBLEM IX.

*To find by phenomena the resistance of a globe moving through a perfectly fluid compressed medium.*

Let A be the weight of the globe in vacuo, B its weight in the resisting medium, D the diameter of the globe, F a space which is to  $\frac{4}{3}D$  as the density of the globe to the density of the medium, that is, as A to A—B, G the time in which the globe falling with the weight B without resistance describes the space F, and H the velocity which the body acquires by that fall. Then H will be the greatest velocity with which the globe can possibly descend with the weight B in the resisting medium, by cor. 2. prop. 38; and the resistance which the globe meets with, when descending with that velocity, will be equal to its weight B: and the resistance it meets with, in any other velocity, will be to the weight B in the duplicate ratio of that velocity to the greatest velocity H, by cor. 1. prop. 38.

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This is the resistance that arises from the inactivity of the matter of the fluid. That resistance which arises from the elasticity, tenacity, and friction of its parts, may be thus investigated.

Let the globe be let fall so that it may descend in the fluid by the weight  $B$ ; and let  $P$  be the time of falling, and let that time be expressed in seconds, if the time  $G$  be given in seconds. Find the absolute Number

$N$  agreeing to the logarithm  $0,4342944819 \frac{2P}{G}$ , and

let  $L$  be the logarithm of the number  $\frac{N-1}{N}$ : and the

velocity acquir'd in falling will be  $\frac{N-1}{N}H$ , and the

height described will be  $\frac{2PF}{G} - 1,3862943611F +$

$4,605170186LF$ . If the fluid be of a sufficient depth, we may neglect the term  $4,605170186LF$ ;

and  $\frac{2PF}{G} - 1,3862943611F$  will be the altitude descri-

bed, nearly. These things appear by prop. 9. book 2. and its corollaries, and are true upon this supposition, that the globe meets with no other resistance but that which arises from the inactivity of matter. Now if it really meet with any resistance of another kind, the descent will be slower, and from the quantity of that retardation will be known the quantity of this new resistance.

That the velocity and descent of a body falling in a fluid might more easily be known, I have composed the following table; the first column of which denotes the times of descent, the second shews the velocities acquir'd in falling, the greatest velocity being 100000000, the third exhibits the spaces described by falling in those times,  $2F$  being the space which the body describes in the time  $G$  with the greatest velocity,

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and

and the fourth gives the spaces described with the greatest velocity in the same times. The numbers in the fourth column are  $\frac{2P}{G}$ , and by subducting the number 1,3862944—4,6051702L, are found the numbers in the third column; and these numbers must be multiplied by the space F to obtain the spaces described in falling. A fifth column is added to all these, containing the spaces described in the same times by a body falling in vacuo with the force of B its comparative weight.

The Times P	Velocities of the body falling in the fluid.	The spaces descri- bed in falling in the fluid.	The spaces described with the greatest mo- tion.	The spaces de- scribed by falling in vacuo.
0,001 G	99999 $\frac{2}{3}$	0,000001 F	0,002 F	0,000001 F
0,01 G	999967	0,0001 F	0,02 F	0,0001 F
0,1 G	9966799	0,0099834 F	0,2 F	0,01 F
0,2 G	19737532	0,0397361 F	0,4 F	0,04 F
0,3 G	29131261	0,0886815 F	0,6 F	0,09 F
0,4 G	37994896	0,1559070 F	0,8 F	0,16 F
0,5 G	46211716	0,2402290 F	1,0 F	0,25 F
0,6 G	53704957	0,3402706 F	1,2 F	0,36 F
0,7 G	60436778	0,4545405 F	1,4 F	0,49 F
0,8 G	66403677	0,5815071 F	1,6 F	0,64 F
0,9 G	71629787	0,7196609 F	1,8 F	0,81 F
1 G	76159416	0,8675617 F	2 F	1 F
2 G	96402758	2,6500055 F	4 F	4 F
3 G	99505475	4,6186570 F	6 F	9 F
4 G	99932930	6,6143765 F	8 F	16 F
5 G	99990920	8,6137964 F	10 F	25 F
6 G	99998771	10,6137179 F	12 F	36 F
7 G	99999834	12,6137073 F	14 F	49 F
8 G	99999980	14,6137059 F	16 F	64 F
9 G	99999997	16,6137057 F	18 F	81 F
10 G	99999999 $\frac{1}{3}$	18,6137056 F	20 F	100 F

## SCHOLIUM.

In order to investigate the resistances of fluids from experiments, I procured a square wooden vessel, whose length and breadth on the inside was 9 inches *English* measure, and its depth 9 foot  $\frac{1}{2}$ ; this I filled with rain-water: and having provided globes made up of wax, and lead included therein, I noted the times of the descents of these globes, the height through which they descended being 112 inches. A solid cubic foot of *English* measure contains 76 pounds *Troy* weight of rain-water; and a solid inch contains  $\frac{1}{3}\frac{2}{6}$  ounces *Troy* weight or  $253\frac{1}{3}$  grains; and a globe of water of one inch in diameter contains 132,645 grains in air, or 132,8 grains in vacuo; and any other globe will be as the excess of its weight in vacuo above its weight in water.

EXPER. I. A globe whose weight was  $156\frac{1}{4}$  grains in air, and 77 grains in water, described the whole height of 112 inches in 4 seconds. And, upon repeating the experiment, the globe spent again the very same time of 4 seconds in falling.

The weight of this globe in vacuo is  $156\frac{1}{3}\frac{1}{8}$  grains; and excess of this weight above the weight of the globe in water is  $79\frac{1}{3}\frac{1}{8}$  grains. Hence the diameter of the globe appears to be 0,84224 parts of an inch. Then it will be, as that excess to the weight of the globe in vacuo, so is the density of the water to the density of the globe; and so is  $\frac{2}{3}$  parts of the diameter of the globe (*viz.* 2,24597 inches) to the space 2F, which will be therefore 4,4256 inches. Now a globe falling in vacuo with its whole weight of  $156\frac{1}{3}\frac{1}{8}$  grains in one second of time will describe  $193\frac{1}{3}$  inches; and falling in water in the same time with the weight of 77 grains without resistance, will describe 95,219 inches; and in the time G which is to one second of time

time in the subduplicate ratio of the space F, or of 2,2128 inches to 95,219 inches, will describe 2,2128 inches, and will acquire the greatest velocity H with which it is capable of descending in water. Therefore the time G is 0,"15244. And in this time G with that greatest velocity H, the globe will describe the space 2F, which is 4,4256 inches; and therefore in 4 seconds will describe a space of 116,1245 inches. Subtract the space 1,3862944F or 3,0676 inches, and there will remain a space of 113,0569 inches, which the globe falling thro' water in a very wide vessel will describe in 4 seconds. But this space, by reason of the narrowness of the wooden vessel beforementioned, ought to be diminished in a ratio compounded of the subduplicate ratio of the orifice of the vessel to the excess of this orifice above half a great circle of the globe, and of the simple ratio of the same orifice to its excess above a great circle of the globe, that is, in a ratio of 1 to 0,9914. This done, we have a space of 112,08 inches, which a globe falling thro' the water in this wooden vessel in 4 seconds of time ought nearly to describe by this theory: but it described 112 inches by the experiment.

EXPER. 2. Three equal globes, whose weights were severally  $76 \frac{1}{3}$  grains in air, and  $5 \frac{1}{6}$  grains in water, were let fall successively; and every one fell thro' the water in 15 seconds of time, describing in its fall a height of 112 inches.

By computation, the weight of each globe in vacuo is  $76 \frac{1}{2}$  grains; the excess of this weight above the weight in water, is 71 grains  $\frac{1}{8}$ ; the diameter of the globe 0,81296 of an inch:  $\frac{2}{3}$  parts of this diameter 2,16789 inches; the space 2F is 2,3217 inches; the space which a globe of  $5 \frac{1}{6}$  grains in weight would describe in one second without resistance, 12,808 inches, and the time G 0",301056. Therefore the globe with the greatest velocity it is capable of receiving

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from



from a weight of  $5 \frac{1}{6}$  grains in its descent thro' water, will describe in the time  $0''{,}301056$  the space of  $2,3217$  inches; and in  $15$  seconds the space  $115,678$  inches. Subtract the space  $1,3862944F$  or  $1,609$  inches, and there remains the space  $114,069$  inches; which therefore the falling globe ought to describe in the same time, if the vessel were very wide. But because our vessel was narrow, the space ought to be diminished by about  $0,895$  of an inch. And so the space will remain  $113,174$  inches, which a globe falling in this vessel ought nearly to describe in  $15$  seconds by the theory. But by the experiment it described  $112$  inches. The difference is not sensible.

EXPER. 3. Three equal globes, whose weights were severally  $121$  grains in air, and  $1$  grain in water, were successively let fall; and they fell thro' the water in the times  $46''$ ,  $47''$ , and  $50''$ , describing a height of  $112$  inches.

By the theory these globes ought to have fallen in about  $40''$ . Now whether their falling more slowly were occasion'd from hence, that in slow motions the resistance arising from the force of inactivity, does really bear a less proportion to the resistance arising from other causes; or whether it is to be attributed to little bubbles that might chance to stick to the globes, or to the rarefaction of the wax by the warmth of the weather, or of the hand that let them fall; or, lastly, whether it proceeded from some insensible errors in weighing the globes in the water, I am not certain. Therefore the weight of the globe in water should be of several grains, that the experiment may be certain, and to be depended on.

EXPER. 4. I began the foregoing experiments to investigate the resistances of fluids, before I was acquainted with the theory laid down in the propositions immediately preceding. Afterwards, in order to examine the theory after it was discovered, I procured a wooden

wooden vessel, whose breadth on the inside was  $8\frac{2}{3}$  inches, and its depth 15 feet and  $\frac{1}{3}$ . Then I made four globes of wax, with lead included, each of which weighed  $139\frac{1}{4}$  grains in air, and  $7\frac{1}{8}$  grains in water. These I let fall, measuring the times of their falling in the water with a pendulum oscillating to half seconds. The globes were cold, and had remained so some time, both when they were weighed and when they were let fall; because warmth rarefies the wax, and by rarefying it diminishes the weight of the globe in the water; and wax, when rarefied, is not instantly reduced by cold to its former density. Before they were let fall, they were totally immersed under water, lest, by the weight of any part of them that might chance to be above the water, their descent should be accelerated in its beginning. Then, when after their immersion they were perfectly at rest, they were let go with the greatest care, that they might not receive any impulse from the hand that let them down. And they fell successively in the times of  $47\frac{1}{2}$ ,  $48\frac{1}{2}$ , 50 and 51 oscillations, describing a height of 15 feet and 2 inches. But the weather was now a little colder than when the globes were weighed, and therefore I repeated the experiment another day; and then the globes fell in the times of 49,  $49\frac{1}{2}$ , 50 and 53; and at a third trial in the times of  $49\frac{1}{2}$ , 50, 51 and 53 oscillations. And by making the experiment several times over, I found that the globes fell mostly in the times of  $49\frac{1}{2}$  and 50 oscillations. When they fell slower, I suspect them to have been retarded by striking against the sides of the vessel.

Now, computing from the theory, the weight of the globe in vacuo is  $139\frac{1}{4}$  grains. The excess of this weight above the weight of the globe in water  $132\frac{1}{8}$  grains, the diameter of the globe 0,99868 of an inch,  $\frac{1}{3}$  parts of the diameter 2,66315 inches, the space 2F 2,8066 inches, the space which a globe weighing  $7\frac{1}{8}$  grains falling without resistance describes

in a second of time 9,88164 inches, and the time  $60^{\circ}, 376843$ . Therefore the globe with the greatest velocity with which it is capable of descending thro' the water by the force of a weight of  $7\frac{1}{8}$  grains will in the time  $60^{\circ}, 376843$  describe a space of 2,8066 inches, and in one second of time a space of 7,44766 inches, and in the time  $25''$ , or in 50 oscillations the space 186,1915 inches. Subtract the space 1,386294 F or 1,9454 inches, and there will remain the space 184,2461 inches, which the globe will describe in that time in a very wide vessel. Because our vessel was narrow, let this space be diminished in a ratio compounded of the subduplicate ratio of the orifice of the vessel to the excess of this orifice above half a great circle of the globe, and of the simple ratio of the same orifice to its excess above a great circle of the globe; and we shall have the space of 181,86 inches, which the globe ought by the theory to describe in this vessel in the time of 50 oscillations, nearly. But it described the space of 182 inches, by experiment, in  $49\frac{1}{2}$  or 50 oscillations.

EXPER. 5. Four globes, weighing  $154\frac{1}{8}$  grains in air, and  $21\frac{1}{2}$  grains in water, being let fall several times, fell in the times of  $28\frac{1}{2}$ , 29,  $29\frac{1}{2}$ , and 30, and sometimes of 31, 32, and 33 oscillations, describing a height of 15 feet and 2 inches.

They ought by the theory to have fallen in the time of 29 oscillations, nearly.

EXPER. 6. Five globes, weighing  $212\frac{3}{8}$  grains in air, and  $79\frac{1}{2}$  in water, being several times let fall, fell in the times of 15,  $15\frac{1}{2}$ , 16, 17, and 18 oscillations, describing a height of 15 feet and 2 inches.

By the theory they ought to have fallen in the time of 15 oscillations, nearly.

EXPER. 7. Four globes weighing  $293\frac{1}{8}$  grains in air, and 35 grains  $\frac{7}{8}$  in water, being let fall several times, fell in the times of  $29\frac{1}{2}$ , 30,  $30\frac{1}{2}$ , 31, 32, and 33 oscillations, describing a height of 15 feet and 1 inch and  $\frac{1}{2}$ . By

By the theory they ought to have fallen in the time of 28 oscillations, nearly.

In searching for the cause that occasioned these globes of the same weight and magnitude to fall, some swifter and some slower, I hit upon this; that the globes, when they were first let go and began to fall, oscillated about their centres, that side which chanced to be the heavier descending first, and producing an oscillating motion. Now by oscillating thus, the globe communicates a greater motion to the water, than if it descended without any oscillations; and by this communication loses part of its own motion with which it should descend; and therefore as this oscillation is greater or less it will be more or less retarded. Besides the globe always recedes from that side of itself which is descending in the oscillation, and by so receding comes nearer to the sides of the vessel so as even to strike against them sometimes. And the heavier the globes are, the stronger this oscillation is; and the greater they are, the more is the water agitated by it. Therefore to diminish this oscillation of the globes, I made new ones of lead and wax, sticking the lead in one side of the globe very near its surface; and I let fall the globe in such a manner, that as near as possible, the heavier side might be lowest at the beginning of the descent. By this means the oscillations became much less than before, and the times in which the globes fell were not so unequal: as in the following experiments.

EXPER. 8. Four globes weighing 139 grains in air and  $6\frac{1}{2}$  in water, were let fall several times, and fell mostly in the time of 51 oscillations, never in more than 52, or in fewer than 50; describing a height of 182 inches.

By the theory they ought to fall in about the time of 52 oscillations.

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EXPER. 9. Four globes weighing  $273\frac{1}{4}$  grains in air, and  $140\frac{1}{4}$  in water, being several times let fall, fell in never fewer than 12, and never more than 13 oscillations, describing a height of 182 inches.

These globes by the theory ought to have fallen in the time of  $11\frac{1}{3}$  oscillations, nearly.

EXPER. 10. Four globes, weighing 384 grains in air and  $119\frac{1}{2}$  in water, being let fall several times, fell in the times of  $17\frac{1}{4}$ , 18,  $18\frac{1}{2}$ , and 19 oscillations, describing a height of  $181\frac{1}{2}$  inches. And when they fell in the time of 19 oscillations, I sometimes heard them hit against the sides of the vessel before they reached the bottom.

By the theory they ought to have fallen in the time of  $15\frac{2}{3}$  oscillations, nearly.

EXPER. 11. Three equal globes, weighing 48 grains in the air, and  $3\frac{2}{3}$  in water, being several times let fall, fell in the times of  $43\frac{1}{2}$ , 44,  $44\frac{1}{2}$ , 45 and 46 oscillations, and mostly in 44 and 45, describing a height of 182 inches  $\frac{1}{2}$ , nearly.

By the theory they ought to have fallen in the time of 46 oscillations and  $\frac{1}{3}$ , nearly.

EXPER. 12. Three equal globes, weighing 141 grains in air and  $4\frac{1}{8}$  in water, being let fall several times, fell in the times of 61, 62, 63, 64 and 65 oscillations, describing a space of 182 inches.

And by the theory they ought to have fallen in  $64\frac{1}{2}$  oscillations, nearly.

From these experiments it is manifest, that when the globes fell slowly, as in the second, fourth, fifth, eighth, eleventh, and twelfth experiments, the times of falling are rightly exhibited by the theory; but when the globes fell more swiftly as in the sixth, ninth, and tenth experiments, the resistance was somewhat greater than in the duplicate ratio of the velocity. For the globes in falling oscillate a little; and this oscillation, in those globes that are light and fall slowly,

slowly, soon ceases by the weakness of the motion; but in greater and heavier globes, the motion being strong, it continues longer; and is not to be checked by the ambient water, till after several oscillations. Besides, the more swiftly the globes move, the less are they pressed by the fluid at their hinder parts; and if the velocity be perpetually increased, they will at last leave an empty space behind them, unless the compression of the fluid be increased at the same time. For the compression of the fluid ought to be increased (by Prop. 32 and 33.) in the duplicate ratio of the velocity, in order to preserve the resistance in the same duplicate ratio. But because this is not done, the globes that move swiftly are not so much pressed at their hinder parts as the others; and by the defect of this pressure it comes to pass that their resistance is a little greater than in a duplicate ratio of their velocity.

So that the theory agrees with the phenomena of bodies falling in water; it remains that we examine the phenomena of bodies falling in air.

EXPER. 13. From the top of *St. Paul's Church in London* in *June 1710.* there were let fall together two glass globes, one full of quicksilver, the other of air; and in their fall they described a height of 220 *English* feet. A wooden table was suspended upon iron hinges on one side, and the other side of the same was supported by a wooden pin. The two globes lying upon this table were let fall together by pulling out the pin by means of an iron wire reaching from thence quite down to the ground; so that, the pin being removed, the table, which had then no support but the iron hinges, fell downwards; and turning round upon the hinges, gave leave to the globes to drop off from it. At the same instant, with the same pull of the iron wire that took out the pin, a pendulum oscillating to seconds was let go, and began to oscillate. The diameters and weights

of the globes, and their times of falling, are exhibited in the following table.

<i>The globes filled with mercury.</i>			<i>The globes full of air.</i>		
<i>Weights.</i>	<i>Diameters.</i>	<i>Times in falling.</i>	<i>Weights.</i>	<i>Diameters.</i>	<i>Times in falling.</i>
908 Grains	0,8 of an inch	4"	510 Grains	5,1 inches	8" $\frac{1}{2}$
983	0,8	4—	642	5,2	8
866	0,8	4	599	5,1	8
747	0,75	4+	515	5,0	8 $\frac{1}{2}$
808	0,75	4	483	5,0	8 $\frac{1}{2}$
784	0,75	4+	641	5,2	8

But the times observed must be corrected; for the globes of mercury (by *Galileo's* theory) in 4 seconds of time, will describe 257 *English* feet, and 220 feet in only 3" 42". So that the wooden table, when the pin was taken out, did not turn upon its hinges so quickly as it ought to have done; and the slowness of that revolution hindered the descent of the globes at the beginning. For the globes lay about the middle of the table, and indeed were rather nearer to the axis upon which it turned, than to the pin. And hence the times of falling were prolonged about 18"; and therefore ought to be corrected by subducting that excess, especially in the larger globes, which, by reason of the largeness of their diameters, lay longer upon the revolving table than the others. This being done, the times in which the six larger globes fell, will come forth 8" 12", 7" 42", 7" 42", 7" 57", 8" 12", and 7" 42".

Therefore the fifth in order among the globes that were full of air, being 5 inches in diameter, and 483 grains in weight, fell in 8" 12", describing a space of 220 feet. The weight of a bulk of water equal to this globe is 16600 grains; and the weight of an equal bulk of air is  $\frac{16600}{86000}$  grains, or 19  $\frac{1}{10}$  grains; and therefore the weight of the globe in vacuo is 502  $\frac{1}{10}$  grains; and

and this weight is to the weight of a bulk of air equal to the globe as  $502 \frac{1}{10}$  to  $19 \frac{1}{10}$ , and so is  $2F$  to  $\frac{2}{3}$  of the diameter of the globe, that is, to  $13 \frac{1}{3}$  inches. Whence  $2F$  becomes 28 feet 11 inches. A globe falling in vacuo with its whole weight of  $502 \frac{1}{10}$  grains, will in one second of time describe  $193 \frac{1}{3}$  inches as above; and with the weight of 483 grains will describe 185,905 inches; and with that weight 483 grains in vacuo will describe the space  $F$  or 14 feet  $5 \frac{1}{2}$  inches, in the time of  $57'' 58'''$ , and acquire the greatest velocity it is capable of descending with in the air. With this velocity the globe in  $8'' 12'''$  of time will describe 245 feet and  $5 \frac{1}{3}$  inches. Subtract  $1,3863F$  or 20 feet and  $\frac{1}{2}$  an inch, and there remain 225 feet 5 inches. This space therefore the falling globe ought by the theory to describe in  $8'' 12'''$ . But by the experiment it described a space of 220 feet. The difference is insensible.

By like calculations applied to the other globes full of air, I composed the following table.

The weights of the globes.	The diameters.	The times of falling from a height of 220 foot.	The spaces which they would describe by the theory.	The Excesses.
510 grains	5,1 inches	8'' 12'''	226 foot 11 inches	6 foot 11 inches
642	5,2	7 42	230 9	10 9
599	5,1	7 42	227 10	7 10
515	5	7 57	224 5	4 5
483	5	8 12	225 5	5 5
641	5,2	7 42	230 7	10 7

EXPER. 14. Anno 1719. in the month of July, Dr. Desaguliers made some experiments of this kind again, by forming hogs bladders into sphærical orbs; which was done by means of a concave wooden sphere, which the bladders, being wetted well first, were put into. After that, being blown full of air, they were obliged



obliged to fill up the spherical cavity that contained them ; and then, when dry, were taken out. These were let fall from the lantern on the top of the cupola of the same church ; namely, from a height of 272 feet ; and at the same moment of time there was let fall a leaden globe whose weight was about 2 pounds Troy weight. And in the mean time some persons standing in the upper part of the church where the globes were let fall, observed the whole times of falling ; and others standing on the ground observed the differences of the times between the fall of the leaden weight, and the fall of the bladder. The times were measured by pendulums oscillating to half seconds. And one of those that stood upon the ground had a machine vibrating four times in one second ; and another had another machine accurately made with a pendulum vibrating four times in a second also. One of those also who stood at the top of the church had a like machine. And these instruments were so contrived, that their motions could be stopped or renewed at pleasure. Now the leaden globe fell in about four seconds and  $\frac{1}{4}$  of time ; and from the addition of this time to the difference of time above spoken of, was collected the whole time in which the bladder was falling. The times which the five bladders spent in falling after the leaden globe had reached the ground were the first time,  $14\frac{3}{4}"$ ,  $12\frac{3}{4}"$ ,  $14\frac{5}{8}"$ ,  $17\frac{3}{4}"$ , and  $16\frac{7}{8}"$  ; and the second time  $14\frac{1}{2}"$ ,  $14\frac{1}{4}"$ ,  $14"$ ,  $19"$  and  $16\frac{1}{4}"$ . Add to these  $4\frac{1}{4}"$ , the time in which the leaden globe was falling, and the whole times in which the five bladders fell, were, the first time  $19"$ ,  $17"$ ,  $18\frac{7}{8}"$ ,  $22"$  and  $21\frac{1}{8}"$  ; and the second time,  $18\frac{1}{4}"$ ,  $18\frac{1}{2}"$ ,  $18\frac{1}{4}"$ ,  $23\frac{1}{4}"$  and  $21"$ . The times observed at the top of the church were, the first time,  $19\frac{3}{8}"$ ,  $17\frac{1}{4}"$ ,  $18\frac{3}{4}"$ ,  $22\frac{5}{8}"$  and  $21\frac{5}{8}"$  ; and the second time,  $19"$ ,  $18\frac{1}{8}"$ ,  $18\frac{1}{8}"$ ,  $24"$  and  $21\frac{1}{4}"$ . But the bladders did not always fall di-

directly down, but sometimes fluttered a little in the air, and waved to and fro as they were descending. And by these motions the times of their falling were prolonged, and increased by half a second sometimes, and sometimes by a whole second. The second and fourth bladder fell most directly the first time, and the first and third the second time. The fifth bladder was wrinkled, and by its wrinkles was a little retarded. I found their diameters by their circumferences measured with a very fine thread wound about them twice. In the following table I have compared the experiments with the theory; making the density of air to be to the density of rain-water as 1 to 860, and computing the spaces which by the theory the globes ought to describe in falling.

<i>The weights of the bladders.</i>	<i>The diameters.</i>	<i>The times of falling from a height of 272 foot.</i>	<i>The spaces which by the theory ought to have been described in these times.</i>	<i>The difference between the theory and the experiments.</i>
128 grains	5,28 inches	19"	27 1 foot 11 inches	— 0 foot 1 inch
156	5,19	17	272 0 $\frac{1}{2}$	+ 0 0 $\frac{1}{2}$
137 $\frac{1}{4}$	5,3	18 $\frac{1}{2}$	272 7	+ 0 7
97 $\frac{1}{2}$	5,26	22	277 4	+ 5 4
99 $\frac{1}{8}$	5	21 $\frac{1}{8}$	282 0	+ 10 0

Our theory therefore exhibits rightly, within a very little, all the resistance that globes moving either in air or in water meet with; which appears to be proportional to the densities of the fluids in globes of equal velocities and magnitudes.

In the scholium subjoined to the sixth section, we shewed by experiments of pendulums, that the resistances of equal and equally swift globes moving in air, water, and quicksilver, are as the densities of the fluids. We here prove the same more accurately by experiments of bodies falling in air and water. For pendulums at each oscillation excite a motion in the fluid always

ways contrary to the motion of the pendulum in its return ; and the resistance arising from this motion, as also the resistance of the thread by which the pendulum is suspended, makes the whole resistance of a pendulum greater than the resistance deduced from the experiments of falling bodies. For by the experiments of pendulums described in that scholium, a globe of the same density as water in describing the length of its semidiameter in air would lose the  $\frac{1}{3342}$  part of its motion. But by the theory delivered in this seventh section, and confirmed by experiments of falling bodies, the same globe in describing the same length would lose only a part of its motion equal to  $\frac{1}{4586}$  supposing the density of water to be to the density of air as 860 to 1. Therefore the resistances were found greater by the experiments of pendulums (for the reasons just mentioned) than by the experiments of falling globes ; and that in the ratio of about 4 to 3. But yet since the resistances of pendulums oscillating in air, water, and quicksilver, are alike increased by like causes, the proportion of the resistances in these mediums will be rightly enough exhibited by the experiments of pendulums, as well as by the experiments of falling bodies. And from all this it may be concluded, that the resistances of bodies, moving in any fluids whatsoever, tho' of the most extreme fluidity, are, *ceteris paribus*, as the densities of the fluids.

These things being thus established, we may now determine what part of its motion any globe projected in any fluid whatsoever would nearly lose in a given time. Let  $D$  be the diameter of the globe, and  $V$  its velocity at the beginning of its motion, and  $T$  the time in which a globe with the velocity  $V$  can describe in vacuo a space that is to the space  $\frac{1}{2} D$  as the density of the globe to the density of the fluid ; and the globe projected in that fluid will, in any other time

time  $t$ , lose the part  $\frac{tV}{T+t}$ , the part  $\frac{TV}{T+t}$  remaining; and will describe a space, which may be to that described in the same time in vacuo with the uniform velocity  $V$ , as the logarithm of the number  $\frac{T+t}{T}$  multiplied by the number 2,302585093 is to

the number  $\frac{t}{T}$ , by cor. 7. prop. 35. In flow motions the resistance may be a little less, because the figure of a globe is more adapted to motion than the figure of a cylinder described with the same diameter. In swift motions the resistance may be a little greater, because the elasticity and compression of the fluid do not increase in the duplicate ratio of the velocity. But these little niceries I take no notice of.

And tho' air, water, quicksilver, and the like fluids, by the division of their parts in infinitum, should be subtilized and become mediums infinitely fluid; nevertheless, the resistance they would make to projected globes would be the same. For the resistance consider'd in the preceding propositions, arises from the inactivity of the matter; and the inactivity of matter is essential to bodies, and always proportional to the quantity of matter. By the division of the parts of the fluid, the resistance arising from the tenacity and friction of the parts may be indeed diminished; but the quantity of matter will not be at all diminished by this division; and if the quantity of matter be the same, its force of inactivity will be the same; and therefore the resistance here spoken of will be the same, as being always proportional to that force. To diminish this resistance, the quantity of matter in the spaces thro' which the bodies move must be diminished. And therefore the celestial spaces, thro' which the globes of the Planets and Comets are perpetually passing towards all parts,

with the utmost freedom, and without the least sensible diminution of their motion, must be utterly void of any corporeal fluid, excepting perhaps some extremely rare vapours, and the rays of light.

Projectiles excite a motion in fluids as they pass thro' them; and this motion arises from the excess of the pressure of the fluid at the fore-parts of the projectile above the pressure of the same at the hinder parts; and cannot be less in mediums infinitely fluid, than it is in air, water, and quicksilver, in proportion to the density of matter in each. Now this excess of pressure does, in proportion to its quantity, not only excite a motion in the fluid, but also acts upon the projectile so as to retard its motion: and therefore the resistance in every fluid is as the motion excited by the projectile in the fluid; and cannot be less in the most subtile æther in proportion to the density of that æther, than it is in air, water, and quicksilver, in proportion to the densities of those fluids.



S E C.



## SECTION VIII.

### *Of motion propagated thro' fluids.*

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#### PROPOSITION XLI. THEOREM XXXII.

*A pressure is not propagated thro' a fluid in rectilinear directions, unless where the particles of the fluid lie in a right line. Pl. 8.*

Fig. 1.

If the particles *a, b, c, d, e*, lie in a right line, the pressure may be indeed directly propagated from *a* to *e*; but then the particle *e* will urge the obliquely posited particles *f* and *g* obliquely, and those particles *f* and *g* will not sustain this pressure, unless they be supported by the particles *h* and *k* lying beyond them; but the particles that support them, are also pressed by them; and those particles cannot sustain that pressure, without being supported by, and pressing upon, those particles that lie still farther, as *l* and *m*, and so on in infinitum. Therefore the pressure, as soon as it is propagated to particles that lie out of right lines, begins to deflect towards one hand and t'other, and will be propagated obliquely in infinitum; and after it has begun to be propagated obliquely, if it reaches more distant particles lying out of the right line, it will deflect again on each hand; and this it will do as

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often

often as it lights on particles that do not lie exactly in a right line. *Q. E. D.*

COR. If any part of a pressure, propagated thro' a fluid from a given point, be intercepted by any obstacle; the remaining part, which is not intercepted, will deflect into the spaces behind the obstacle. This may be demonstrated also after the following manner. Let a pressure be propagated from the point *A* (*Pl. 8. Fig. 2.*) towards any part, and, if it be possible, in rectilinear directions; and the obstacle *NBCK* being perforated in *BC*, let all the pressure be intercepted but the coniform part *APQ* passing thro' the circular hole *BC*. Let the cone *APQ* be divided into frustums by the transverse planes *de*, *fg*, *hi*. Then while the cone *ABC*, propagating the pressure, urges the conic frustum *degf* beyond it on the superficies *de*, and this frustum urges the next frustum *fgih* on the superficies *fg*, and that frustum urges a third frustum, and so in infinitum; it is manifest (by the third law) that the first frustum *degf* is, by the reaction of the second frustum *fgih*, as much urged and pressed on the superficies *fg*, as it urges and presses that second frustum. Therefore the frustum *degf* is compressed on both sides, that is, between the cone *Ade* and the frustum *fhig*; and therefore (by case 6. prop. 19.) cannot preserve its figure, unless it be compressed with the same force on all sides. Therefore with the same force with which it is pressed on the superficies *de*, *fg*, it will endeavour to break forth at the sides *df*, *eg*; and there (being not in the least tenacious or hard, but perfectly fluid) it will run out, expanding itself, unless there be an ambient fluid opposing that endeavour. Therefore, by the effort it makes to run out, it will press the ambient fluid, at its sides *df*, *eg*, with the same force that it does the frustum *fgih*; and therefore the pressure will be propagated as much from the sides *df*, *eg* into the spaces *NO*, *KL* this way and that way, as it

is

is propagated from the superficies  $fg$  towards  $PQ$ .  
 $Q.E.D.$

PROPOSITION XLII. THEOREM XXXIII.

*All motion propagated thro' a fluid, diverges from a rectilinear progress into the unmoved spaces.* Pl. 8. Fig. 3.

CASE I. Let a motion be propagated from the point  $A$  thro' the hole  $BC$ , and, if it be possible, let it proceed in the conic space  $BCQP$  according to right lines diverging from the point  $A$ . And let us first suppose this motion to be that of waves in the surface of standing water; and let  $de, fg, hi, kl, \&c.$  be the tops of the several waves, divided from each other by as many intermediate valleys or hollows. Then, because the water in the ridges of the waves is higher than in the unmoved parts of the fluid  $KL, NO$ , it will run down from off the tops of those ridges  $e, g, i, l, \&c. d, f, h, k, \&c.$  this way and that way towards  $KL$  and  $NO$ ; and because the water is more depressed in the hollows of the waves than in the unmoved parts of the fluid  $KL, NO$ , it will run down into those hollows out of those unmoved parts. By the first deflux the ridges of the waves will dilate themselves this way and that way, and be propagated towards  $KL$  and  $NO$ . And because the motion of the waves from  $A$  towards  $PQ$  is carried on by a continual deflux from the ridges of the waves into the hollows next to them; and therefore cannot be swifter than in proportion to the celerity of the descent; and the descent of the water on each side towards  $KL$  and  $NO$  must be performed with the same velocity; it follows, that the dilatation of the waves on each side towards  $KL$  and  $NO$  will be propagated with the same velocity as the waves them-

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selves



selves go forward with, directly from  $A$  to  $PQ$ . And therefore the whole space this way and that way towards  $KL$  and  $NO$  will be filled by the dilated waves  $rfgr$ ,  $shis$ ,  $tklt$ ,  $umnv$ , &c. *Q.E.D.* That these things are so, any one may find by making the experiment in still water.

CASE 2. Let us suppose that  $de$ ,  $fg$ ,  $hi$ ,  $kl$ ,  $mn$ , represent pulses successively propagated from the point  $A$  thro' an elastic medium. Conceive the pulses to be propagated by successive condensations and rarefactions of the medium, so that the densest part of every pulse may occupy a sphaerical superficies described about the centre  $A$ , and that equal intervals intervene between the successive pulses. Let the lines  $de$ ,  $fg$ ,  $hi$ ,  $kl$ , &c. represent the densest parts of the pulses, propagated thro' the hole  $BC$ ; and because the medium is denser there, than in the spaces on either side towards  $KL$  and  $NO$ , it will dilate itself as well towards those spaces  $KL$ ,  $NO$  on each hand, as towards the rare intervals between the pulses; and thence the medium becoming always more rare next the intervals, and more dense next the pulses, will partake of their motion. And because the progressive motion of the pulses arises from the perpetual relaxation of the denser parts towards the antecedent rare intervals; and since the pulses will relax themselves on each hand towards the quiescent parts of the medium  $KL$ ,  $NO$ , with very near the same celerity; therefore the pulses will dilate themselves on all sides into the unmoved parts  $KL$ ,  $NO$ , with almost the same celerity with which they are propagated directly from the centre  $A$ ; and therefore will fill up the whole space  $KLON$ . *Q.E.D.* And we find the same by experience also in sounds, which are heard tho' a mountain interpose; and if they come into a chamber thro' the window, dilate themselves into all the parts of the room, and are heard in every corner; and not as reflected from the opposite walls, but directly

rectly propagated from the window, as far as our sense can judge.

CASE 3. Let us suppose lastly, that a motion of any kind is propagated from  $A$  thro' the hole  $BC$ . Then since the cause of this propagation is, that the parts of the medium that are near the centre  $A$  disturb and agitate those which lie farther from it; and since the parts which are urged are fluid, and therefore recede every way towards those spaces where they are less pressed, they will by consequence recede towards all the parts of the quiescent medium; as well to the parts on each hand, as  $KL$  and  $NO$ , as to those right before as  $PQ$ : and by this means all the motion, as soon as it has passed thro' the hole  $BC$ , will begin to dilate itself, and from thence, as from its principle and centre, will be propagated directly every way. *Q.E.D.*

### PROPOSITION XLIII. THEOREM XXXIV.

*Every tremulous body in an elastic medium propagates the motion of the pulses on every side right forward; but in a non-elastic medium excites a circular motion.*

CASE 1. The parts of the tremulous body alternately going and returning, do in going urge and drive before them those parts of the medium that lie nearest, and by that impulse compress and condense them; and in returning suffer those compressed parts to recede again and expand themselves. Therefore the parts of the medium that lie nearest to the tremulous body, move to and fro by turns, in like manner as the parts of the tremulous body itself do; and for the same cause that the parts of this body agitate these parts of the medium, these parts being agitated by like tremors,

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will

will in their turn agitate others next to themselves, and these others agitated in like manner, will agitate those that lie beyond them, and so on in infinitum. And in the same manner as the first parts of the medium were condensed in going, and relaxed in returning, so will the other parts be condensed every time they go, and expand themselves every time they return. And therefore they will not be all going and all returning at the same instant, (for in that case they would always preserve determined distances from each other, and there could be no alternate condensation and rarefaction;) but since in the places where they are condensed, they approach to, and in the places where they are rarefied, recede from, each other; therefore some of them will be going while others are returning; and so on in infinitum. The parts so going, and in their going condensed, are pulses, by reason of the progressive motion with which they strike obstacles in their way; and therefore the successive pulses produced by a tremulous body, will be propagated in rectilinear directions; and that at nearly equal distances from each other, because of the equal intervals of time in which the body, by its several tremors, produces the several pulses. And tho' the parts of the tremulous body go and return in some certain and determinate direction, yet the pulses propagated from thence thro' the medium, will dilate themselves towards the sides, by the foregoing proposition; and will be propagated on all sides from that tremulous body, as from a common centre, in superficies nearly sphaerical and concentrical. An example of this we have in waves excited by shaking a finger in water, which proceed not only forwards and backwards agreeably to the motion of the finger, but spread themselves in the manner of concentric circles all round the finger, and are propagated on every side. For the gravity of the water supplies the place of elastic force.

CASE

CASE 2. If the medium be not elastic, then, because its parts cannot be condensed by the pressure arising from the vibrating parts of the tremulous body, the motion will be propagated in an instant towards the parts where the medium yields most easily, that is, to the parts which the tremulous body leaves for some time vacuous behind it. The case is the same with that of a body projected in any medium whatever. A medium yielding to projectiles does not recede in infinitum, but with a circular motion comes round to the spaces which the body leaves behind it. Therefore as often as a tremulous body tends to any part, the medium yielding to it comes round in a circle to the parts which the body leaves; and as often as the body returns to the first place, the medium will be driven from the place it came round to, and return to its original place. And tho' the tremulous body be not firm and hard, but every way flexible; yet if it continue of a given magnitude, since it cannot impel the medium by its tremors any where without yielding to it somewhere else; the medium receding from the parts where it is pressed, will always come round in a circle to the parts that yield to it. *Q. E. D.*

COR. 'Tis a mistake therefore to think, as some have done, that the agitation of the parts of flame conduces to the propagation of a pressure in rectilinear directions thro' an ambient medium. A pressure of that kind must be derived, not from the agitation only of the parts of flame, but from the dilatation of the whole.

PRO-

## PROPOSITION XLIV. THEOREM XXXV.

*If water ascend and descend alternately in the erected legs KL, MN of a canal or pipe; and a pendulum be constructed, whose length between the point of suspension and the centre of oscillation is equal to half the length of the water in the canal: I say, that the water will ascend and descend in the same times in which the pendulum oscillates. Pl. 8. Fig. 4.*

I measure the length of the water along the axes of the canal and its legs, and make it equal to the sum of those axes; and take no notice of the resistance of the water, arising from its attrition by the sides of the canal. Let therefore  $AB$ ,  $CD$  represent the mean height of the water in both legs; and when the water in the leg  $KL$  ascends to the height  $EF$ , the water will descend in the leg  $MN$  to the height  $GH$ . Let  $P$  be a pendulous body,  $VP$  the thread,  $V$  the point of suspension,  $RPOS$  the cycloid which the pendulum describes,  $P$  its lowest point,  $PQ$  an arc equal to the height  $AE$ . The force, with which the motion of the water is accelerated and retarded alternately, is the excess of the weight of the water in one leg above the weight in the other; and therefore, when the water in the leg  $KL$  ascends to  $EF$ , and in the other leg descends to  $GH$ , that force is double the weight of the water  $EABF$ , and therefore is to the weight of the whole water as  $AE$  or  $PQ$  to  $VP$  or  $PR$ . The force also with which the body  $P$  is accelerated or retarded in any place as  $Q$  of a cycloid, is (by cor. prop. 51.) to its whole weight, as its distance  $PQ$  from the lowest place  $P$  to the length  $PR$  of the cycloid. Therefore the motive forces of the water and pendulum, descri-

bing

being the equal spaces  $AE$ ,  $PQ$  are as the weights to be moved ; and therefore if the water and pendulum are quiescent at first, those forces will move them in equal times, and will cause them to go and return together with a reciprocal motion. *Q. E. D.*

COR. 1. Therefore the reciprocations of the water in ascending and descending, are all performed in equal times, whether the motion be more or less intense or remiss.

COR. 2. If the length of the whole water in the canal be of  $6\frac{1}{2}$  feet of *French* measure, the water will descend in one second of time, and will ascend in another second, and so on by turns in infinitum ; for a pendulum of  $3\frac{1}{8}$  such feet in length will oscillate in one second of time.

COR. 3. But if the length of the water be increased or diminished, the time of the reciprocation will be increased or diminished in the subduplicate ratio of the length.

## PROPOSITION XLV. THEOREM XXXVI.

*The velocity of waves is in the subduplicate ratio of the breadths.*

This follows from the construction of the following proposition.

## PROPOSITION XLVI. PROBLEM X.

*To find the velocity of waves.*

Let a pendulum be constructed, whose length between the point of suspension and the centre of oscillation is equal to the breadth of the waves ; and in the time that the pendulum will perform one single oscillation,

lation, the waves will advance forward nearly a space equal to their breadth.

That which I call the breadth of the waves is the transverse measure lying between the deepest part of the hollows, or the tops of the ridges. Let *ABCDEF* (*Pl. 8. Fig. 5.*) represent the surface of stagnant water ascending and descending in successive waves; and let *A, C, E*, &c. be the tops of the waves; and let *B, D, F*, &c. be the intermediate hollows. Because the motion of the waves is carried on by the successive ascent and descent of the water, so that the parts thereof, as *A, C, E*, &c. which are highest at one time, become lowest immediately after; and because the motive force, by which the highest parts descend and the lowest ascend, is the weight of the elevated water, that alternate ascent and descent will be analogous to the reciprocal motion of the water in the canal, and observe the same laws as to the times of its ascent and descent; and therefore (by prop. 44.) if the distances between the highest places of the waves *A, C, E*, and the lowest *B, D, F*, be equal to twice the length of any pendulum, the highest parts *A, C, E*, will become the lowest in the time of one oscillation, and in the time of another oscillation will ascend again. Therefore between the passage of each wave, the time of two oscillations will intervene; that is, the wave will describe its breadth in the time that pendulum will oscillate twice; but a pendulum of four times that length, and which therefore is equal to the breadth of the waves, will just oscillate once in that time. *Q. E. I.*

COR. 1. Therefore waves, whose breadth is equal to  $3\frac{1}{8}$  French feet, will advance thro' a space equal to their breadth in one second of time; and therefore in one minute will go over a space of  $183\frac{1}{3}$  feet; and in an hour a space of 11000 feet, nearly.

COR. 2. And the velocity of greater or less waves will be augmented or diminished in the subduplicate ratio of their breadth.

These







These things are true upon the supposition, that the parts of water ascend or descend in a right line; but in truth, that ascent and descent is rather performed in a circle; and therefore I propose the time defined by this proposition as only near the truth.

# PROPOSITION XLVII. THEOREM XXXVII.

*If pulses are propagated thro' a fluid, the several particles of the fluid, going and returning with the shortest reciprocal motion, are always accelerated or retarded according to the law of the oscillating pendulum. Pl. 9. Fig. 1.*

Let  $AB, BC, CD, \&c.$  represent equal distances of successive pulses;  $ABC$  the line of direction of the motion of the successive pulses, propagated from  $A$  to  $B$ ;  $E, F, G$  three physical points of the quiescent medium situate in the right line  $AC$  at equal distances from each other;  $Ee, Ff, Gg$  equal spaces of extreme shortness, thro' which those points go and return with a reciprocal motion in each vibration;  $\epsilon, \phi, \gamma$  any intermediate places of the same points; and  $EF, FG$  physical lineolæ, or linear parts of the medium lying between those points, and successively transfer'd into the places  $\epsilon\phi, \phi\gamma$ , and  $e\phi, fg$ . Let there be drawn the right line  $PS$  equal to the right line  $Ee$ . Bisect the same in  $O$ , and from the centre  $O$ , with the interval  $OP$ , describe the circle  $SIPi$ . Let the whole time of one vibration, with its proportional parts, be expounded by the whole circumference of this circle and its parts; in such sort, that when any time  $PH$  or  $PHSh$  is compleated, if there be let fall to  $PS$  the perpendicular  $HL$  or  $hl$ , and there be taken  $E\epsilon$  equal to  $PL$  or  $Pl$ , the physical point  $E$  may be found in  $\epsilon$ .

A

A point as  $E$  moving according to this law with a reciprocal motion; in its going from  $E$  thro'  $\epsilon$  to  $e$ , and returning again thro'  $\epsilon$  to  $E$ , will perform its several vibrations with the same degrees of acceleration and retardation with those of an oscillating pendulum. We are now to prove, that the several physical points of the medium will be agitated with such a kind of motion. Let us suppose then, that a medium hath such a motion excited in it from any cause whatsoever, and consider what will follow from thence.

In the circumference  $PHSh$  let there be taken the equal arcs  $HI, IK$ , or  $hi, ik$ , having the same ratio to the whole circumference as the equal right lines  $EF, FG$  have to  $BC$  the whole interval of the pulses. Let fall the perpendiculars  $IM, KN$  or  $im, kn$ ; then because the points  $E, F, G$  are successively agitated with like motions, and perform their entire vibrations composed of their going and return, while the pulse is transferr'd from  $B$  to  $C$ ; if  $PH$  or  $PHSh$  be the time elapsed since the beginning of the motion of the point  $E$ , then will  $PI$  or  $PHSi$  be the time elapsed since the beginning of the motion of the point  $F$ , and  $PK$  or  $PHSk$  the time elapsed since the beginning of the motion of the point  $G$ ; and therefore  $E\epsilon, F\phi, G\gamma$  will be respectively equal to  $PL, PM, PN$ , while the points are going, and to  $Pi, Pm, Pn$ , when the points are returning. Therefore  $\epsilon\gamma$  or  $EG + G\gamma - E\epsilon$  will, when the points are going, be equal to  $EG - LN$ , and in their return equal to  $EG + ln$ . But  $\epsilon\gamma$  is the breadth or expansion of the part  $EG$  of the medium in the place  $\epsilon\gamma$ ; and therefore the expansion of that part in its going, is to its mean expansion as  $EG - LN$  to  $EG$ ; and in its return as  $EG + ln$  or  $EG - LN$  to  $EG$ . Therefore since  $LN$  is to  $KH$  as  $IM$  to the radius  $OP$ , and  $KH$  to  $EG$  as the circumference  $PHShP$  to  $BC$ ; that is, if we put  $V$  for the radius of a circle whose circumference is equal to  $BC$

*BC* the interval of the pulses, as *OP* to *V*; and, *ex æquo*, *LN* to *EG* as *IM* to *V*; the expansion of the part *EG* or of the physical point *F* in the place  $\epsilon\gamma$  to the mean expansion of the same part in its first place *EG*, will be as *V—IM* to *V* in going, and as  $V-\frac{1}{2}im$  to *V* in its return. Hence the elastic force of the point *F* in the place  $\epsilon\gamma$  to its mean elastic force in the place *EG*, is as  $\frac{1}{V-IM}$  to  $\frac{1}{V}$  in its going, and as

$\frac{1}{V-\frac{1}{2}im}$  to  $\frac{1}{V}$  in its return. And by the same reason-

ing the elastic forces of the physical points *E* and *G* in going, are as  $\frac{1}{V-HL}$  and  $\frac{1}{V-KN}$  to  $\frac{1}{V}$ ; and the

difference of the forces to the mean elastic force of the medium, as  $\frac{HL-KN}{VV-V \times HL-V \times KN-\frac{1}{2}HL \times KN}$

to  $\frac{1}{V}$ ; that is, as  $\frac{HL-KN}{VV}$  to  $\frac{1}{V}$ , or as *HL—*

*KN* to *V*; if we suppose (by reason of the very short extent of the vibrations) *HL* and *KN* to be indefinitely less than the quantity *V*. Therefore since the quantity *V* is given, the difference of the forces is as *HL—KN*; that is, (because *HL—KN* is proportional to *HK*, and *OM* to *OI* or *OP*; and because *HK* and *OP* are given) as *OM*; that is, if *Ff* be bisected in  $\Omega$ , as  $\Omega\phi$ . And for the same reason the difference of the elastic forces of the physical points  $\epsilon$  and  $\gamma$  in the return of the physical lineola  $\epsilon\gamma$ , is as  $\Omega\phi$ . But that difference (that is, the excess of the elastic force of the point  $\epsilon$  above the elastic force of the point  $\gamma$ ) is the very force by which the intervening physical lineola  $\epsilon\gamma$  of the medium is accelerated in going, and retarded in returning; and therefore the accelerative force of the physical lineola  $\epsilon\gamma$  is

3.

as

as its distance from  $\Omega$  the middle place of the vibration. Therefore (by prop. 38. book 1.) the time is rightly expounded by the arc  $PI$ ; and the linear part of the medium  $\epsilon\gamma$  is moved according to the law above-mentioned, that is, according to the law of a pendulum oscillating; and the case is the same of all the linear parts of which the whole medium is compounded.  
*Q. E. D.*

**COR.** Hence it appears that the number of the pulses propagated is the same with the number of the vibrations of the tremulous body, and is not multiplied in their progress. For the physical lineola  $\epsilon\gamma$  as soon as it returns to its first place is at rest; neither will it move again, unless it receives a new motion, either from the impulse of the tremulous body, or of the pulses propagated from that body. As soon therefore as the pulses cease to be propagated from the tremulous body, it will return to a state of rest; and move no more.

**PROPOSITION XLVIII. THEOREM XXXVIII.**

*The velocities of pulses propagated in an elastic fluid, are in a ratio compounded of the subduplicate ratio of the elastic force directly, and the subduplicate ratio of the density inversely; supposing the elastic force of the fluid to be proportional to its condensation.*

**CASE I.** If the mediums be homogeneous, and the distances of the pulses in those mediums be equal amongst themselves, but the motion in one medium is more intense than in the other: the contractions and dilatations of the correspondent parts will be as those motions. Not that this proportion is perfectly accurate. However, if the contractions and dilatations are not exceedingly intense, the error will not be sensible; and there-

therefore this proportion may be consider'd as physically exact. Now the motive elastic forces are as the contractions and dilatations ; and the velocities generated in the same time in equal parts are as the forces. Therefore equal and corresponding parts of corresponding pulses will go and return together, thro' spaces proportional to their contractions and dilatations, with velocities that are as those spaces : and therefore the pulses, which in the time of one going and returning advance forwards a space equal to their breadth, and are always succeeding into the places of the pulses that immediately go before them, will, by reason of the equality of the distances, go forward in both mediums with equal velocity.

CASE 2. If the distances of the pulses or their lengths are greater in one medium than in another ; let us suppose that the correspondent parts describe spaces ; in going and returning, each time proportional to the breadths of the pulses : then will their contractions and dilatations be equal. And therefore if the mediums are homogeneous, the motive elastic forces, which agitate them with a reciprocal motion, will be equal also. Now the matter to be moved by these forces is as the breadth of the pulses ; and the space thro' which they move every time they go and return, is in the same ratio. And moreover, the time of one going and returning, is in a ratio compounded of the subduplicate ratio of the matter, and the subduplicate ratio of the space ; and therefore is as the space. But the pulses advance a space equal to their breadths in the times of going once and returning once, that is, they go over spaces proportional to the times ; and therefore are equally swift.

CASE 3. And therefore in mediums of equal density and elastic force, all the pulses are equally swift. Now if the density or the elastic force of the medium were augmented, then because the motive force is increased

in the ratio of the elastic force, and the matter to be moved is increased in the ratio of the density; the time which is necessary for producing the same motion as before, will be increased in the subduplicate ratio of the density, and will be diminished in the subduplicate ratio of the elastic force. And therefore the velocity of the pulses will be in a ratio compounded of the subduplicate ratio of the density of the medium inversely, and the subduplicate ratio of the elastic force directly. *Q. E. D.*

This proposition will be made more clear from the construction of the following problem.

### PROPOSITION XLIX. PROBLEM XI.

*The density and elastic force of a medium being given, to find the velocity of the pulses.*

Suppose the medium to be press'd by an incumbent weight after the manner of our air; and let *A* be the height of a homogeneous medium, whose weight is equal to the incumbent weight and whose density is the same with the density of the compressed medium in which the pulses are propagated. Suppose a pendulum to be constructed, whose length between the point of suspension and the centre of oscillation is *A*: and in the time in which that pendulum will perform one entire oscillation composed of its going and returning, the pulse will be propagated right onwards, thro' a space equal to the circumference of a circle described with the radius *A*.

For letting those things stand which were constructed in Prop. 47. if any physical line as *EF* (*Pl. 9. Fig. 1.*) describing the space *PS* in each vibration, be acted on in the extremities *P* and *S* of every going and return that it makes by an elastic force that is equal to its weight; it will perform its several vibrations in the

the time in which the same might oscillate in a cycloid, whose whole perimeter is equal to the length  $PS$ : and that because equal forces will impel equal corpuscles thro' equal spaces in the same or equal times. Therefore since the times of the oscillations are in the subduplicate ratio of the lengths of the pendulums, and the length of the pendulum is equal to half the arc of the whole cycloid; the time of one vibration would be to the time of the oscillation of a pendulum, whose length is  $A$ , in the subduplicate ratio of the length  $\frac{1}{2}PS$  or  $PO$  to the length  $A$ . But the elastic force, with which the physical lineola  $EG$  is urged, when it is found in its extreme places  $P, S$ , was (in the demonstration of prop. 47.) to its whole elastic force as  $HL - KN$  to  $V$ , that is, (since the point  $K$  now falls upon  $P$ ) as  $HK$  to  $V$ : and all that force, or, which is the same thing, the incumbent weight by which the lineola  $EG$  is compress'd, is to the weight of the lineola as the altitude  $A$  of the incumbent weight to  $EG$  the length of the lineola; and therefore, *ex aequo*, the force with which the lineola  $EG$  is urged in the places  $P$  and  $S$ , is to the weight of that lineola as  $HK \times A$  to  $V \times EG$ ; or as  $PO \times A$  to  $VV$ ; because  $HK$  was to  $EG$  as  $PO$  to  $V$ . Therefore since the times, in which equal bodies are impelled thro' equal spaces, are reciprocally in the subduplicate ratio of the forces, the time of one vibration, produced by the action of that elastic force, will be to the time of a vibration, produced by the impulse of the weight, in a subduplicate ratio of  $VV$  to  $PO \times A$ ; and therefore to the time of the oscillation of a pendulum whose length is  $A$ , in the subduplicate ratio of  $VV$  to  $PO \times A$ , and the subduplicate ratio of  $PO$  to  $A$  conjunctly; that is, in the entire ratio of  $V$  to  $A$ . But in the time of one vibration composed of the going and returning of the pendulum, the pulse will be propagated right onwards thro' a space equal to its breadth  $BC$ : Therefore the time in which a pulse runs over the space

$N$ 
 $BC$



$BC$ , is to the time of one oscillation composed of the going and returning of the pendulum, as  $V$  to  $A$ , that is, as  $BC$  to the circumference of a circle whose radius is  $A$ . But the time in which the pulse will run over the space  $BC$ , is to the time in which it will run over a length equal to that circumference, in the same ratio; and therefore in the time of such an oscillation, the pulse will run over a length equal to that circumference.  
*Q. E. D.*

COR. 1. The velocity of the pulses is equal to that which heavy bodies acquire by falling with an equally accelerated motion, and in their fall describing half the altitude  $A$ . For the pulse will, in the time of this fall, supposing it to move with the velocity acquired by that fall, run over a space that will be equal to the whole altitude  $A$ ; and therefore in the time of one oscillation composed of one going and return, will go over a space equal to the circumference of a circle described with the radius  $A$ : for the time of the fall is to the time of oscillation, as the radius of a circle to its circumference.

COR. 2. Therefore since that altitude  $A$  is as the elastic force of the fluid directly, and the density of the same inversely; the velocity of the pulses will be in a ratio compounded of the subduplicate ratio of the density inversely, and the subduplicate ratio of the elastic force directly.

## PROPOSITION L. PROBLEM XII.

*To find the distances of the pulses.*

Let the number of the vibrations of the body, by whose tremor the pulses are produced, be found to any given time. By that number divide the space which a pulse can go over in the same time, and the part found will be the breadth of one pulse. *Q. E. I.*

SCHOL.

## SCHOLIUM.

The last propositions respect the motions of light and sounds. For since light is propagated in right lines, it is certain that it cannot consist in action alone, (by Prop. 41 and 42.) As to sounds, since they arise from tremulous bodies, they can be nothing else but pulses of the air propagated thro' it, (by Prop. 43.) And this is confirmed by the tremors, which sounds, if they be loud and deep, excite in the bodies near them, as we experience in the sound of drums. For quick and short tremors are less easily excited. But it is well known, that any sounds, falling upon strings in unison with the sonorous bodies, excite tremors in those strings. This is also confirmed from the velocity of sounds. For since the specific gravities of rain-water and quick-silver are to one another as about 1 to  $13\frac{1}{3}$ , and when the mercury in the barometer is at the height of 30 inches of our measure, the specific gravities of the air and of rain-water are to one another as about 1 to 870: therefore the specific gravity of air and quick-silver are to each other as 1 to 11890. Therefore when the height of the quick-silver is at 30 inches, a height of uniform air, whose weight would be sufficient to compress our air to the density we find it to be of, must be equal to 356700 inches or 29725 feet of our measure. And this is that very height of the medium, which I have called A in the construction of the foregoing proposition. A circle whose radius is 29725 feet is 186768 feet in circumference. And since a pendulum  $39\frac{1}{2}$  inches in length compleats one oscillation, composed of its going and return, in two seconds of time, as is commonly known; it follows that a pendulum 29725 feet or 356700 inches in length will perform a like oscillation in  $190\frac{1}{4}$  seconds. Therefore

in that time a sound will go right onwards 186768 feet, and therefore in one second 979 feet.

But in this computation we have made no allowance for the crassitude of the solid particles of the air, by which the sound is propagated instantaneously. Because the weight of air is to the weight of water as 1 to 870, and because salts are almost twice as dense as water; if the particles of air are supposed to be of near the same density as those of water or salt, and the rarity of the air arises from the intervals of the particles; the diameter of one particle of air will be to the interval between the centres of the particles, as 1 to about 9 or 10, and to the interval between the particles themselves as 1 to 8 or 9. Therefore to 979 feet, which, according to the above calculation, a sound will advance forward in one second of time, we may add  $\frac{22}{9}$ , or about 109 feet, to compensate for the crassitude of the particles of the air; and then a sound will go forward about 1088 feet in one second of time.

Moreover, the vapors floating in the air, being of another spring, and a different tone, will hardly, if at all, partake of the motion of the true air in which the sounds are propagated. Now if these vapors remain unmoved, that motion will be propagated the swifter thro' the true air alone, and that in the subduplicate ratio of the defect of the matter. So if the atmosphere consist of ten parts of true air and one part of vapors, the motion of sounds will be swifter in the subduplicate ratio of 11 to 10, or very nearly in the entire ratio of 21 to 20, than if it were propagated thro' eleven parts of true air: and therefore the motion of sounds above discovered must be encreased in that ratio. By this means the sound will pass thro' 1142 feet in one second of time.

These things will be found true in spring and autumn, when the air is rarefied by the gentle warmth of those seasons, and by that means its elastic force becomes

comes somewhat more intense. But in winter, when the air is condensed by the cold, and its elastic force is somewhat remitted, the motion of sounds will be slower in a subduplicate ratio of the density ; and on the other hand, swifter in the summer.

Now by experiments it actually appears that sounds do really advance in one second of time about 1142 feet of *English* measure, or 1070 feet of *French* measure.

The velocity of sounds being known, the intervals of the pulses are known also. For M. *Sauveur*, by some experiments that he made, found that an open pipe about five *Paris* feet in length, gives a sound of the same tone with a viol-string that vibrates a hundred times in one second. Therefore there are near 100 pulses in a space of 1070 *Paris* feet, which a sound runs over in a second of time ; and therefore one pulse fills up a space of about  $10\frac{7}{10}$  *Paris* feet, that is, about twice the length of the pipe. From whence it is probable, that the breadths of the pulses, in all sounds made in open pipes, are equal to twice the length of the pipes.

Moreover, from the corollary of prop. 47. appears the reason, why the sounds immediately cease with the motion of the sonorous body, and why they are heard no longer when we are at a great distance from the sonorous bodies, than when we are very near them. And besides, from the foregoing principles it plainly appears how it comes to pass that sounds are so mightily encreased in speaking-trumpets. For all reciprocal motion uses to be encreased by the generating cause at each return. And in tubes hindering the dilatation of the sounds, the motion decays more slowly, and recurs more forcibly ; and therefore is the more encreased by the new motion impressed at each return, And these are the principal phenomena of sounds.



## SECTION IX.

*Of the circular motion of fluids.*

## HYPOTHESIS.

*The resistance, arising from the want of lubricity in the parts of a fluid, is, cæteris paribus, proportional to the velocity with which the parts of the fluid are separated from each other.*

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## PROPOSITION LI. THEOREM XXXVIII.

*If a solid cylinder infinitely long, in an uniform and infinite fluid, revolve with an uniform motion about an axis given in position, and the fluid be forced round by only this impulse of the cylinder, and every part of the fluid persevere uniformly in its motion; I say, that the periodic times of the parts of the fluid are as their distances from the axis of the cylinder.*

Let *AFL* (Pl. 9. Fig. 2.) be a cylinder turning uniformly about the axis *S*, and let the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, &c. divide the fluid into innumerable concentric cylindric solid orbs of the same

same thickness. Then, because the fluid is homogeneous, the impressions which the contiguous orbs make upon each other mutually, will be (by the hypothesis) as their translations from each other, and as the contiguous superficies upon which the impressions are made. If the impression made upon any orb be greater or less on its concave, than on its convex side, the stronger impression will prevail, and will either accelerate or retard the motion of the orb, according as it agrees with, or is contrary to the motion of the same. Therefore, that every orb may persevere uniformly in its motion, the impressions made on both sides must be equal, and their directions contrary. Therefore since the impressions are as the contiguous superficies, and as their translations from one another; the translations will be inversely as the superficies, that is, inversely as the distances of the superficies from the axis. But the differences of the angular motions about the axis, are as those translations applied to the distances, or as the translations directly and the distances inversely; that is, joining these ratio's together, as the squares of the distances inversely. Therefore if there be erected the lines  $Aa$ ,  $Bb$ ,  $Cc$ ,  $Dd$ ,  $Ee$ , &c. perpendicular to the several parts of the infinite right line  $SABCDEQ$  and reciprocally proportional to the squares of  $SA$ ,  $SB$ ,  $SC$ ,  $SD$ ,  $SE$ , &c. and thro' the extremities of those perpendiculars there be supposed to pass an hyperbolic curve; the sums of the differences, that is, the whole angular motions, will be as the correspondent sums of the lines  $Aa$ ,  $Bb$ ,  $Cc$ ,  $Dd$ ,  $Ee$ , that is, (if to constitute a medium uniformly fluid, the number of the orbs be encreased and their breadth diminished in infinitum) as the hyperbolic area's  $AaQ$ ,  $BbQ$ ,  $CcQ$ ,  $DdQ$ ,  $EeQ$ , &c. analogous to the sums. And the times, reciprocally proportional to the angular motions, will be also reciprocally proportional to those areas. Therefore the periodic time of any particle as  $D$ , is reciprocally as the area  $DdQ$ , that is, (as

appears

appears from the known methods of quadratures of curves) directly as the distance  $SD$ . *Q.E.D.*

COR. 1. Hence the angular motions of the particles of the fluid are reciprocally as their distances from the axis of the cylinder, and the absolute velocities are equal.

COR. 2. If a fluid be contained in a cylindric vessel of an infinite length, and contain another cylinder within, and both the cylinders revolve about one common axis, and the times of their revolutions be as their semidiameters, and every part of the fluid perseveres in its motion: the periodic times of the several parts will be as the distances from the axis of the cylinders.

COR. 3. If there be added or taken away any common quantity of angular motion from the cylinder and fluid moving in this manner; yet because this new motion will not alter the mutual attrition of the parts of the fluid, the motion of the parts among themselves will not be changed. For the translations of the parts from one another depend upon the attrition. Any part will persevere in that motion, which, by the attrition made on both sides with contrary directions, is no more accelerated than it is retarded.

COR. 4. Therefore if there be taken away from this whole system of the cylinders and the fluid, all the angular motion of the outward cylinder, we shall have the motion of the fluid in a quiescent cylinder.

COR. 5. Therefore if the fluid and outward cylinder are at rest, and the inward cylinder revolve uniformly; there will be communicated a circular motion to the fluid, which will be propagated by degrees thro' the whole fluid; and will go on continually encreasing, till such time as the several parts of the fluid acquire the motion determined in cor. 4.

COR. 6. And because the fluid endeavours to propagate its motion still farther, its impulse will carry the outmost cylinder also about with it, unless the cylinder

be violently detained ; and accelerate its motion till the periodic times of both cylinders become equal among themselves. But if the outward cylinder be violently detained, it will make an effort to retard the motion of the fluid ; and unless the inward cylinder preserve that motion by means of some external force impressed thereon, it will make it cease by degrees.

All these things will be found true, by making the experiment in deep standing water,

## PROPOSITION LII. THEOREM XL.

*If a solid sphere, in an uniform and infinite fluid, revolves about an axis given in position with an uniform motion, and the fluid be forced round by only this impulse of the sphere ; and every part of the fluid perseveres uniformly in its motion : I say, that the periodic times of the parts of the fluid are as the squares of their distances from the centre of the sphere.*

CASE I, Let  $AFL$  be a sphere turning uniformly about the axis  $S$ , and let the concentric circles  $BGM$ ,  $CHN$ ,  $DIO$ ,  $EKP$ , &c. divide the fluid into innumerable concentric orbs of the same thickness. Suppose those orbs to be solid ; and because the fluid is homogeneous, the impressions which the contiguous orbs make one upon another, will be (by the supposition) as their translations from one another, and the contiguous superficies upon which the impressions are made. If the impression upon any orb be greater or less upon its concave than upon its convex side ; the more forcible impression will prevail, and will either accelerate or retard the velocity of the orb, according as it is directed



rected with a conspiring or contrary motion to that of the orb. Therefore that every orb may persevere uniformly in its motion, it is necessary that the impressions made upon both sides of the orb should be equal, and have contrary directions. Therefore since the impressions are as the contiguous superficies, and as their translations from one another; the translations will be inversely as the superficies, that is, inversely as the squares of the distances of the superficies from the centre. But the differences of the angular motions about the axis are as those translations applied to the distances, or as the translations directly and the distances inversely; that is, by compounding those ratio's, as the cubes of the distances inversely. Therefore, if upon the several parts of the infinite right line  $SABCDEQ$  there be erected the perpendiculars  $Aa, Bb, Cc, Dd, Ee$ , &c. reciprocally proportional to the cubes of  $SA, SB, SC, SD, SE$ , &c. the sums of the differences, that is, the whole angular motions, will be as the corresponding sums of the lines  $Aa, Bb, Cc, Dd, Ee$ , &c. that is, (if to constitute an uniformly fluid medium the number of the orbs be encreased and their thickness diminished in infinitum) as the hyperbolic areas  $AaQ, BbQ, CcQ, DdQ, EeQ$ , &c. analogous to the sums; and the periodic times being reciprocally proportional to the angular motions, will be also reciprocally proportional to those areas. Therefore the periodic time of any orb  $DIO$  is reciprocally as the area  $DdQ$ , that is, (by the known methods of quadratures) directly as the square of the distance  $SD$ . Which was first to be demonstrated.

CASE 2. From the centre of the sphere let there be drawn a great number of indefinite right lines, making given angles with the axis, exceeding one another by equal differences; and, by these lines revolving about the axis, conceive the orbs to be cut into innumerable annuli: then will every annulus have four an-

nuli contiguous to it, that is, one on its inside, one on its outside, and two on each hand. Now each of these annuli cannot be impelled equally and with contrary directions by the attrition of the interior and exterior annuli unless the motion be communicated according to the law which we demonstrated in case 1. This appears from that demonstration. And therefore any series of annuli, taken in any right line extending itself in infinitum from the globe, will move according to the law of case 1. except we should imagine it hindered by the attrition of the annuli on each side of it. But now in a motion, according to this law, no such attrition is, and therefore cannot be any obstacle to the motion's persevering according to that law. If annuli at equal distances from the centre revolve either more swiftly or more slowly near the poles than near the ecliptic; they will be accelerated if slow, and retarded if swift, by their mutual attrition; and so the periodic times will continually approach to equality, according to the law of case 1. Therefore this attrition will not at all hinder the motion from going on according to the law of case 1. and therefore that law will take place; that is, the periodic times of the several annuli will be as the squares of their distances from the centre of the globe. Which was to be demonstrated in the second place.

CASE 3. Let now every annulus be divided by transverse sections into innumerable particles constituting a substance absolutely and uniformly fluid; and because these sections do not at all respect the law of circular motion, but only serve to produce a fluid substance, the law of circular motion will continue the same as before. All the very small annuli will either not at all change their asperity and force of mutual attrition upon account of these sections, or else they will change the same equally. Therefore the proportion of the causes remaining the same, the proportion of the effects will remain

remain the same also ; that is, the proportion of the motions and the periodic times. *Q.E.D.* But now as the circular motion, and the centrifugal force thence arising, is greater at the ecliptic than at the poles, there must be some cause operating to retain the several particles in their circles ; otherwise the matter that is at the ecliptic will always recede from the centre, and come round about to the poles by the outside of the vortex, and from thence return by the axis to the ecliptic with a perpetual circulation.

COR. 1. Hence the angular motions of the parts of the fluid about the axis of the globe, are reciprocally as the squares of the distances from the centre of the globe, and the absolute velocities are reciprocally as the same squares applied to the distances from the axis.

COR. 2. If a globe revolve with a uniform motion about an axis of a given position in a similar and infinite quiescent fluid with an uniform motion, it will communicate a whirling motion to the fluid like that of a vortex, and that motion will by degrees be propagated onwards in infinitum ; and this motion will be increased continually in every part of the fluid, till the periodical times of the several parts become as the squares of the distances from the centre of the globe.

COR. 3. Because the inward parts of the vortex are by reason of their greater velocity continually pressing upon and driving forwards the external parts, and by that action are perpetually communicating motion to them, and at the same time those exterior parts communicate the same quantity of motion to those that lie still beyond them, and by this action preserve the quantity of their motion continually unchanged ; it is plain that the motion is perpetually transferred from the centre to the circumference of the vortex, till it is quite swallowed up and lost in the boundless extent of that circumference. The matter between any two spherical superficies concentric to the vortex will never be accelerated.

celerated; because that matter will be always transferring the motion it receives from the matter nearer the centre to that matter which lies nearer the circumference.

COR. 4. Therefore in order to continue a vortex in the same state of motion, some active principle is required, from which the globe may receive continually the same quantity of motion which it is always communicating to the matter of the vortex. Without such a principle it will undoubtedly come to pass that the globe and the inward parts of the vortex, being always propagating their motion to the outward parts, and not receiving any new motion, will gradually move slower and slower, and at last be carried round no longer.

COR. 5. If another globe should be swimming in the same vortex at a certain distance from its centre, and in the mean time by some force revolve constantly about an axis of a given inclination; the motion of this globe will drive the fluid round after the manner of a vortex; and at first this new and small vortex will revolve with its globe about the centre of the other; and in the mean time its motion will creep on, farther and farther, and by degrees be propagated in infinitum, after the manner of the first vortex. And for the same reason that the globe of the new vortex was carried about before by the motion of the other vortex, the globe of this other will be carried about by the motion of this new vortex, so that the two globes will revolve about some intermediate point, and by reason of that circular motion mutually fly from each other, unless some force restrains them. Afterwards, if the constantly impressed forces, by which the globes persevere in their motions, should cease, and every thing be left to act according to the laws of mechanics, the motion of the globes will languish by degrees, (for the reason assigned in cor. 3 and 4.) and the vortices at last will quite stand still.

COR.

COR. 6. If several globes in given places should constantly revolve with determined velocities about axes given in position, there would arise from them as many vortices going on in infinitum. For upon the same account that any one globe propagates its motion in infinitum, each globe apart will propagate its own motion in infinitum also; so that every part of the infinite fluid will be agitated with a motion resulting from the actions of all the globes. Therefore the vortices will not be confined by any certain limits, but by degrees run mutually into each other; and by the mutual actions of the vortices on each other, the globes will be perpetually moved from their places, as was shewn in the last corollary; neither can they possibly keep any certain position among themselves, unless some force restrains them. But if those forces, which are constantly impressed upon the globes to continue these motions, should cease; the matter (for the reason assigned in cor. 3 and 4.) will gradually stop, and cease to move in vortices.

COR. 7. If a similar fluid be inclosed in a sphaerical vessel, and by the uniform rotation of a globe in its centre, is driven round in a vortex; and the globe and vessel revolve the same way about the same axis, and their periodical times be as the squares of the semidiameters; the parts of the fluid will not go on in their motions without acceleration or retardation, till their periodical times are as the squares of their distances from the centre of the vortex. No constitution of a vortex can be permanent but this.

COR. 8. If the vessel, the inclosed fluid, and the globe, retain this motion, and revolve besides with a common angular motion about any given axis; because the mutual attrition of the parts of the fluid is not changed by this motion, the motions of the parts among each other will not be changed. For the translations of the parts among themselves depend upon this attrition.

Any

Any part will persevere in that motion, in which its attrition on one side retards it just as much as its attrition on the other side accelerates it.

COR. 9. Therefore if the vessel be quiescent, and the motion of the globe be given, the motion of the fluid will be given. For conceive a plane to pass thro' the axis of the globe, and to revolve with a contrary motion; and suppose the sum of the time of this revolution and of the revolution of the globe to be to the time of the revolution of the globe, as the square of the semidiameter of the vessel to the square of the semidiameter of the globe; and the periodic times of the parts of the fluid in respect of this plane will be as the squares of their distances from the centre of the globe.

COR. 10. Therefore if the vessel move about the same axis with the globe, or with a given velocity about a different one, the motion of the fluid will be given. For if from the whole system we take away the angular motion of the vessel, all the motions will remain the same among themselves as before, by cor. 8. and those motions will be given by cor. 9.

COR. 11. If the vessel and the fluid are quiescent, and the globe revolves with an uniform motion, that motion will be propagated by degrees through the whole fluid to the vessel, and the vessel will be carried round by it, unless violently detained; and the fluid and the vessel will be continually accelerated till their periodic times become equal to the periodic times of the globe. If the vessel be either withheld by some force, or revolve with any constant and uniform motion, the medium will come by little and little to the state of motion defined in cor. 8. 9. 10. nor will it ever persevere in any other state. But if then the forces, by which the globe and vessel revolve with certain motions, should cease, and the whole system be left to act according to the mechanical laws, the vessel and globe, by means of the intervening fluid, will act upon

each other, and will continue to propagate their motions through the fluid to each other, till their periodic times become equal among themselves, and the whole system revolves together like one solid body.

### SCHOLIUM.

In all these reasonings, I suppose the fluid to consist of matter of uniform density and fluidity. I mean that the fluid is such, that a globe placed any where therein may propagate with the same motion of its own, at distances from it self continually equal, similar and equal motions in the fluid, in the same interval of time. The matter by its circular motion endeavours to recede from the axis of the vortex; and therefore presses all the matter that lies beyond. This pressure makes the attrition greater, and the separation of the parts more difficult; and by consequence diminishes the fluidity of the matter. Again, if the parts of the fluid are in any one place denser or larger than in the others, the fluidity will be less in that place, because there are fewer superficies where the parts can be separated from each other. In these cases I suppose the defect of the fluidity to be supplied by the smoothness or softness of the parts, or some other condition; otherwise the matter where it is less fluid, will cohere more, and be more sluggish, and therefore will receive the motion more slowly, and propagate it farther than agrees with the ratio above assigned. If the vessel be not spherical, the particles will move in lines, not circular, but answering to the figure of the vessel, and the periodic times will be nearly as the squares of the mean distances from the centre. In the parts between the centre and the circumference, the motions will be slower where the spaces are wide, and swifter where narrow; but yet the particles will not tend to the circumference

cumference at all the more for their greater swiftness. For they then describe arcs of less curvity, and the conatus of receding from the centre is as much diminished by the diminution of this curvature, as it is augmented by the increase of the velocity. As they go out of narrow into wide spaces they recede a little farther from the centre, but in doing so are retarded; and when they come out of wide into narrow spaces they are again accelerated; and so each particle is retarded and accelerated by turns for ever. These things will come to pass in a rigid vessel. For the state of vortices in an infinite fluid is known by cor. 6. of this proposition.

I have endeavoured in this proposition to investigate the properties of vortices, that I might find whether the celestial phænomena can be explained by them. For the phænomenon is this, that the periodic times of the Planets revolving about Jupiter, are in the sesquuplicate ratio of their distances from Jupiter's centre; and the same rule obtains also among the Planets that revolve about the Sun. And these rules obtain also with the greatest accuracy, as far as has been yet discovered by astronomical observation. Therefore, if those Planets are carried round in vortices revolving about Jupiter and the Sun, the vortices must revolve according to that law. But here we found the periodic times of the parts of the vortex to be in the duplicate ratio of the distances from the centre of motion; and this ratio cannot be diminished and reduced to the sesquuplicate, unless either the matter of the vortex be more fluid, the farther it is from the centre, or the resistance arising from the want of lubricity in the parts of the fluid, should, as the velocity with which the parts of the fluid are separated goes on increasing, be augmented with it in a greater ratio than that in which the velocity increases. But neither of these suppositions seem reasonable. The more gross and less fluid parts

O 2

will



will tend to the circumference, unless they are heavy towards the centre. And tho', for the sake of demonstration, I proposed, at the beginning of this Section, an hypothesis that the resistance is proportional to the velocity, nevertheless, 'tis in truth probable that the resistance is in a less ratio than that of the velocity. Which granted, the periodic times of the parts of the vortex will be in a greater than the duplicate ratio of the distances from its centre. If, as some think, the vortices move more swiftly near the centre, then slower to a certain limit, then again swifter near the circumference, certainly neither the sesquuplicate, nor any other certain and determinate ratio can obtain in them. Let philosophers then see how that phenomenon of the sesquuplicate ratio can be accounted for by vortices.

### PROPOSITION LIII. THEOREM XLI.

*Bodies, carried about in a vortex and returning in the same orb, are of the same density with the vortex, and are moved according to the same law with the parts of the vortex, as to velocity and direction of motion.*

For if any small part of the vortex, whose particles or physical points preserve a given situation among each other, be supposed to be congealed; this particle will move according to the same law as before, since no change is made either in its density, *vis insita*, or figure. And again, if a congealed or solid part of the vortex be of the same density with the rest of the vortex, and be resolved into a fluid, this will move according to the same law as before, except in so far as its particles now become fluid, may be moved among themselves. Neglect therefore the motion of the particles

ticles among themselves, as not at all concerning the progressive motion of the whole, and the motion of the whole will be the same as before. But this motion will be the same with the motion of other parts of the vortex at equal distances from the centre; because the solid, now resolved into a fluid, is become perfectly like to the other parts of the vortex. Therefore a solid, if it be of the same density with the matter of the vortex, will move with the same motion as the parts thereof, being relatively at rest in the matter that surrounds it. If it be more dense, it will endeavour more than before to recede from the centre; and therefore overcoming that force of the vortex, by which, being as it were kept in equilibrio, it was retained in its orbit, it will recede from the centre, and in its revolution describe a spiral, returning no longer into the same orbit. And by the same argument, if it be more rare it will approach to the centre. Therefore it can never continually go round in the same orbit, unless it be of the same density with the fluid. But we have shewn in that case, that it would revolve according to the same law with those parts of the fluid that are at the same or equal distances from the centre of the vortex.

COR. 1. Therefore a solid revolving in a vortex, and continually going round in the same orbit, is relatively quiescent in the fluid that carries it.

COR. 2. And if the vortex be of an uniform density, the same body may revolve at any distance from the centre of the vortex.

#### SCHOLIUM.

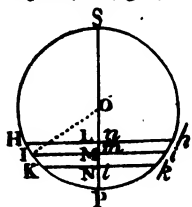
Hence it is manifest, that the Planets are not carried round in corporeal vortices. For according to the *Copernican* hypothesis, the Planets going round the Sun,

O 3

revolve

revolve in ellipses, having the Sun in their common focus; and by radii drawn to the sun describe areas proportional to the times. But now the parts of a vortex can never revolve with such a motion. Let *AD*, *BE*, *CF*, (*Pl. 9. Fig. 3.*) represent three orbits described about the Sun *S*, of which let the utmost circle *CF* be concentric to the Sun; and let the aphelia of the two innermost be *A*, *B*; and their perihelia *D*, *E*. Therefore a body revolving in the orb *CF*, describing, by a radius drawn to the Sun, areas proportional to the times, will move with an uniform motion. And according to the laws of astronomy, the body revolving in the orb *BE* will move slower in its aphelion *B*, and swifter in its perihelion *E*; whereas, according to the laws of mechanics, the matter of the vortex ought to move more swiftly in the narrow space between *A* and *C*, than in the wide space between *D* and *F*; that is, more swiftly in the aphelion than in the perihelion. Now these two conclusions contradict each other. So at the beginning of the sign of Virgo, where the aphelion of Mars is at present, the distance between the orbits of Mars and Venus is to the distance between the same orbits at the beginning of the sign of Pisces, as about 3 to 2; and therefore the matter of the vortex between those orbits ought to be swifter at the beginning of Pisces, than at the beginning of Virgo, in the ratio of 3 to 2. For the narrower the space is, thro' which the same quantity of matter passes in the same time of one revolution, the greater will be the velocity with which it passes thro' it. Therefore if the Earth being relatively at rest in this celestial matter should be carried round by it, and revolve together with it about the Sun, the velocity of the Earth at the beginning of Pisces would be to its velocity at the beginning of Virgo in a sesquialteral ratio. Therefore the Sun's apparent diurnal motion at the beginning of Virgo, ought to be above 70 minutes; and at the beginning of Pisces less than 48 minutes,

*Fig. 1 p. 173. 78.*



*Fig. 2.*  
*p.184.87.*

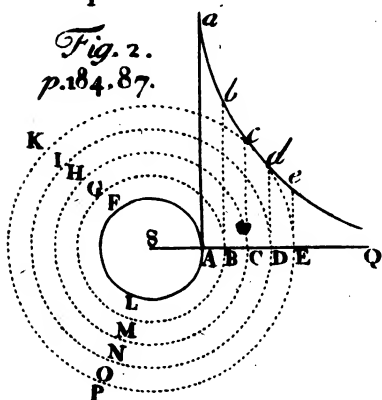
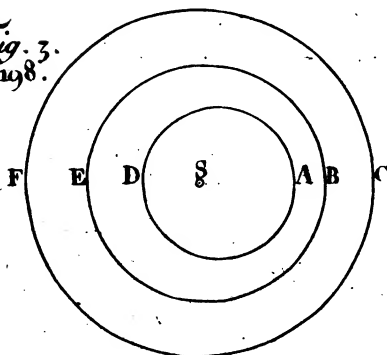


Fig. 3.  
p. 198.





minutes. Whereas on the contrary that apparent motion of the Sun is really greater at the beginning of Pisces than at the beginning of Virgo, as experience testifies; and therefore the earth is swifter at the beginning of Virgo than at the beginning of Pisces. So that the hypothesis of vortices is utterly irreconcilable with astronomical phenomena, and rather serves to perplex than explain the heavenly motions. How these motions are performed in free spaces without vortices, may be understood by the first book; and I shall now more fully treat of it in the following book *of the System of the World.*

OF THE  
SYSTEM  
OF THE  
WORLD.

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BOOK III.

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**I**N the preceding books I have laid down the principles of philosophy; principles, not philosophical, but mathematical; such, to wit, as we may build our reasonings upon in philosophical enquiries. These principles are, the laws and conditions of certain motions, and powers or forces, which chiefly have respect to philosophy. But lest they should have appeared of themselves dry and barren, I have illustrated them here and there, with some philosophical scholiums, giving an account of such things, as are of more general nature, and which philosophy seems chiefly to be founded on; such as the density and the resistance of bodies,

bodies, spaces void of all bodies, and the motion of light and sounds. It remains, that from the same principles, I now demonstrate the frame of the System of the World. Upon this subject, I had indeed compos'd the third book in a popular method, that it might be read by many. But afterwards considering that such as had not sufficiently enter'd into the principles, could not easily discern the strength of the consequences, nor lay aside the prejudices to which they had been many years accustomed; therefore to prevent the disputes which might be rais'd upon such accounts, I chose to reduce the substance of that book into the form of propositions (in the mathematical way) which should be read by those only, who had first made themselves matters of the principles establish'd in the preceding books. Not that I would advise any one to the previous study of every proposition of those books. For they abound with such as might cost too much time, even to readers of good mathematical learning. It is enough if one carefully reads the definitions, the laws of motion, and the first three sections of the first book. He may then pass on to this book, of the System of the World, and consult such of the remaining propositions of the first two books, as the references in this, and his occasions, shall require.

Segun el final de este preface, para comprender este libro III, sobre el sistema del mundo, es necesario estudiar bien (primamente) las 9 Definiciones.



Las leyes del movimiento y las 11 primeras secciones del libro I, segun las ordenas las siguientes proposiciones de los dos primeros libros a que se haga referen-





T H E  
R U L E S  
O F

REASONING in PHILOSOPHY.  
*razonamiento*

---

R U L E I.

*We are to admit no more causes of natural things, than such as are both true and sufficient to explain their appearances.*

To this purpose the philosophers say, that Nature do's nothing in vain, and more is in vain, when less will serve; For Nature is pleas'd with simplicity, and affects not the pomp of superfluous causes.

R U L E II.

*Therefore to the same <sup>minima</sup> natural effects we must, (as far as possible, assign the same causes.*

As to respiration in a man, and in a beast; the descent of stones in *Europe* and in *America*; the light of our culinary fire and of the Sun; the reflection of light in the Earth, and in the Planets.

R U L E

RULE III.

The qualities of bodies, which admit neither in-  
tension nor remission of degrees, and which are  
found to belong to all bodies within the reach  
of our experiments, are to be esteemed the uni-  
versal qualities of all bodies whatsoever.

For since the qualities of bodies are only known to  
us by experiments, we are to hold for universal, all  
such as universally agree with experiments; and such  
as are not liable to diminution, can never be quite  
(taken away). We are certainly not to relinquish  
the evidence of experiments for the sake of dreams  
and vain fictions of our own devising; nor are we to  
recede from the analogy of Nature, which uses to be  
simple, and always consonant to itself. We no other-  
ways know the extension of bodies, than by our  
senses, nor do these reach it in all bodies; but because  
we perceive extension in all that are sensible, therefore  
we ascribe it universally to all others also. That  
abundance of bodies are hard we learn by experience.  
And because the hardness of the whole arises from the  
hardness of the parts, we therefore justly infer the hard-  
ness of the undivided particles not only of the bodies  
we feel but of all others. That all bodies are im-  
penetrable, we gather not from reason, but from sen-  
sation. The bodies which we handle we find impen-  
etrable, and thence conclude impenetrability to be an  
universal property of all bodies whatsoever. That all  
bodies are moveable, and endow'd with certain powers  
(which we call the *vires inertia*) of persevering in their  
motion or in their rest, we only infer from the like  
properties observ'd in the bodies which we have seen.  
The extension, hardness, impenetrability, mobility,  
and *vis inertia* of the whole, result from the exten-  
sion

sion, hardness, impenetrability, mobility, and *vires inertiae* of the parts: and thence we conclude the least particles of all bodies to be also all extended, and hard, and impenetrable, and moveable, and endow'd with their proper *vires inertiae*. And this is the foundation of all philosophy. Moreover, that the divided but contiguous particles of bodies may be separated from one another, is matter of observation; and, in the particles that remain undivided, our (minds are able) to distinguish yet lesser parts, as is mathematically demonstrated. But whether the parts so distinguish'd, and not yet divided, may, by the powers of nature, be actually divided and separated from one another, we cannot certainly determine. Yet had we the proof of but one experiment, that any undivided particle, in breaking a hard and solid body, suffer'd a division, we might by virtue of this rule, conclude, that the undivided as well as the divided particles, may be divided and actually separated to infinity.

Lastly, If it universally appears, by experiments and astronomical observations, that all bodies about the Earth, gravitate towards the Earth; and that in proportion to the quantity of matter which they severally contain; that the Moon likewise, according to the quantity of its matter, gravitates towards the Earth; that on the other hand our Sea gravitates towards the Moon; and all the Planets mutually one towards another; and the Comets in like manner towards the Sun; we must, in consequence of this rule, universally allow, that all bodies whatsoever are endow'd with a principle of mutual gravitation. For the argument from the appearances concludes with more force for the universal gravitation of all bodies, than for their impenetrability; of which among those in the celestial regions, we have no experiments, nor any manner of observation. Not that I affirm gravity to be essential to bodies. By their *vis insita* I mean nothing but their *vis inertiae*. This

no demonstration  
 de viâ  
 -a manâ

is immutable. Their gravity is diminished as they recede from the Earth. *recedere*

# RULE IV.

In experimental philosophy we are to <sup>consider</sup> look upon <sup>reminds</sup> propositions collected by general induction from <sup>verba de re</sup> phenomena as accurately or very nearly true, <sup>etiam</sup> notwithstanding any contrary hypotheses that <sup>omne</sup> may be imagined, till such time as other <sup>matte</sup> phenomena occur, by which they may either be <sup>occurit</sup> made more accurate, or <sup>explanatio</sup> liable to exceptions.

This rule we must follow that the argument of induction may not be evaded by hypotheses. *evadido*

THE



## THE

## PHÆNOMENA OF APPEARANCES.

## PHÆNOMENON I.

*That the circumjovial planets, by radij drawn to Jupiter's center, describe areas proportional to the <sup>centro</sup>times of description, and that their periodic times, the fixed Stars being at rest, are in the sesquiplicate proportion of their distances from its center.*

**T**HIS we know from astronomical observations. For the orbits of these planets differ but insensibly from circles concentric to Jupiter; and their motions in those circles are found to be uniform. And all astronomers agree, that their periodic times are in the sesquiplicate proportion of the semidiameters of their orbits: and so it manifestly appears from the following table.

*The periodic times of the Satellites of Jupiter.*

1<sup>d</sup>. 18<sup>h</sup>. 27'. 34".    3<sup>d</sup>. 13<sup>h</sup>. 13'. 42".    7<sup>d</sup>. 3<sup>h</sup>. 42'. 36".  
16<sup>d</sup>. 16<sup>h</sup>. 32'. 9".

(1) Sesquiplicate proportion (relación)  
de los cuadrados de las distancias  
de los planetas al Sol, o de los satélites  
a Júpiter, de las distancias medias  
de los planetas al Sol, o de los satélites  
a Júpiter, es de 3<sup>a</sup>

*The distances of the Satellites from Jupiter's center.*

*From the observations of*

Borelli

Townley *by the Microm.*

Cassini *by the Telescope.*

Cassini *by the eclips. of the satel.*

1	2	3	4	} <i>semi- am. of Jupiter.</i>
$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	
5, 52	8, 78	13, 47	24, 72	
5	8	13	23	
$5\frac{2}{3}$	9	$14\frac{2}{3}$	$25\frac{1}{6}$	
From the periodic times.	5, 667	9, 017	14, 384	25, 299

Mr. *Pound* has determined by the help of excellent micrometers, the diameters of Jupiter and the elongation of its satellites after the following manner. The greatest heliocentric elongation of the fourth satellite from Jupiter's centre was taken with a micrometer in a 15 foot telescope, and at the mean distance of Jupiter from the Earth was found about  $8'. 16''$ . The elongation of the third satellite was taken with a micrometer in a telescope of 123 feet, and at the same distance of Jupiter from the Earth was found  $4'. 42''$ . The greatest elongations of the other satellites at the same distance of Jupiter from the Earth, are found from the periodic times to be  $2'. 56''. 47''$ . and  $1'. 51''. 6'''$ .

The diameter of Jupiter taken with the micrometer in an 123 foot telescope several times, and reduced to Jupiter's mean distance from the Earth, proved always less than  $40''$ , never less than  $38''$ , generally  $39''$ . This diameter in shorter telescopes is  $40''$ , or  $41''$ . For Jupiter's light is a little dilated by the unequal refrangibility of the rays, and this dilatation bears a less ratio to the diameter of Jupiter in the longer and more perfect telescopes, than in those which are shorter and less perfect. The times in which two satellites, the first and the third, passed over Jupiter's body, were observed, from the beginning of the ingress to the beginning of the egress, and from the complete ingress to the complete egress, with the long telescope. And from the transit of the first satellite, the diameter of Jupiter at its mean distance

distance from the Earth, (came forth)  $37\frac{1}{8}''$ , and from the transit of the third  $37\frac{1}{8}''$ . There was observed also the time in which the shadow of the first satellite pass'd over Jupiter's body, and thence the diameter of Jupiter at its mean distance from the Earth came out about  $37''$ . Let us suppose its diameter to be  $37\frac{1}{4}''$  very nearly, and then the greatest elongations of the first, second, third and fourth satellite will be respectively equal to 5,965, 9,494, 15,141, and 26,63 semidiameters of Jupiter.

## PHÆNOMENON. II.

*That the circumsaturnal planets, by radij drawn to Saturn's center, describe areas proportional to the times of description, and that their periodic times, the fixed Stars being at rest, are in the sesquiplicate proportion of their distances from its centre.*

For as *Cassini* from his own observations has determin'd, their distances from Saturn's centre, and their periodic times are as follow.

### *The periodic times of the satellites of Saturn.*

$1^d. 21^h. 18'. 27''.$   $2^d. 17^h. 41'. 22''.$   $4^d. 12^h. 25'. 12''.$   
 $15^d. 22^h. 41'. 14''.$   $79^d. 7^h. 48'. 00''.$

### *The distances of the satellites from Saturn's center, in semidiameters of its Ring.*

From observations  $1\frac{1}{2}\frac{2}{5}.$   $2\frac{1}{2}.$   $3\frac{1}{2}.$  8. 24.  
 From the periodic times. 1, 93. 2, 47. 3, 45. 8. 23, 35.

The greatest elongation of the fourth satellite from Saturn's centre is commonly determined from the observations to be eight of those semidiameters very nearly.

y. But the greatest elongation of this satellite from Saturn's centre, when taken with an excellent micrometer in M. *Huygens's* telescope of 123 feet, appeared to be eight semidiameters and  $\frac{1}{10}$  of a semidiameter. And from this observation and the periodic times, the distances of the satellites from Saturn's centre in semidiameters of the Ring are 2,1. 2,69. 3,75. 8,7. and 25,35. The diameter of Saturn observed in the same telescope was found to be to the diameter of the Ring as 3 to 7, and the diameter of the Ring, *May* 28, 29. 1719. was found to be 43". And thence the diameter of the Ring when Saturn is at its mean distance from the Earth is 42", and the diameter of Saturn 18". These things appear so in very long and excellent telescopes, because in such telescopes the apparent magnitudes of the heavenly bodies bear a greater proportion to the dilatation of light in the extremities of those bodies, than in shorter telescopes. If we then reject all the spurious light, the diameter of Saturn will not amount to more than 16".

## PHÆNOMENON III.

*That the five primary Planets, Mercury, Venus, Mars, Jupiter and Saturn, with their several orbits, encompass the Sun.*

That Mercury and Venus revolve about the Sun, is evident from their moon-like appearances. When they shine out with a full face, they are in respect of us, beyond or above the Sun; when they appear half-full, they are about the same height on one side or other of the Sun; when horn'd, they are below or between, us and the Sun, and they are sometimes, when directly under, seen like spots traversing the Sun's disk. That Mars surrounds the Sun, is as plain from its full face when near its conjunction with the Sun, and from the gibbous figure which it shews in its quadratures. And the same



same thing is demonstrable of Jupiter and Saturn, from their appearing full in all situations; for the shadows of their satellites that appear sometimes upon their disks make it plain that the light they shine with, is not their own, but borrowed from the Sun.

## PHÆNOMENON IV.

*That the fixed Stars being at rest, the periodic times of the five primary Planets, and (whether of the Sun about the Earth, or) of the Earth about the Sun, are in the sesquiquiplicate proportion of their mean distances from the Sun.*

This proportion, first observ'd by *Kepler*, is now receiv'd by all astronomers. For the periodic times are the same, and the dimensions of the orbits are the same, whether the Sun revolves about the Earth, or the Earth about the Sun. And as to the measures of the periodic times, all astronomers are agreed about them. But for the dimensions of the orbits, *Kepler* and *Bullialdus*, above all others, have determin'd them from observations with the greatest accuracy: and the mean distances corresponding to the periodic times, differ but insensibly from those which they have assign'd, and for the most part fall in between them; as we may see from the following Table.

*The periodic times, with respect to the fixed Stars, of the Planets and Earth revolving about the Sun, in days and decimal parts of a day.*

♂	♂	♂	♂	♀
10759, 275.	4332, 514.	686, 9785.	365, 2565.	224, 6176.
♂				
87, 9692.				

The

*The mean distances of the Planets and of the Earth from the Sun.*

	h	u	8
According to Kepler	951000.	519650.	152350.
To Bullialdus	954198.	522520.	152350.
To the periodic Times	954006.	520096.	152369.

	8	9	8
According to Kepler	100000.	72400.	38806.
To Bullialdus	100000.	72398.	38585.
To the periodic times	100000.	72333.	38710.

As to Mercury and Venus, there can be no doubt about their distances from the Sun; for they are determin'd by the elongations of those Planets from the Sun. And for the distances of the superior Planets, all dispute is cut off by the eclipses of the satellites of Jupiter. For, by those eclipses, the position of the shadow, which Jupiter projects, is determin'd; whence we have the heliocentric longitude of Jupiter. And from its heliocentric and geocentric longitudes compar'd together, we determine its distance.

## PHÆNOMENON V.

*Then the primary Planets, by radij drawn to the Earth, describe areas no wise proportional to the times: But that the areas, which they describe by radij drawn to the Sun, are proportional to the times of description.*

For to the Earth they appear sometimes direct, sometimes stationary, now and sometimes retrograde. But from the Sun they are always seen direct, and to proceed with a motion nearly uniform, that is to say, a little swifter in the perihelion and a little slower in the aphelion.

libre: con-  
-cider  
entre  
ayuda

aphelion distances, so as to maintain an equality in the description of the areas. This is a noted proposition among astronomers, and particularly demonstrable in Jupiter, from the eclipses of his satellites; by the help of which eclipses, as we have said, the heliocentric lon-gitudes of that Planet, and its distances from the Sun are determined.

## PHÆNOMENON VI.

*That the Moon by a radius drawn to the Earth's centre, describes an area proportional to the time of description.*

de la lune

un peu de  
l'air

This we gather from the apparent motion of the Moon, compar'd with its apparent diameter. It is true that the motion of the Moon is a little disturb'd by the action of the Sun. But in (laying down) these phænomena, I neglect those small and inconsiderable errors.

supra

THE

# THE PROPOSITIONS.

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## PROPOSITION I. THEOREM I.

*That the forces by which the circumjovial Planets are continually drawn off from recti-<sup>deriad.</sup> linear motions, and retain'd in their proper orbits, tend to Jupiter's centre; and are reciprocally as the squares of the distances of the places of those Planets from that centre.*

**T**HE former part of this proposition appears from phæn. 1. and prop. 2. or 3. book 1. The latter from phæn. 1. and cor. 6. prop. 4. of the same book.

The same thing we are to understand of the Planets<sup>extended</sup> which encompass Saturn, by phæn. 2.  
*circumdan*

## PROPOSITION II. THEOREM II.

*That the forces by which the primary Planets are continually drawn off from rectilinear motions, and retain'd in their proper orbits, tend to the Sun ; and are reciprocally as the squares of the distances of the places of those Planets from the Sun's centre.*

The former part of the proposition is manifest from phæn. 5. and prop. 2. book 1. The latter from phæn. 4. and cor. 6. prop. 4. of the same book. But this part of the proposition is, with great accuracy, demonstrable from the quiescence of the aphelion points. For a very small aberration from the reciprocal duplicate proportion, would (by cor. 1. prop. 45. book 1.) produce a motion of the apsides, sensible enough in every single revolution, and in many of them enormously great.

## PROPOSITION III. THEOREM III.

*That the force by which the Moon is retain'd in its orbit, tends to the Earth ; and is reciprocally as the square of the distance of its place from the Earth's centre.*

The former part of the proposition is evident from phæn. 6. and prop. 2. or 3. book 1. The latter from the very slow motion of the Moon's Apogee ; which in every single revolution amounting but to  $3^{\circ} 3'$ . in consequentia, may be neglected. For (by cor. 1. prop. 45. book 1.) it appears, that if the distance of the Moon from the Earth's centre, is to the semidiameter of the Earth,

Earth, as D to 1; the force, from which such a motion will result, is reciprocally as  $D^2 \frac{4}{3}$ , i. e. reciprocally as the power of D, whose exponent is  $2 \frac{4}{3}$ , that is to say, in the proportion of the distance something greater than reciprocally duplicate, but which comes  $59 \frac{1}{4}$  times nearer to the duplicate than to the triplicate proportion. But in regard that this motion is owing to the action of the Sun, (as we shall afterwards shew) it is here to be neglected. The action of the Sun, attracting the Moon from the Earth, is nearly as the Moon's distance from the Earth; and therefore (by what we have shewed in cor. 2. pr. 45. book 1.) is to the centripetal force of the Moon, as 2 to 357,45, or nearly so; that is, as 1 to  $178 \frac{2}{3}$ . And if we neglect so inconsiderable a force of the Sun, the remaining force, by which the Moon is retained in its orb, will be reciprocally as  $D^2$ . This will yet more fully appear from comparing this force with the force of gravity, as is done in the next proposition.

COR. If we augment the mean centripetal force by which the Moon is retained in its orb, first in the proportion of  $177 \frac{2}{3}$  to  $178 \frac{2}{3}$ , and then in the duplicate proportion of the semidiameter of the Earth to the mean distance of the centres of the Moon and Earth; we shall have the centripetal force of the Moon at the surface of the Earth; supposing this force, in descending to the Earth's surface, continually to increase in the reciprocal duplicate proportion of the height.

#### PROPOSITION IV. THEOREM IV.

*That the Moon gravitates towards the Earth; and, by the force of gravity is continually (drawn off) from a rectilinear motion, and retained in its orbit.*

The mean distance of the Moon from the Earth in the syzygies in semidiameters of the Earth, is, according

ding to *Ptolomy* and most Astronomers, 59, according  
 to *Vendelin* and *Huygens* 60, to *Copernicus*  $60 \frac{1}{3}$ , to *Street*  
 60  $\frac{2}{3}$ , and to *Tycho* 56  $\frac{1}{2}$ . But *Tycho*, and all that fol-  
 low his tables of refraction, making the refractions of  
 the Sun and Moon (altogether against the nature of  
 light) to exceed the refractions of the fixt Stars, and  
 that by four or five minutes near the Horizon, did  
 thereby increase the Moon's horizontal parallax, by  
 a like number of minutes, that is, by a twelfth, or fif-  
 teenth part of the whole parallax. Correct this error,  
 and the distance will become about  $60 \frac{1}{2}$  semidiameters  
 of the Earth, near to what others have assigned. Let  
 us assume the mean distance of 60 diameters in the  
 syzygies; and suppose one revolution of the Moon, in  
 respect of the fixt stars, to be completed in  $27^d. 7^h. 43'$ ,  
 as Astronomers have determined; and the circumference  
 of the Earth to amount to 123249600 *Paris feet*, as  
 the *French* have found by mensuration. And now if  
 we imagine the Moon, deprived of all motion, to be  
 let go, so as to descend towards the Earth with the  
 impulse of all that force by which (by cor. prop. 3.)  
 it is retained in its orb; it will, in the space of one  
 minute of time, describe in its fall 15  $\frac{1}{2}$  *Paris feet*.  
 This we gather by a calculus, founded either upon prop.  
 36. book 1. or (which comes to the same thing) up-  
 on cor. 9. prop. 4. of the same book. For the versed  
 sine of that arc, which the Moon, in the space of one  
 minute of time, would by its mean motion describe at  
 the distance of 60 semidiameters of the Earth, is nearly  
 15  $\frac{1}{2}$  *Paris feet*, or more accurately 15 feet, 1 inch,  
 and 1 line  $\frac{4}{9}$ . Wherefore, since that force, in ap-  
 proaching to the Earth, increases in the reciprocal du-  
 plicate proportion of the distance, and, upon that ac-  
 count, at the surface of the Earth, is  $60 \times 60$  times  
 greater, than at the Moon; a body in our regions,  
 falling with that force, ought, in the space of one mi-  
 nute of time, to describe  $60 \times 60 \times 15 \frac{1}{2}$  *Paris feet*,  
 and,

and, in the space of one second of time, to describe  $15 \frac{1}{2}$  of those feet; or more accurately 15 feet, 1 inch, and 1 line  $\frac{2}{3}$ . And with this very force we actually find that bodies here upon Earth do really descend. *again*  
 For a pendulum oscillating seconds in the latitude of *Paris*, will be 3 *Paris* feet, and 8 lines  $\frac{1}{2}$  in length, as *longitude*  
 Mr. *Huygens* has observed. And the space which a heavy body describes by falling in one second of time, is to half the length of this pendulum, in the duplicate ratio of the circumference of a circle to its diameter, (as Mr. *Huygens* has also shewn) and is therefore 15 *Paris* feet, 1 inch, 1 line  $\frac{2}{3}$ . And therefore the force by which the Moon is retained in its orbit becomes, at the very surface of the Earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by rule 1. & 2.) the force by which the Moon is retained in its orbit, is that very same force, which we commonly call gravity. For, were gravity *si quera*  
 another force different from that, then bodies descending to the Earth with the joint impulse of both forces would fall with a double velocity, and in the space of one second of time would describe  $30 \frac{1}{2}$  *Paris* feet; *combined*  
 altogether against experience. *contra experientiam*

*not complete* This calculus is founded on the hypothesis of the Earth's standing still. For, if both Earth and Moon *immobilis*  
 move about the *fixed* Sun, and at the same time about their common centre of gravity; the distance of the centres of the Moon and Earth from one another, will be  $60 \frac{1}{2}$  semidiameters of the Earth; as may be found by a computation from prop. 60. book 1.

### SCHOLIUM.

The demonstration of this proposition may be more diffusely explained after the following manner. Suppose several moons to revolve about the Earth, as in the system



stem of Jupiter or Saturn; the periodic times of these moons (by the argument of induction) would observe the same law which *Kepler* found to obtain among the Planets; and therefore their centripetal forces would be reciprocally as the squares of the distances from the centre of the Earth, by prop. 1. of this book. Now if the lowest of these were very small, and were so near the Earth as almost to touch the tops of the highest mountains; the centripetal force <sup>cent</sup> in its orb, would be very nearly equal of any terrestrial bodies that should be at the tops of those mountains, as may be seen by the foregoing computation. Therefore the moon should be deserted by its centripetal force, and so be directed <sup>cent</sup> upwards therein, it would descend <sup>cent</sup> that with the same velocity as heavy bodies fall with, upon the tops of those very mountains, because of the equality of the forces both to descend. And if the lowest moon would descend, were directed <sup>cent</sup> to the Earth, as we find terrestrial bodies do to the tops of mountains, it would then descend with the same velocity, as being impelled by both these together. Therefore since both the centripetal force of the moons, respect the centre of the Earth, (as the gravity of heavy bodies, and the centripetal force of the moons, respect the centre of the Earth, are similar and equal between themselves, by rule 1. and 2.) have one and the same force, therefore the force which retains the moon in its orb, is that very force which we call gravity, because otherwise this little moon at a certain distance, must either be without gravity, or fall as swiftly as heavy bodies use to do.

PROP

PROPOSITION V. THEOREM V.

*That the circumjovial Planets gravitate towards Jupiter; the circumsaturnal towards Saturn; the circumsolar towards the Sun; and by the forces of their gravity are (drawn <sup>or viate</sup> off) from rectilinear motions, and retained in curvilinear orbits.*

For the revolutions of the circumjovial Planets about Jupiter, of the circumsaturnal about Saturn, and of Mercury and Venus, and the other circumsolar Planets about the Sun, are appearances of the same sort with the revolution of the Moon about the Earth; and therefore by rule 2. must be owing to the same sort of causes; especially since it has been demonstrated, that the forces, upon which those revolutions depend, tend to the centres of Jupiter, of Saturn, and of the Sun; and that those forces, in receding from Jupiter, <sup>apart from</sup> from Saturn, and from the Sun, decrease in the same proportion, and according to the same law, as the force of gravity does in receding from the Earth.

COR. 1. There is therefore a power of gravity tending to all the Planets. For doubtless Venus, Mercury, and the rest, <sup>includes</sup> are bodies of the same sort with Jupiter and Saturn. And since all attraction (by law 3.) is mutual, Jupiter will therefore gravitate towards all <sup>of the</sup> his own satellites, Saturn towards his, the Earth towards the Moon, and the Sun towards all the primary Planets. <sup>for major proportion</sup>

COR. 2. The force of gravity, which tends to any one Planet, is reciprocally as the square of the distance of places from that Planet's centre.

COR. 3. All the Planets do mutually gravitate towards one another, by cor. 1. and 2. And hence it

it is, that Jupiter and Saturn, when near their conjunction, by their mutual attractions sensibly disturb each other's motions. So the Sun disturbs the motions of the Moon; and both Sun and Moon disturb our Sea, as we shall hereafter explain. *= explicat*  
*mar* *manifeste*  
*in lo futuro*

## SCHOLIUM.

The force which retains the celestial bodies in their orbits, has been hitherto called centripetal force. But *it* being now made plain, that it can be no other than a gravitating force, we shall hereafter call it gravity. For the cause of that centripetal force, which retains the Moon in its orbit, will extend it self to all the Planets by rule 1. 2. and 4.  
*ita aliter*  
*le successive*

## PROPOSITION VI. THEOREM VI.

*That all bodies gravitate towards every <sup>cademo</sup> Planet; and that the Weights of bodies towards any the same Planet, at equal distances from the centre of the Planet, are proportional to the quantities of matter which they severally contain.*  
*discretemente continen*

It has been, now of a long time, observed by others, that all sorts of heavy bodies, (allowance being made for the inequality of retardation, which they suffer from a small power of resistance in the air) descend to the Earth from equal heights in equal times: and that equality of times we may distinguish to a great accuracy, by the help of pendulums. I tried the thing in gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I provided two wooden boxes, round and equal.  
*pendulo* *de mader* *capas* *qual*

qual. I filled the one with wood, and suspended an equal weight of gold (as exactly as I could) in the centre of oscillation of the other. The boxes hanging by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally receiving the resistance of the air. And placing the one by the other, I observed them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by cor. 1. and 6. prop. 24. book 2.) was to the quantity of matter in the wood, as the action of the motive force (or *vis motrix*) upon all the gold, to the action of the same upon all the wood; that is, as the weight of the one to the weight of the other. And the like happened in the other bodies. By these experiments, in bodies of the same weight, I could manifestly have discovered a difference of matter less than the thousandth part of the whole, had any such been. But, without all doubt, the nature of gravity towards the Planets, is the same as towards the Earth. For, should we imagine our terrestrial bodies removed to the orb of the Moon, and there, together with the Moon, deprived of all motion, to be let go, so as to fall together towards the Earth: it is certain, from what we have demonstrated before, that, in equal times, they would describe equal spaces with the Moon, and of consequence are to the Moon, in quantity of matter, as their weights to its weight. Moreover, since the satellites of Jupiter perform their revolutions in times which observe the sesquiplicate proportion of their distances from Jupiter's centre, their accelerative gravities towards Jupiter will be reciprocally as the squares of their distances from Jupiter's centre; that is, equal, at equal distances. And therefore, these satellites, if supposed to fall towards Jupiter from equal heights, would describe equal spaces in equal times, in like manner as heavy bodies do on our Earth. And by the

same argument, if the circumsolar Planets were sup-  
 posed to be let fall at equal distances from the Sun, they  
 would, in their descent towards the Sun, describe equal  
 spaces in equal times. But forces, which equally ac-  
 celerate unequal bodies, must be as those bodies; that  
 is to say, the weights of the Planets *towards the Sun*  
 must be as their quantities of matter. Further, that  
 the weights of Jupiter and of his satellites towards the  
 Sun are proportional to the several quantities of their  
 matter, appears from the exceeding regular motions of  
 the satellites, (by cor. 3. prop. 65. book 1.) For if some  
 of those bodies were more strongly attracted to the  
 Sun in proportion to their quantity of matter, than  
 others; the motions of the satellites would be disturbed  
 by that inequality of attraction (by cor. 2. prop. 65.  
 book 1.) If, at equal distances from the Sun, any sa-  
 tellite in proportion to the quantity of its matter, did  
 gravitate towards the Sun, with a force greater than  
 Jupiter in proportion to his, according to any given  
 proportion, suppose of  $d$  to  $e$ ; then the distance be-  
 tween the centres of the Sun and of the satellite's orbit  
 would be always greater than the distance between the  
 centres of the Sun and of Jupiter, nearly in the sub-  
 duplicate of that proportion; as by some computations  
 I have found. And if the satellite did gravitate towards  
 the Sun with a force, lesser in the proportion of  $e$   
 to  $d$ , the distance of the centre of the satellite's orb  
 from the Sun, would be less than the distance of the  
 centre of Jupiter from the Sun, in the subduplicate of  
 the same proportion. Therefore if, at equal distances  
 from the Sun, the accelerative gravity of any satellite  
 towards the Sun were greater or less than the accele-  
 rative gravity of Jupiter towards the Sun, but by one  
 $\frac{1}{n}$  part of the whole gravity; the distance of the  
 centre of the satellite's orbit from the Sun would be  
 greater or less than the distance of Jupiter from the  
 Sun, by one  $\frac{1}{n}$  part of the whole distance; that is,

$\frac{1}{n}$   
 by

by

by a fifth part of the distance of the utmost satellite *mas distante* from the centre of Jupiter; an excentricity of the orbit, which would be very sensible. But the orbits of the satellites are concentric to Jupiter, and therefore the accelerative gravities of Jupiter, and of all its satellites towards the Sun, are equal among themselves. And by the same argument, the weights of Saturn and of his satellites towards the Sun, at equal distances from the Sun, are as their several quantities of matter: and the weights of the Moon and of the Earth towards the Sun, are either none, or accurately proportional to the masses of matter which they contain. But some they are by cor. 1. and 3. prop. 5.

But further, the weights of all the parts of every *adema* Planet towards any other Planet, are one to another as the matter in the several parts. For if some parts *hiicere* did gravitate more, others less, than for the quantity of their matter; then the whole Planet, according to the sort of parts with which it most *abundat* abounds, would gravitate more or less, than in proportion to the quantity of matter in the whole. Nor is it of any *ni* moment, whether these parts are external or internal. For, if, for example, we should imagine the terrestrial bodies with us to be raised up to the orb of the Moon, to be there compared with its body: If the weights of such bodies were to the weights of the external parts of the Moon, as the quantities of matter in the one and in the other respectively; but to the weights of the internal parts, in a greater or less proportion, then likewise the weights of those bodies would be to the weight of the whole Moon, in a greater or less proportion; *consequenter* against what we have shewed above.

COR. 1. Hence the weights of bodies do not depend upon their forms and textures. For if the weights could be altered with the forms, they would be greater or less, according to the variety of forms, in equal matter; altogether against experience.

*not completely confutatio*

COR.

*decrease*  
*entire*  
*made*  
 COR. 2. Universally, all bodies about the Earth gravitate towards the Earth; and the weights of all, at equal distances from the Earth's centre, are as the quantities of matter which they severally contain. This is the quality of all bodies, within the reach of our experiments; and therefore, (by rule 3.) to be affirmed of all bodies whatsoever. If the *ether*, or any other body, were either altogether void of gravity, or were to gravitate less in proportion to its quantity of matter; then, because (according to *Aristotle, Des Cartes*, and others) there is no difference betwixt that and other bodies, but in *mere* form of matter, by a successive change from form to form, it might be changed at last into a body of the same condition with those which gravitate most in proportion to their quantity of matter; and, on the other hand, the heaviest bodies, acquiring the first form of that body, might by degrees, quite lose their gravity. And therefore the weights would depend upon the forms of bodies, and with those forms might be changed, contrary to what was proved in the preceding corollary.

*clearer*  
*clearer*  
*no matter*  
*no matter*  
 COR. 3. All spaces are not equally Full. For if all spaces were equally full, then the specific gravity of the fluid which fills the region of the air, on account of the extreme density of the matter, would fall *relatively* short of the specific gravity of quick-silver, or gold, for any other the most dense body; and therefore, neither gold, nor any other body, could descend in air. *no matter* For bodies do not descend in fluids, unless they are specifically heavier than the fluids. And if the quantity of matter in a given space, can, by any rarefaction, be diminished, what should hinder a diminution to infinity? *no matter* *impedire*

*concedere*  
 COR. 4. If all the solid particles of all bodies are of the same density, nor can be rarified without pores a void space or vacuum must be granted. By bodies *of*

of the same density, I mean those, whose *vires inertiae* are in the proportion of their *bulks*. = *volumen*

COR. 5. The power of gravity is of a different nature from the power of magnetism. For the magnetic attraction is not as the *matter* attracted. Some bodies are attracted more by the magnet, others less; most bodies not at all. The power of magnetism, in one and the same body, may be increased and diminished; and is sometimes *far* stronger, for the quantity of matter, than the power of gravity; and in receding from the magnet, decreases not in the duplicate, but almost in the triplicate proportion of the distance, as nearly as I could judge from some rude observations. *lat*

# PROPOSITION VII. THEOREM VII.

*That there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain.*

That all the Planets mutually gravitate one towards another, we have prov'd before; as well as that the force of gravity towards every one of them, consider'd apart, is reciprocally as the square of the distance of places from the centre of the planet. And thence (by prop. 69. book. 1. and its corollaries) it follows, that the gravity tending towards all the Planets, is proportional to the matter which they contain.

Moreover, since all the parts of any planet *A* gravitate towards any other planet *B*; and the gravity of every part is to the gravity of the whole, as the matter of the part to the matter of the whole; and (by law 3.) to every action corresponds an equal re-action: therefore the planet *B* will, on the other hand, gravitate towards all the parts of the planet *A*; and its gravity towards any one part will be to the gravity towards the whole,

VOL. II.

Q

whole,



whole, as the matter of the part to the matter of the whole. *Q. E. D.*

*wige*  
*dan*  
*pendida*  
*underando*  
*yponda*  
COR. 1. Therefore the force of gravity towards any whole planet, arises from, and is compounded of, the forces of gravity towards all its parts. Magnetic and electric attractions afford us examples of this. For all attraction towards the whole arises from the attractions towards the several parts. The thing may be easily understood in gravity, if we consider a greater planet, as form'd of a number of lesser planets, meeting together in one globe. For hence it would appear that the force of the whole must arise from the forces of the component parts. If it is objected, that, according to this law, all bodies with us must mutually gravitate one towards another, whereas no such gravitation any where appears: I answer, that since the gravitation towards these bodies is to the gravitation towards the whole Earth, as these bodies are to the whole Earth, the gravitation towards them must be (far less) than to fall under the observation of our senses. *much more cal*

COR. 2. The force of gravity towards the several equal particles of any body, is reciprocally as the square of the distance of places from the particles; as appears from cor. 3. prop. 74. book 1.

### PROPOSITION. VIII. THEOREM VIII.

*de*  
In two spheres mutually gravitating each towards the other, if the matter in places on all sides round about and equidistant from the centres, is similar; the weight of either sphere towards the other, will be reciprocally as the square of the distance between their centres.

After I had found that the force of gravity towards a whole planet did arise from, and was compounded of the *betw* *face*

the forces of gravity towards all its parts; and towards every one part, was in the reciprocal proportion of the squares of the distances from the part: I was yet in doubt, whether that reciprocal duplicate proportion did accurately hold, or but nearly so, in the total force compounded of so many partial ones. For it might be that the proportion which accurately enough took place in greater distances, should be wide of the truth near the surface of the planet, where the distances of the particles are unequal, and their situation dissimilar. But by the help of prop. 75. and 76. book 1. and their corollaries, I was at last satisfy'd of the truth of the proposition, as it now lies before us.

COR. I. Hence we may find and compare together the weights of bodies towards different planets. For the weights of bodies revolving in circles about planets, are (by cor. 2. prop. 4. book 1.) as the diameters of the circles directly, and the squares of their periodic times reciprocally; and their weights at the surfaces of the planets, or at any other distances from their centres, are (by this prop.) greater or less, in the reciprocal duplicate proportion of the distances. Thus from the periodic times of Venus, revolving about the Sun, in  $224^d. 16\frac{1}{4}^h.$  of the utmost circumjovial satellite revolving about Jupiter, in  $16^d. 16\frac{1}{2}^h$ ; of the Hugonian satellite about Saturn in  $15^d. 22\frac{2}{3}^h$ ; and of the Moon about the Earth in  $27^d. 7^h. 43'$ ; compared with the mean distance of Venus from the Sun, and with the greatest heliocentric elongations of the outmost circumjovial satellite from Jupiter's centre,  $8', 16''$ . of the Hugonian satellite from the centre of Saturn,  $3'. 4''$ , and of the Moon from the Earth,  $10'. 33''$ ; by computation I found, that the weight of equal bodies, at equal distances from the centres of the Sun, of Jupiter, of Saturn, and of the Earth, towards the Sun, Jupiter, Saturn, and the Earth, were one to another, as  $1, \frac{1}{1067}, \frac{1}{3021},$  and  $\frac{1}{169282}$  respectively. Then because as

Q. 2

the

the distances are increased or diminished, the weights are diminished or increased in a duplicate ratio; the weights of equal bodies towards the Sun, Jupiter, Saturn, and the Earth, at the distances 10000, 997, 791 and 109 from their centres, that is, at their very superficies, will be as 10000, 943, 529 and 435 respectively. How much the weights of bodies are at the superficies of the Moon, will be shewn hereafter.

*or to find* COR. 2. Hence likewise we discover the quantity of matter in the several Planets. For their quantities of matter are as the forces of gravity at equal distances from their centres, that is, in the Sun, Jupiter, Saturn, and the Earth, as 1,  $\frac{1}{1067}$ ,  $\frac{1}{3021}$ , and  $\frac{1}{169282}$  respectively. If the parallax of the Sun be taken greater or less than 10'', 30'', the quantity of matter in the Earth must be augmented or diminished in the triplicate of that proportion.

COR. 3. Hence also we find the densities of the Planets. For (by prop. 72. book I.) the weights of equal and similar bodies towards similar spheres, are, at the surfaces of those spheres, as the diameters of the spheres. And therefore the densities of dissimilar spheres are as those weights applied to the diameters of the spheres. But the true diameters of the Sun, Jupiter, Saturn, and the Earth, were one to another as 10000, 997, 791 and 109; and the weights towards the same, as 10000, 943, 529, and 435 respectively; and therefore their densities are as 100,  $94\frac{1}{2}$ , 67 and 400. The density of the Earth, which comes out by this computation, does not depend upon the parallax of the Sun, but is determined by the parallax of the Moon, and therefore is here truly defin'd. The Sun therefore is a little denser than Jupiter, and Jupiter than Saturn, and the Earth four times denser than the Sun; for the Sun, by its great heat, is kept in a sort of a rarefy'd state. The Moon is denser than the Earth, as shall appear afterwards.

COR.

COR. 4. The smaller the Planets are, they are, *ceteris paribus*, of so much the greater density. For so the powers of gravity on their several surfaces, come nearer to equality. They are likewise, *ceteris paribus*, of the greater density, as they are nearer to the Sun. So Jupiter is more dense than Saturn, and the Earth than Jupiter. For the Planets were to be placed at different distances from the Sun, that according to their degrees of density, they might enjoy a greater or less proportion of the Sun's heat. Our water, if it were remov'd as far as the orb of Saturn, would be converted into ice, and in the orb of Mercury would quickly fly away in vapour. For the light of the Sun, to which its heat is proportional, is <sup>seven</sup> times denser in the orb of the Mercury than with us: and by the thermometer I have found, that a sevenfold heat of our summer-sun will make water boil. Nor are we to doubt, that the matter of Mercury is adapted to its heat, and is therefore more dense than the matter of our Earth; since, in a denser matter, the operations of nature require a stronger heat,

PROPOSITION IX. THEOREM IX.

*That the force of gravity, consider'd downwards from the surface of the planets, decreases nearly in the proportion of the distances from their centres,*

If the matter of the planet were of an uniform density, this proposition would be accurately true, (by prop. 73. book 1.) The error therefore can be no greater than what may arise from the inequality of the density.

Q 3

PRO 3

## PROPOSITION X. THEOREM X.

*That the motions of the Planets in the heavens  
may subsist an exceeding long time.* <sup>ucllo</sup>  
<sup>exercicio</sup>

In the scholium of prop. 40. book 2. I have shew'd  
that a globe of water, frozen into ice, and moving free-  
ly in our air, in the time that it would describe the  
length of its semidiameter, would lose by the resistance  
of the air  $\frac{1}{4386}$  part of its motion. And the same  
proportion holds nearly in all globes, (how great soever,)  
and mov'd with whatever velocity. But that our globe  
of earth is of greater density than it would be if the  
whole consisted of water only, I thus make out. If  
the whole consisted of water only, whatever was of  
less density than water, because of its less specific gra-  
vity, wou'd emerge and float above. And upon this  
account, if a globe of terrestrial matter, cover'd on all  
sides with water, was less dense than water, it would  
emerge somewhere; and the subsiding water falling  
back, would be gathered to the opposite side. And  
such is the condition of our Earth, which in a great  
measure is covered with seas. The Earth, if it was not  
for its greater density, would emerge from the seas, and,  
according to its degree of levity, would be raised more  
or less above their surface, the water of the seas flowing  
backwards to the opposite side. By the same argument,  
the spots of the Sun, which float upon the lucid mat-  
ter thereof, are lighter than that matter. And however  
the Planets have been form'd, while they were yet in  
fluid masses, all the heavier matter subsided to the cen-  
tre. Since therefore the common matter of our Earth  
on the surface thereof, is about twice as heavy as wa-  
ter, and a little lower, in mines, is found about three  
or four, or even five times more heavy; it is probable,  
that the quantity of the whole matter of the Earth may  
be

be five or six times greater than if it consisted all of water; especially since I have before shew'd, that the Earth is about four times more <sup>condensed</sup> dense than Jupiter. If therefore Jupiter is a little more dense than water, in the space of thirty days, in which that planet describes <sup>circumference</sup> the length of 459 of its semidiameters, it would, in a medium of the same density with our air, lose almost a tenth part of its motion. But since the resistance of <sup>mediums</sup> mediums decreases in proportion to their weight or density, so that water, which is  $13\frac{1}{2}$  times lighter than quicksilver, resists less in that proportion; and air, which is 860 times lighter than water, resists less in the same proportion: Therefore in the heavens, where the weight of the medium, in which the Planets move, is immensely diminished, the resistance will almost <sup>vanish</sup> vanish.

It is shewn in the scholium of prop. 22. book 2. that at the height of 200 miles above the Earth, the air is more rare than it is at the superficies of the Earth, in the ratio of 30 to 0,0000000000003998, or as 75000000000000 to 1 nearly. And hence the planet Jupiter, revolving in a medium of the same density with that superior air, would not lose by the resistance of the medium the 1000000th part of its motion in 1000000 years. In the spaces near the Earth, the resistance is produced only by the air, exhalations and vapours. When these are carefully exhausted by the air pump from under the receiver, heavy bodies fall within the receiver with perfect freedom, and without the least sensible resistance; gold <sup>itself</sup> and the <sup>lightest</sup> down, let fall together, will descend with equal velocity; and though they fall through a space of four, six, and eight feet, they will come to the bottom at the same time; as appears from experiments. And therefore the celestial regions being perfectly void of air and exhalations, the Planets and Comets meeting no sensible resistance in those spaces,

spaces, will continue their motions through them for an immense tract of time.

### HYPOTHESIS I.

*That the centre of the system of the world is immoveable.*

*consider* This is acknowledg'd by all, while some contend that the Earth, others, that the Sun is fix'd in that centre. Let us see what may from hence follow,

### PROPOSITION XI. THEOREM XI.

*That the common centre of gravity of the Earth, the Sun, and all the Planets is immoveable.*

For (by cor. 4. of the laws) that centre either is at rest, or moves uniformly forward in a right line. But if that centre mov'd, the centre of the world would move also, against the hypothesis.

### PROPOSITION XII. THEOREM XII.

*That the Sun is agitated by a perpetual motion, but never recedes far from the common centre of gravity of all the Planets.*

For since (by cor. 2. prop. 8.) the quantity of matter in the Sun, is to the quantity of matter in Jupiter, as 1067 to 1; and the distance of Jupiter from the Sun, is to the semidiameter of the Sun, in a proportion but a small matter greater; the common centre of gravity of Jupiter and the Sun, will fall upon a point a little without the surface of the Sun. By the same argument, since the quantity of matter in the Sun is to the quantity of matter in Saturn, as 3021 to 1; and the

the distance of Saturn from the Sun is to the semidiameter of the Sun in a proportion but a small matter less; <sup>10 diameters</sup> the common centre of gravity of Saturn and the Sun will fall upon a point a little within the surface of the Sun. And pursuing the principles of this computation, we should find that tho' the Earth and all the Planets <sup>anneau</sup> were plac'd on one side of the Sun, the distance of the common centre of gravity of all from the centre of the Sun would scarcely amount to one diameter of the Sun. <sup>10 diameters</sup> In other cases, the distances <sup>unproportion</sup> of those centres is always less. And therefore, since that centre of gravity is in perpetual rest, the Sun, according to the various positions of the Planets, must perpetually be moved <sup>each</sup> every way, but will <sup>in direction</sup> never recede far from that centre.

COR. Hence the common centre of gravity of the Earth, the Sun, and all the Planets is to be esteem'd <sup>estimated</sup> the Centre of the World. For since the Earth, the Sun and all the Planets, mutually gravitate one towards another, and are therefore, according to their powers of gravity, in perpetual agitation, as the laws of motion require; it is plain that their moveable centres cannot be taken for the immoveable centre of the world. If that body were to be plac'd in the centre, towards which other bodies gravitate most, (according to common opinion) that privilege <sup>convenient</sup> ought to be allow'd to the Sun. But since the Sun it self is mov'd, <sup>convenient</sup> a fixt point is to be chosen, from which the centre of the Sun recedes, <sup>least</sup> least, and from which it would recede yet less, <sup>le moins</sup> if the body of the Sun were denser and greater, and therefore less apt to be mov'd. <sup>le plus</sup>



## PROPOSITION XIII. THEOREM XIII.

*The Planets move in ellipses which have their common focus in the centre of the Sun; and, by radij drawn to that centre, they describe areas proportional to the times of description.*

We have discours'd above of these motions from the phenomena. Now that we know the principles on which they depend, from those principles we deduce the motions of the heavens *a priori*. Because the weights of the Planets towards the Sun, are reciprocally as the squares of their distances from the Sun's centre; if the Sun was at rest, and the other Planets did not mutually act one upon another, their orbits would be ellipses, having the Sun in their common focus; and they would describe areas proportional to the times of description by prop. 1 & 11. and cor. 1. prop. 13. book 1. But the mutual actions of the Planets one upon another, are so very small, that they may be neglected. And by prop. 66. book 1. they less disturb the motions of the Planets around the Sun in motion, than if those motions were perform'd about the Sun at rest.

It is true, that the action of Jupiter upon Saturn is not to be neglected. For the force of gravity towards Jupiter is to the force of gravity towards the Sun as 1 to 1067; and therefore in the conjunction of Jupiter and Saturn, because the distance of Saturn from Jupiter is to the distance of Saturn from the Sun, almost as 4 to 9; the gravity of Saturn towards Jupiter, will be to the gravity of Saturn towards the Sun, as 81 to  $16 \times 1067$ ; or, as 1 to about 211. And hence arises a perturbation of the orb of Saturn in every conjunction of this Planet with Jupiter, so sensible that astronomers

mers are puzzled with it. As the Planet is differently *confundit* situated in these conjunctions, its excentricity is sometimes augmented, sometimes diminish'd; its aphelion is sometimes carry'd forwards, sometimes backwards, and *transporta* its mean motion is by turns accelerated and retarded. Yet the whole error in its motion about the Sun, *sin embargo* tho' arising from so great a force, may be almost avoided (except in the mean motion) by placing the lower *en el foco mas bajo* focus of its orbit in the common centre of gravity of Jupiter and the Sun, (according to prop. 67. book 1.) and therefore that error when it is greatest, scarcely *escasamente* exceeds two minutes. And the greatest error in the mean motion, scarcely exceeds two minutes yearly. But *anualmente* in the conjunction of Jupiter and Saturn, the accelerative forces of gravity of the Sun towards Saturn, of Jupiter towards Saturn, and of Jupiter toward the Sun, are almost as 16, 81 and  $\frac{16 \times 81 \times 3021}{25}$  or 156609; and *cent* therefore the difference of the forces of gravity of the Sun towards Saturn, and of Jupiter towards Saturn, is to the force of gravity of Jupiter towards the Sun, as 65 to 156609, or as 1 to 2409. But the greatest power of Saturn to disturb the motion of Jupiter is proportional to this difference; and therefore the perturbation of the orbit of Jupiter is much less than that of Saturn's. The perturbations of the other orbits are yet *mucho* far less, except that the orbit of the Earth is sensibly disturb'd by the Moon. The common centre of gravity of the Earth and Moon moves in an ellipse about the Sun in the focus thereof, and by a radius drawn *de ella* to the Sun, describes areas proportional to the times of description. But the Earth in the mean time by a menstrual motion is revolv'd about this common centre.

PRO-

## PROPOSITION XIV. THEOREM XIV.

*The aphelions and nodes of the orbits of the Planets are fixt.*

The aphelions are immoveable, by prop. 11. book 1. and so are the planes of the orbits by prop. 1. of the same book. And if the planes are fixt, the nodes must be so too. It is true, that some inequalities may arise from the mutual actions of the Planets and Comets in their revolutions. But these will be so small that they may be here (pass'd by.) = omit it as

COR. 1. The fixt Stars are immoveable, seeing they keep the same position to the aphelions and nodes of the Planets.

COR. 2. And since these Stars are liable to no sensible parallax from the annual motion of the Earth, they can have no force, because of their immense distance, to produce any sensible effect in our system, Not to mention, that the fixt Stars, every where promiscuously dispers'd in the heavens, by their contrary attractions destroy their mutual actions, by prop. 70. book 1.

## SCHOLIUM,

Since the Planets near the Sun (*viz.* Mercury, Venus, the Earth and Mars) are so small that they can act but with little force upon each other; therefore their aphelions and nodes must be fixt, excepting in so far as they are disturb'd by the actions of Jupiter and Saturn, and other higher bodies. And hence we may find, by the theory of gravity, that their aphelions move a little *in consequentia*, in respect of the fixed Stars, and that in the sesquiplicate proportion of their several

several distances from the Sun. So that if the aphelion of Mars, in the space of an hundred years, is carried <sup>then</sup> 33'. 20". *in consequentia*, in respect of the fixed Stars; the aphelions of the Earth, of Venus, and of Mercury, will, in an hundred years be carried forwards 17'. 40". 10'. 53". and 4'. 16". respectively. But these motions are so inconsiderable, that we have neglected them in this proposition.

### PROPOSITION XV. THEOREM I.

*To find the principal diameters of the orbits of the Planets.*

They are to be taken in the subsequeplicate proportion of the periodic times by prop. 15. book 1. and then to be severally augmented in the proportion of the sum of the masses of matter in the Sun and each Planet to the first of two mean proportionals betwixt that sum and the quantity of matter in the Sun, by prop. 60. <sup>primo</sup> book 1.

### PROPOSITION XVI. PROBLEM II.

*To find the eccentricities and aphelions of the Planets.*

This problem is resolved by prop. 18. book 1.

PRO-

## PROPOSITION XVII. THEOREM XV.

*That the diurnal motions of the Planets are uniform, and that the libration of the Moon arises from its diurnal motion.*

*univ.*

The proposition is prov'd from the first law of motion, and cor. 22. prop. 66. book 1. Jupiter, with respect to the fixed Stars, revolves in  $9^h. 56'$ . Mars in  $24^h. 39'$ . Venus in about  $23^h.$  the Earth in  $23^h. 56'$ . the Sun in  $25 \frac{1}{2}$  days, and the Moon in 27 days 7 hours  $43'$ . These things appear by the phænomena. The spots in the Sun's body return to the same situation on the Sun's disk, with respect to the Earth in  $27 \frac{1}{2}$  days; and therefore with respect to the fixed Stars the Sun revolves in about  $25 \frac{1}{2}$  days. But because the lunar day, arising from its uniform revolution about its axis is menstrual, *that is, equal to the time of its periodic revolution in its orb*, therefore the same face of the Moon will be always nearly turned to the upper focus of its orb; but, as the situation of that focus requires, will deviate a little, to one side and to the other, from the Earth in the lower focus; and this is the libration in longitude. For the libration in latitude arises from the Moon's latitude, and the inclination of its axis to the plane of the ecliptic. This theory of the libration of the Moon, Mr. N. Mercator in his astronomy, published at the beginning of the Year 1676, explained more fully out of the letters I sent him. The utmost <sup>satellite</sup> of Saturn seems to revolve about its axis with a motion like this of the Moon, respecting Saturn continually with the same face. For in its revolution round Saturn, as often as it comes to the eastern part of its orbit, it is scarcely visible, and generally quite disappears; which is like to be occasioned by

by some spots in that part of its body, which is then turned toward the Earth, as M. Cassini has observed. So also the utmost satellite of Jupiter seems to revolve about its axis with a like motion, because in that part of its body which is turned from Jupiter, it has a spot, which always appears as if it were in Jupiter's own body, whenever the satellite passes between Jupiter and our eye.

PROPOSITION XVIII. THEOREM XVI.

*That the axes of the Planets are less than the diameters drawn perpendicular to the axes.*

The equal gravitation of the parts on all sides would give a sphaerical figure to the Planets, if it was not for their diurnal revolution in a circle. By that circular motion it comes to pass that the parts receding from the axe endeavour to ascend about the equator. And therefore if the matter is in a fluid state, by its ascent towards the equator it will enlarge the diameters there, and by its descent towards the poles it will shorten the axe. So the diameter of Jupiter, (by the concurring observations of astronomers) is found shorter betwixt pole and pole, than from east to west. And by the same argument, if our Earth was not higher about the equator than at the poles, the Seas would subside about the poles, and rising towards the equator, would lay all things there under water.

PROPOSITION XIX. PROBLEM III.

*To find the proportion of the axe of a Planet to the diameters perpendicular thereto.*

Our countryman Mr. Norwood, measuring a distance of 905751 feet of London measure between London and York,

*York* in 1635, and observing the difference of latitudes to be  $2^{\circ}. 28'$ , determined the measure of one degree to be 367196 feet of *London* measure, that is 57300 *Paris* toises. M. *Picart* measuring an arc of one degree, and  $22'. 55''$ . of the meridian between *Amiens* and *Malvoisine*, found an arc of one degree to be 57060 *Paris* toises. M. *Cassini* the father measured the distance upon the meridian from the town of *Colloure* in *Roussillon* to the observatory of *Paris*: And his son added the distance from the observatory to the <sup>citadel</sup> of *Dunkirk*. The whole distance was 486156 <sup>citadel</sup> toises, and the difference of the latitudes of *Colloure* and *Dunkirk* was 8 degrees, and  $31'. 11 \frac{1}{6}''$ . Hence an arc of one degree appears to be 57061 *Paris* toises. And from these measures we conclude, that the circumference of the Earth is 123249600, and its semidiameter 19615800 *Paris* feet, upon the supposition that the Earth is of a spherical figure.

In the latitude of *Paris* a heavy body falling in a second of time, describes 15 *Paris* feet, 1 inch, 1 line as above, that is, 2173 lines  $\frac{1}{2}$ . The weight of the body is diminished by the weight of the ambient air. Let us suppose the weight lost thereby to be  $\frac{1}{1000}$  part of the whole weight; <sup>weight</sup> then that heavy body falling *in vacuo* will describe a height of 2174 lines in one second of time.

A body in every sidereal day of  $23^h. 56'. 4''$ . uniformly revolving in a circle at the distance of 19615800 feet from the centre, in one second of time describes an arc of 1433, 46 feet; the versed sine of which is 0,05236561 feet, or 7,54064 lines. And therefore the force with which bodies descend in the latitude of *Paris* is to the centrifugal force of bodies in the equator arising from the diurnal motion of the Earth, as 2174 to 7,54064.

The centrifugal force of bodies in the equator, is to the centrifugal force with which bodies recede directly from

from the Earth in the latitude of *Paris*  $48^{\circ} 50' 10''$ .  
 in the duplicate proportion of the radius to the co-  
 sine of the latitude, that is, as 7,54064 to 3,267.  
 Add this force to the force with which bodies descend  
 by their weight in the latitude of *Paris*, and a body,  
 in the latitude of *Paris*, falling by its whole undimi-  
 nished force of gravity, in the time of one second, will  
 describe 2177,267 lines, or 15 *Paris* feet, 1 inch, and  
 5,267 lines. And the total force of gravity in that  
 latitude will be to the centrifugal force of bodies in  
 the equator of the Earth, as 2177,267 to 7,54064,  
 or as 289 to 1.

Wherefore if *APBQ* (*Pl. 10. Fig. 1.*) represent the fi-  
 gure of the Earth, now no longer sphaerical, but generated *above*  
 by the rotation of an ellipsis about its lesser axe; and  
*ACQqca* a canal <sup>itself</sup> full of water, reaching from the *at cause and*  
 pole *Qq* to the centre *Cc*, and <sup>thence</sup> rising to the *eleo and ore*  
 equator *Aa*: The weight of the <sup>side all</sup> water in the leg of *pierna*  
 the canal *ACca*, will be to the weight of water in the  
 other leg *QCcq*, as 289 to 288, because the centri-  
 fugal force, arising from the circular motion, sustains *additione*  
 and (takes off) one of the 289 parts of the weight (in *distinge*  
 the one leg) and the weight of 288 in the other sus-  
 tains the rest. But by computation (from cor. 2.  
 prop. 91. book 1.) I find, that if the matter of the  
 Earth was all uniform, and without any motion, and  
 its axe *PQ* were to the diameter *AB*, as 100 to 101;  
 the force of gravity in the place *Q*, towards the Earth,  
 would be to the force of gravity in the same place *Q*  
 towards a sphere describ'd about the centre *C* with  
 the radius *PC*, or *QC*, as 126 to 125. And by the  
 same argument, the force of gravity in the place *A*  
 towards the sphaeroid, generated by the rotation of the  
 ellipse *APBQ* about the axe *AB*, is to the force of  
 gravity in the same place *A*, towards the sphere de-  
 scrib'd about the centre *C* with the radius *AC*, as  
 125 to 126. But the force of gravity in the place *A*,



towards the Earth, is a mean proportional betwixt the forces of gravity towards that sphaeroid and this sphere; because the sphere, by having its diameter  $PQ$  diminished, in the proportion of 101 to 100, is transformed into the figure of the Earth; and this figure, by having a third diameter perpendicular to the two diameters  $AB$  and  $PQ$  diminish'd in the same proportion, is converted into the said sphaeroid; and the force of gravity in  $A$ , in either case, is diminish'd nearly in the same proportion. Therefore the force of gravity in  $A$ , towards the sphere describ'd about the centre  $C$ , with the radius  $AC$ , is to the force of gravity in  $A$ , towards the Earth, as 126 to  $125\frac{1}{2}$ . And the force of gravity in the place  $Q$ , towards the sphere describ'd about the centre  $C$  with the radius  $QC$ , is to the force of gravity in the place  $A$ , towards the sphere describ'd about the centre  $C$ , with the radius  $AC$ , in the proportion of the diameters, (by prop. 72. book 1.) that is, as 100 to 101. If therefore we compound those three proportions 126 to  $125\frac{1}{2}$ ,  $126$  to  $125\frac{1}{2}$ , and 100 to 101; into one: The force of gravity in the place  $Q$  towards the Earth, will be to the force of gravity in the place  $A$  towards the Earth, as  $126 \times 126 \times 100$  to  $125 \times 125\frac{1}{2} \times 101$ ; or as 501 to 500.

Now since (by cor. 3. prop. 91. book 1.) the force of gravity in either leg of the canal  $ACca$ , or  $QCcq$ , is as the distance of the places from the centre of the Earth, if those legs are conceived to be divided by transverse, parallel, and equidistant surfaces, into parts proportional to the wholes, the weights of any number of parts in the one leg  $ACca$ , will be to the weights of the same number of parts in the other leg, as their magnitudes and the accelerative forces of their gravity conjunctly, that is, as 101 to 100, and 500 to 501, or as 505 to 501. And therefore if the centrifugal force of every part in the leg  $ACca$ , arising from the diurnal motion, was to the weight of the same part, as

4 to

4 to 505, so that from the weight of every part, conceived to be divided into 505 parts, the centrifugal force might (take off) four of those parts, the weights <sup>be the same</sup> would remain equal in each leg, and therefore the fluid would rest in an equilibrium. But the centrifugal force of every part is to the weight of the same part as 1 to 289; that is, the centrifugal force which should be  $\frac{4}{505}$  parts of the weight, is only  $\frac{1}{71.25}$  part thereof. And therefore, I say, by the rule of proportion, that if the centrifugal force  $\frac{4}{505}$  make the height of the water in the leg *ACca* to exceed the height of the water in the leg *QCcq*, by one  $\frac{1}{71.25}$  part of its whole height; the centrifugal force  $\frac{1}{71.25}$  will make the excess of the height in the leg *ACca*, only  $\frac{1}{71.25}$  part of the height of the water in the other leg *QCcq*. And therefore the diameter of the Earth at the equator, is to its diameter from pole to pole, as 230 to 229. And since the mean semidiameter of the Earth, according to *Picart's* mensuration, is 19615800 *Paris* feet, or 3923,16 miles (reckoning 5000 feet <sup>calculated</sup> to a mile) the Earth will be higher at the equator, than at the poles, by 85472 feet, or 17  $\frac{1}{5}$  miles. And its height at the equator will be about 19658600 feet, and at the poles 19573000 feet.

If, the density and periodic time of the diurnal revolution remaining the same, the Planet was greater or less than the Earth; the proportion of the centrifugal force to that of gravity, and therefore also of the diameter betwixt the poles to the diameter at the equator, would likewise remain the same. But if the diurnal motion was accelerated or retarded in any proportion, the centrifugal force would be augmented or diminished nearly in the same duplicate proportion; and therefore the difference of the diameters will be increased or diminished in the same duplicate ratio very nearly. And if the density of the Planet was augmented or diminished in any proportion, the force of

R 2

gravity

gravity tending towards it would also be augmented or diminished in the same proportion; and the difference of the diameters contrarywise would be diminished in proportion as the force of gravity is augmented, and augmented in proportion as the force of gravity is diminished. Wherefore, since the Earth, in respect of the fixt Stars, revolves in  $23^h. 56'$ , but Jupiter in  $9^h. 56'$ , and the squares of their periodic times are as 29 to 5, and their densities as 400 to  $94 \frac{1}{2}$ ; the difference of the diameters of Jupiter will be to its lesser

diameter, as  $\frac{29}{5} \times \frac{400}{94 \frac{1}{2}} \times \frac{229}{1}$  to 1, or as 1 to  $9 \frac{1}{3}$  nearly.

Therefore the diameter of Jupiter from east to west, is to its diameter from pole to pole nearly as  $10 \frac{1}{3}$  to  $9 \frac{1}{3}$ . Therefore since its greatest diameter is  $37''$ , its lesser diameter lying between the poles, will be  $33'' 25'''$ . Add thereto about  $3''$  for the irregular refraction of light, and the apparent diameters of this Planet will become  $40''$  and  $36'' 25'''$ : which are to each other as  $11 \frac{1}{6}$  to  $10 \frac{1}{6}$  very nearly. These things are so upon the supposition, that the body of Jupiter is uniformly dense. But now if its body be denser towards the plane of the equator than towards the poles, its diameters may be to each other as 12 to 11, or 13 to 12, or perhaps as 14 to 13.

And *Cassini* observed in the year 1691, that the diameter of Jupiter reaching from east to west, is greater by about a fifteenth part than the other diameter. Mr. *Pound* with his 123 foot telescope, and an excellent micrometer, measured the diameters of Jupiter in the year 1719, and found them as follows.

The times.			Greatest diam.	Leffer diam.	The diam. to each other.		
day. hours.			parts.	parts.	as		
Jan.	28	6	13,40	12,28	12	to	11
Mar.	6	7	13,12	12,20	$13\frac{1}{4}$	to	$12\frac{1}{4}$
Mar.	9	7	13,12	12,08	$12\frac{2}{3}$	to	$11\frac{2}{3}$
Apr.	9	9	12,32	11,48	$14\frac{1}{2}$	to	$13\frac{1}{2}$

So that the theory agrees with the phænomena. *convene*  
For the Planets are more heated by the Sun's rays to- *calentaf -*  
wards their equators, and therefore are a little more  
condensed by that heat, than towards their poles. *calor*

Moreover, that there is a diminution of gravity oc- *admet*  
casioned by the diurnal rotation of the Earth, and  
therefore the Earth rises higher there than it does at  
the poles, (supposing that its matter is uniformly dense) *tube*  
will appear by the experiments of pendulums related  
under the following proposition.

PROPOSITION XX. PROBLEM IV.

To find and compare together the weights of bo-  
dies in the different regions of our Earth.

*aque* Because the weights of the unequal legs of the canal *rama*  
of water *ACQqca*, are equal; and the weights of the *igualant*  
parts proportional to the whole legs, and alike situated *entre*  
in them, are one to another as the weights of the  
wholes, and therefore equal betwixt themselves; the  
weights of equal parts and alike situated in the legs,  
will be reciprocally as the legs, that is, reciprocally as  
230 to 229. And the case is the same in all homo-  
geneous equal bodies alike situated in the legs of the  
canal. Their weights are reciprocally as the legs, that  
is,

R 3

is, reciprocally as the distances of the bodies from the centre of the Earth. Therefore if the bodies are situated in the uppermost parts of the canals, or on the surface of the Earth, their weights will be, one to another, reciprocally as their distances from the centre. And by the same argument, the weights in all other places round the whole surface of the Earth, are reciprocally as the distances of the places from the centre; and therefore, in the hypothesis of the Earth's being a sphæroid, are given in proportion.

Whence arises this theorem, that the increase of weight, in passing from the equator to the poles, is nearly as the versed sine of double the latitude, or, which comes to the same thing, as the square of the right sine of the latitude. And the arcs of the degrees of latitude in the meridian, increase nearly in the same proportion. And therefore, since the latitude of *Paris* is  $48^{\circ}. 50'$ , that of places under the equator,  $00^{\circ}. 00'$ . and that of places under the poles  $90^{\circ}$ ; and the versed sines of double those arcs are 11334, 00000 and 20000, the radius being 10000; and the force of gravity at the pole is to the force of gravity at the equator, as 230 to 229, and the excess of the force of gravity at the pole, to the force of gravity at the equator, as 1 to 229, the excess of the force of gravity in the latitude of *Paris*, will be to the force of gravity at the equator as  $1 \times \frac{11334}{2290000}$  to 229, or as 5667 to 2290000. And therefore the whole forces of gravity in those places will be, one to the other, as 2295667 to 2290000. Wherefore, since the lengths of pendulums vibrating in equal times, are as the forces of gravity, and in the latitude of *Paris*, the length of a pendulum vibrating seconds, is 3 *Paris* feet, and  $8 \frac{1}{2}$  lines, or rather, because of the weight of the air  $8 \frac{1}{9}$  lines; the length of a pendulum vibrating in the same time under the equator, will be shorter by 1,087 lines. And by a like calculus the following table is made.

*Latitude*

<i>Latitude of the place.</i>	<i>Length of the pendulum.</i>		<i>Measure of one degree in the meridian.</i>
Deg.	Feet.	Lines.	Toises.
0	3 .	7,468	56637
5	3 .	7,482	56642
10	3 .	7,526	56659
15	3 .	7,596	56687
20	3 .	7,692	56724
25	3 .	7,812	56769
30	3 .	7,948	56823
35	3 .	8,099	56882
40	3 .	8,261	56945
1	3 .	8,294	56958
2	3 .	8,327	56971
3	3 .	8,361	56984
4	3 .	8,394	56997
45	3 .	8,428	57010
6	3 .	8,461	57022
7	3 .	8,494	57035
8	3 .	8,528	57048
9	3 .	8,561	57061
50	3 .	8,594	57074
55	3 .	8,756	57137
60	3 .	8,907	57196
65	3 .	9,044	57250
70	3 .	9,162	57295
75	3 .	9,258	57332
80	3 .	9,329	57360
85	3 .	9,372	57377
90	3 .	9,387	57382

By this table therefore it appears, that the inequality of degrees is so small, that the figure of the Earth, in

R 4

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geographical matters, may be considered as sphaerical; especially if the Earth be a little denser towards the plane of the equator than towards the poles.

Now several astronomers sent into remote countries to make astronomical observations, have found that pendulum clocks do accordingly move slower near the equator than in our climates. And first of all in the year 1672, M. Richer took notice of it in the island of Cayenne. For when, in the month of August, he was observing the transits of the fixt Stars over the meridian, he found his clock to go slower than it ought in respect of the mean motion of the Sun, at the rate of 2'. 28". a day. Therefore (fitting up) a simple pendulum to vibrate in seconds, which were measured by an excellent clock, he observed the length of that simple pendulum; and this he did over and over every week for ten months together. And upon his return to France, comparing the length of that pendulum, with the length of the pendulum at Paris, (which was 3 Paris feet and  $8\frac{1}{2}$  lines) he found it shorter by  $1\frac{1}{4}$  line.

Afterwards our friend Dr. Halley, about the year 1677, arriving at the island of St. Helen, found his pendulum-clock to go slower there than at London, without marking the difference. But he shortened the rod of his clock, by more than the  $\frac{1}{8}$  of an inch, or  $1\frac{1}{2}$  line. And to effect this, because the length of the screw at the lower end of the rod was not sufficient, he interposed a wooden ring betwixt the nut and the ball.

Then in the year 1682. M. Varin and M. des Hayes, found the length of a simple pendulum vibrating in seconds at the royal observatory of Paris to be 3 feet and  $8\frac{1}{2}$  lines. And by the same method in the island of Goree, they found the length of an isochronal pendulum to be 3 feet and  $6\frac{1}{2}$  lines, differing from the former by two lines. And in the same year, going to the islands

islands of *Guadaloupe* and *Martinico*, they found that the length of an isochronal pendulum in those islands was 3 feet and  $6\frac{1}{2}$  lines.

After this M. Complet, the <sup>wise</sup> son, in the month of *July* 1697, <sup>at</sup> at the royal observatory of *Paris*, <sup>mea</sup> he fitted his <sup>ajunta</sup> pendulum clock to the mean motion of the Sun, that for a considerable time together, the clock agreed <sup>de continuo</sup> with the motion of the Sun. In *November* following, <sup>Paraisba</sup> upon his arrival at *Lisbon*, he found his clock to go slower than before, at the rate of 2'. 13". in 24 hours. And next <sup>12 months</sup> *March* coming to *Paraiba* he found his clock to go slower there than at *Paris*, and at the rate of 4'. 12". in 24 hours. And he affirms, that the pendulum vibrating in seconds was shorter at *Lisbon* by  $2\frac{1}{2}$  lines, and at *Paraiba* by  $3\frac{2}{3}$  lines, than at *Paris*. He had done better to have reckon'd those differences  $1\frac{1}{3}$  and  $2\frac{2}{3}$ . <sup>calculas</sup> For these differences correspond to the differences of the times 2'. 13". and 4'. 12". But this gentleman's observations are so gross, that we cannot confide in them.

In the following years 1699 and 1700. M. *des Hayes*, making another voyage to *America*, determin'd that in the islands of *Cayenne* and *Granada* the length of the pendulum vibrating in seconds was a small matter less than 3 feet and  $6\frac{1}{2}$  lines; that in the island of *St. Christophers*, it was 3 feet and  $6\frac{1}{4}$  lines; and in the island of *St. Domingo*, 3 feet and 7 lines. <sup>haciendo cosa</sup>

And in the year 1704. P. *Feuillé* at *Puerto bello* in *America*, found that the length of the pendulum vibrating in seconds, was 3 *Paris* feet, and only  $5\frac{7}{12}$  lines, that is, almost 3 lines shorter than at *Paris*; but the observation was faulty. For afterwards going to the island of *Martinico*, he found the length of the isochronal pendulum there, 3 *Paris* feet and  $5\frac{1}{2}$  lines. <sup>defectuosa</sup>

Now the latitude of *Paraiba* is 6°. 38'. south. That of *Puerto bello* 9°. 33'. north. And the latitudes of the islands *Cayenne*, *Goree*, *Guadaloupe*, *Martinico*, *Granada*, *St.*



St. *Christophers* and St. *Domingo*, are respectively  $4^{\circ} 55'$ ,  $14^{\circ} 40''$ ,  $14^{\circ} 00'$ ,  $14^{\circ} 44'$ ,  $12^{\circ} 06'$ ,  $17^{\circ} 19'$  and  $19^{\circ} 48'$ , north. And the excesses of the length of the pendulum at *Paris* above the lengths of the isochronal pendulums observ'd in those latitudes, are a little greater than by the table of the lengths of the pendulum above computed. And therefore the Earth is a little higher under the equator than by the preceding calculus, and a little denser at the centre than in mines near the surface, unless perhaps the heats of the torrid zone have a little <sup>also</sup> extended the length of the pendulums.

For M. *Picart* has observ'd, that a rod of iron, which in frosty weather in the winter season was one foot long, when heated by fire, was lengthen'd into 1 foot and  $\frac{1}{4}$  line. Afterwards M. *de la Hire* found that a rod of iron, which in the like winter season was 6 feet long; when expos'd to the heat of the summer Sun, was extended into 6 feet and  $\frac{3}{4}$  line. In the former case the heat was greater than in the latter. But in the latter it was greater than the heat of the external parts of an human body. For metals expos'd to the summer-sun, acquire a very considerable degree of heat. But the rod of a pendulum-clock is never expos'd to the heat of the summer-sun, nor ever acquires a heat equal to that of the external parts of an human body. And therefore though the 3 foot rod of a pendulum clock will indeed be a little longer in the summer than in the winter-season; yet the difference will scarcely amount to  $\frac{1}{4}$  line. Therefore the total difference of the lengths of isochronal pendulums in different climates, cannot be ascrib'd to the difference of heat. Nor indeed to the mistakes of the French astronomers. For although there is not a perfect agreement betwixt their observations, yet the errors are so small that they may be neglected; and in this they all agree, that isochronal pendulums are shorter under the equator than at the royal observatory of *Paris*, by a difference not less

less than  $1 \frac{1}{4}$  line, nor greater than  $2 \frac{2}{3}$  lines. By the observations of M. *Richer* in the island of *Cayenne*, the difference was  $1 \frac{1}{4}$  line. That difference being corrected by those of M. *des Hayes* becomes  $1 \frac{1}{2}$  line or  $1 \frac{1}{4}$  line. By the less accurate observations of others the same was made about two lines. And this disagreement might arise partly from the errors of the observations, partly from the dissimilitude of the internal parts of the Earth, and the height of mountains, partly from the different heats of the air. discrepancy made

I take an iron rod of 3 feet long to be shorter by a sixth part of one line in winter time with us here in *England*, than in the summer. Because of the great heats under the equator, subtract this quantity from the difference of one line and a quarter observ'd by M. *Richer*, and there will remain one line  $\frac{1}{4}$ , which agrees very well with  $1 \frac{87}{100}$  line collected by the theory a little before. M. *Richer* repeated his observations, made in the island of *Cayenne*, every week for 10 months together, and compared the lengths of the pendulum which he had there noted in the iron rods, with the lengths thereof which he observ'd in *France*. This diligence and care seems to have been wanting to the other observers. = gain  
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If this gentleman's observations are to be depended on, the Earth is higher under the equator than at the poles, and that by an excess of about 17 miles: as appeared above by the theory.

PROP.

## PROPOSITION XXI. THEOREM XVII.

*That the equinoctial points go <sup>retrograde</sup> backwards, and that the axe of the Earth, by a nutation in every annual revolution, twice <sup>each</sup> vibrates towards the ecliptic, and <sup>often</sup> (as often) returns to its former position. <sup>times</sup>*

The proposition appears from cor. 20. prop. 66. book 1. But that motion of nutation must be very small, and indeed scarce perceptible.

## PROPOSITION XXII. THEOREM XVIII.

*That all the motions of the Moon, and all the inequalities of those motions, follow from the principles which we have laid down.*

That the greater Planets, while they are carried about the Sun may, in the mean time, carry other lesser Planets, revolving about them; and that those lesser Planets must move in ellipses, which have their foci in the centres of the greater, appears from prop. 65. book 1. But then their motions will be several ways disturbed by the action of the Sun, and they will suffer such inequalities as are observ'd in our Moon. Thus our Moon, (by cor. 2, 3, 4, and 5. prop. 66. book 1.) moves faster, and, by a radius drawn to the Earth, describes an area, greater for the time, and has its orbit less curv'd, and therefore approaches nearer to the Earth, in the syzygies than in the quadratures, excepting in (so far as) these effects are hinder'd by the motion of eccentricity. For (by cor. 9. prop. 66. book 1.) the

the eccentricity is greatest, when the apogee of the Moon is in the syzygies, and least when the same is in the quadratures; and upon this account, the perigeon Moon is swifter, and nearer to us, but the apogee Moon slower, and farther from us, in the syzygies than in the quadratures. Moreover the apogee goes forwards, and the nodes backwards: and this is done, not with a regular, but an unequal motion. For (by cor. 7 and 8. prop. 66. book 1.) the apogee goes more swiftly forwards in its syzygies, more slowly backwards in its quadratures; and, by the excess of its progress above its regress, advances yearly in consequence. But (contrarywise) the nodes (by cor. 11. prop. 66. book 1.) are quiescent in their syzygies, and go fastest back in their quadratures. Further, the greatest latitude of the Moon, (by cor. 10. prop. 66. book 1.) is greater in the quadratures of the Moon, than in its syzygies. And (by cor. 6. prop. 66. book 1.) the mean motion of the Moon is slower in the perihelion of the Earth, than in its aphelion. And these are the principal inequalities (of the Moon) taken notice of by astronomers.

But there are yet other inequalities, not observ'd by former astronomers; by which the motions of the Moon are so disturb'd, that to this day we have not been able to bring them under any certain rule. For the velocities or horary motions of the apogee and nodes of the Moon, and their equations as well as the difference betwixt the greatest eccentricity in the syzygies, and the least eccentricity in the quadratures, and that inequality, which we call the variation, are (by cor. 14. prop. 66. book 1.) in the course of the year, augmented and diminish'd, in the triplicate proportion of the Sun's apparent diameter. And besides (by cor. 1 and 2. lem. 10. and cor. 16. prop. 66. book 1.) the variation is augmented and diminish'd, nearly in the duplicate proportion of the time between the

the quadratures. But in astronomical calculations, *this* inequality is commonly thrown into, and confounded with, the equation of the Moon's centre.

### PROPOSITION XXIII. PROBLEM V.

*To derive the unequal motions of the satellites of Jupiter and Saturn from the motions of our Moon.*

From the motions of our Moon we deduce the corresponding motions of the moons or satellites of Jupiter, in this manner, by cor. 16. prop. 66. book 1. The mean motion of the nodes of the outmost <sup>in a manner</sup> satellite of Jupiter, is to the mean motion of the nodes of our Moon, in a proportion compounded of the duplicate proportion of the periodic time of the Earth about the Sun, to the periodic time of Jupiter about the Sun, and the simple proportion of the periodic time of the satellite about Jupiter to the periodic time of our Moon about the Earth: and therefore those nodes, in the space of an hundred years, are carried  $8^{\circ}. 24'$  backwards, or *in antecedentia*. The mean motions of the nodes of the inner satellites, are to the mean motion of the nodes of the outmost, as their periodic times to the periodic time of the former, by the same corollary, and are thence given. And the motion of the apsis of every satellite *in consequentia*, is to the motion of its nodes *in antecedentia*, as the motion of the apogee of our Moon, to the motion of its nodes (by the same corollary) and is thence given. But the motions of the apsides thus found, must be diminish'd in the proportion of 5 to 9, or of about 1 to 2, on account of a cause, which I cannot here descend to explain. The greatest equations of the nodes, and of the apsides of every satellite, are to the greatest equations of the nodes, and

and apogee of our Moon respectively, as the motions of the nodes and apfides of the satellites, in the time of one revolution of the former equations, to the motions of the nodes and apogee of our Moon, in the time of one revolution of the latter equations. The variation of a satellite, seen from Jupiter, is to the variation of our Moon, in the same proportion, as the whole motions of their nodes respectively, during the times, in which the satellite and our Moon, (after parting from) are revolv'd (again) to the Sun, by the same corollary; and therefore in the outmost satellite, the variation does not exceed 5". 12".

PROPOSITION XXIV. THEOREM XIX.

*That the flux and reflux of the Sea, arise from the actions of the Sun and Moon.*

By cor. 19 and 20. prop. 66, book 1. It appears that the waters of the sea ought twice to rise and twice to fall every day, as well lunar as solar; and that the greatest height of the waters in the open and deep seas, ought to follow the appulse of the luminaries to the meridian of the place, by a less interval than 6 hours; as happens in all that eastern tract of the Atlantic and Ethiopic seas between France and the Cape of Good Hope; and on the coasts of Chili and Peru in the South-Sea; in all which shoars, the flood falls out about the second, third, or fourth hour, unless where the motion propagated from the deep ocean is by the shallowness of the channels, through which it passes to some particular places, retarded to the fifth, sixth, or seventh hour, and even later. The hours I reckon from the appulse of each luminary to the meridian of the place, as well under, as above the horizon; and by the hours of the lunar day, I understand the 24th parts of that time, which the Moon, by its apparent diurnal motion, employs

plays to come about again to the meridian of the place which it left the day before. The force of the Sun or Moon in raising the sea, is greatest in the appulse of the luminary to the meridian of the place. But the force impressed upon the sea at that time continues a little while after the impression, and is afterwards encreas'd by a new, though less, force still acting upon it. This makes the sea rise higher and higher, till this new force becoming too weak to raise it any more, the sea rises to its greatest height. And this will come to pass perhaps in one or two hours, but more frequently near the shores in about three hours, or even more where the sea is shallow.

The two luminaries excite two motions, which will not appear distinctly, but between them will arise one mixt motion compounded out of both. In the conjunction or opposition of the luminaries, their forces will be conjoin'd, and bring on the greatest flood and ebb. In the quadratures the Sun will raise the waters which the Moon depresses, and depress the waters which the Moon raises, and from the difference of their forces, the smallest of all tides will follow. And because (as experience tells us) the force of the Moon is greater than that of the Sun, the greatest height of the waters will happen about the third lunar hour. Out of the syzygies and quadratures, the greatest tide, which by the single force of the Moon ought to fall out at the third lunar hour, and by the single force of the Sun at the third solar hour, by the compounded forces of both must fall out in an intermediate time, that approaches nearer to the third hour of the Moon, than to that of the Sun. And therefore while the Moon is passing from the syzygies to the quadratures, during which time the 3d hour of the Sun precedes the 3d hour of the Moon, the greatest height of the waters will also precede the 3d hour of the Moon; and that, by the greatest interval, a little after the octants of the Moon;

Moon; and by like intervals, the greatest tide will follow <sup>maxima</sup> the 3d lunar hour, while the Moon is passing from the quadratures to the syzygies. Thus it happens in the open sea. For in the mouths of rivers, the greater tides <sup>subsides</sup> come later to their height.

But the effects of the luminaries depend upon their distances from the Earth. For when they are less distant, their effects are greater, and when more distant, their effects are less, and that in the triplicate proportion of their apparent diameter. Therefore it is, that the Sun, in the winter time, being then in its perigee, has a greater effect, and makes the tides in the syzygies <sup>maxima</sup> something greater, and those in the quadratures something less than in the summer season; and every month the Moon, while in the perigee, raises greater tides <sup>maxima</sup> than at the distance of 15 days before or after, when it is in its apogee. Whence it comes to pass, that two highest tides don't follow, one the other, in two immediately succeeding syzygies.

The effect of either <sup>code</sup> <sup>antya</sup> luminary doth likewise depend upon its declination or distance from the equator. For, if the luminary was plac'd at the pole, it would constantly attract all the parts of the waters, without any intension or remission of its action, and could cause no reciprocation of motion. And therefore, as the luminaries decline from the equator towards either pole, they will, by degrees, lose their force, and on this account will excite lesser tides in the solstitial than in the equinoctial syzygies. But in the solstitial quadratures, they will raise greater tides than in the quadratures about the equinoxes; because the force of the Moon then situated in the equator, most exceeds the force of the Sun. Therefore the greatest tides fall out in those syzygies, and the least in those quadratures, which happen about the time of both equinoxes; and the greatest tide in the syzygies is always succeeded by the least tide in the quadratures, as we find by experience.

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But, because the Sun is less distant from the Earth in winter than in summer, it comes to pass that the greatest and least tides more frequently appear before than after the vernal equinox, and more frequently after than before the autumnal.

Moreover, the effects of the luminaries depend upon the latitudes of places. Let *ApEP Pl. 10. Fig. 2.* represent the Earth cover'd with deep waters; *C* its centre; *P, p* its poles; *AE* the equator; *F*, any place without the equator; *Ff*, the parallel of the place; *Dd* the correspondent parallel on the other side of the equator; *L*, the place of the Moon three hours before; *H*, the place of the Earth directly under it; *h*, the opposite place; *K, k* the places at 90 degrees distance; *CH, Ch*, the greatest heights of the sea from the centre of the Earth; and *CK, ck* its least heights: and if with the axes *Hh, Kk*, an ellipsis is describ'd, and by the revolution of that ellipsis about its longer axis *Hh*, a sphæroid *HP Kbpk* is form'd, this sphæroid will nearly represent the figure of the sea; and *CF, Cf, CD, Cd*, will represent the heights of the sea in the places *Ff, Dd*. But further, in the said revolution of the ellipsis any point *N* describes the circle *NM*, cutting the parallels *Ff, Dd*, in any places *RT*; and the equator *AE* in *S*; *CN* will represent the height of the sea in all those places *R, S, T*, situated in this circle. Wherefore in the diurnal revolution of any place *F*, the greatest flood will be in *F*, at the 3d hour after the appulse of the Moon to the meridian above the Horizon; and afterwards the greatest ebb in *Q*, at the 3d hour after the setting of the Moon: and then the greatest flood in *f*, at the 3d hour after the appulse of the Moon to the meridian under the horizon, and lastly, the greatest ebb in *Q*, at the 3d hour after the rising of the Moon; and the latter flood in *f*, will be less than the preceding flood in *F*. For the whole sea is divided into two hemispherical floods, one in the hemisphere *KHk* on the north side, the

the other in the opposite hemisphere *Kkk* which we may therefore call the <sup>great</sup> northern and the southern floods. *meridional* These floods being always opposite the one to the other, come by turns to the meridians of all places; after an interval of 12 lunar hours. And seeing the northern countries partake more of the northern flood, and the southern countries more of the southern flood, thence arise tides, alternately greater and less in all places without the equator, in which the luminaries rise and set. *participa* But the greatest tide will happen, when the Moon declines towards the vertex of the place, about the 3d *marcas* hour after the appulse of the Moon to the meridian above the horizon; and when the Moon changes its declination *verte* to the other side of the equator, that which was the greater tide will be chang'd into a lesser. And the greatest difference of the floods will fall out about the times of the solstices; especially if the ascending node of the Moon is about the first of Aries. So it is found by experience, that the morning tides in winter exceed those of the evening, and the evening tides in summer exceed those of the morning; at *Plymouth* by the height of one foot, but at *Bristol*, by the height of 15 inches, according to the observations of *Colepreſt* and *Sturmy*. *marca* *primero* *matina* *tarde*

But the motions which we have been describing, suffer some alteration from that force of reciprocation, which the waters, being once moved, retain a little while *by their vis insita*. Whence it comes to pass that the tides may continue for some time, tho' the actions of the luminaries should cease. This power of retaining the impress'd motion lessens the difference of the alternate tides and makes those tides which immediately succeed after the syzygies greater, and those which follow next after the quadratures, less. And hence it is, that the alternate tides at *Plymouth* and *Bristol*, don't differ much more one from the other than by the height of a foot or 15 inches, and that the greatest tides of all at

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those ports are not the first but the third after the syzygies. And besides all the motions are retarded in their passage through shallow channels, so that the greatest tides of all in some streights and mouths of rivers, are the fourth or even the fifth after the syzygies.

Farther it may happen that the tide may be propagated from the ocean through different channels towards the same port, and may pass quicker through some channels than through others, in which case the same tide, divided into two or more succeeding one another, may compound new motions of different kinds. Let us suppose two equal tides flowing towards the same port from different places, the one preceding the other by 6 hours; and suppose the first tide to happen at the third hour of the appulse of the Moon to the meridian of the port. If the Moon at the time of the appulse to the meridian was in the equator, every 6 hours alternately there would arise equal floods, which meeting with as many equal ebbs would so ballance one the other, that for that day the water would stagnate and remain quiet. If the Moon then declined from the equator, the tides in the ocean would be alternately greater and less as was said. And from thence two greater and two lesser tides would be alternately propagated towards that port. But the two greater floods would make the greatest height of the waters to fall out in the middle time betwixt both; and the greater and lesser floods would make the waters to rise to a mean height in the middle time between them, and in the middle time between the two lesser floods the waters would rise to their least height. Thus in the space of 24 hours the waters would come, not twice, as commonly, but once only to their greatest, and once only to their least height; and their greatest height, if the Moon declined towards the elevated pole, would happen at the 6 or 30th hour after the appulse of the Moon to the meridian; and when the Moon changed its declination this flood would

would be changed into an ebb. An example of all which Dr. *Halley* has given us, from the observations of seamen in the port of *Batsham* in the kingdom of *Tunquin* in the latitude of  $20^{\circ}. 50'$ . north. In that port, on the day which follows after the passage of the Moon over the equator, the waters stagnate: when the Moon declines to the north they begin to flow and ebb, not twice, as in other ports, but once only every day, and the flood happens at the setting, and the greatest ebb at the rising of the Moon. This tide encreases with the declination of the Moon till the 7th or 8th day; then for the 7 or 8 days following, it decreases at the same rate as it had increased before, and ceases when the Moon changes its declination, crossing over the equator to the south. After which the flood is immediately chang'd into an ebb; and thenceforth the ebb happens at the setting, and the flood at the rising of the Moon; till the Moon again passing the equator changes its declination. There are two inlets to this port, and the neighbouring channels, one from the seas of *China*, between the continent and the island of *Luconia*, the other from the *Indian sea*, between the continent and the island of *Borneo*. But whether there be really two tides propagated through the said channels, one from the *Indian sea* in the space of 12 hours, and one from the sea of *China* in the space of 6 hours, which therefore happening at the 3d and 9th lunar hours, by being compounded together, produce those motions, or whether there be any other circumstances in the state of those seas, I leave to be determin'd by observations on the neighbouring shoars.

Thus I have explain'd the causes of the motions of the Moon and of the Sea. Now it is fit to subjoin something concerning the quantity of those motions.

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## PROPOSITION XXV. PROBLEM VI.

*To find the forces with which the Sun disturbs the motions of the Moon.* Pl. 10. Fig. 3.

Let  $S$  represent the Sun,  $T$  the Earth,  $P$  the Moon,  $CADB$  the Moon's orbit. In  $SP$  take  $SK$  equal to  $ST$ ; and let  $SL$  be to  $SK$ , in the duplicate<sup>(1)</sup> proportion of  $SK$  to  $SP$ ; draw  $LM$  parallel to  $PT$ ; and if  $ST$  or  $SK$  is suppos'd to represent the accelerated force of gravity of the Earth towards the Sun,  $SL$  will represent the accelerative force of gravity of the Moon towards the Sun. But that force is compounded of the parts  $SM$  and  $LM$ , of which the force  $LM$ , and that part of  $SM$  which is represented by  $TM$ , disturb the motion of the Moon, as we have shew'd in prop. 66. book 1. and its corollaries. Forasmuch as the Earth and Moon are revolv'd about their common centre of gravity, the motion of the Earth about that centre will be also disturb'd by the like forces, but we may consider the sums both of the forces and of the motions as in the Moon, and represent the sum of the forces by the lines  $TM$  and  $ML$ , which are analogous to them both. The force  $ML$  (in its mean quantity) is, to the centripetal force by which the Moon may be retain'd in its orbit revolving about the Earth at rest at the distance  $PT$ , in the duplicate proportion of the periodic time of the Moon about the Earth, to the periodic time of the Earth about the Sun (by cor. 17. prop. 66. book 1.) that is in the duplicate proportion of  $27^d. 7^h. 43'$ . to  $365^d. 6^h. 9'$ ; or as 1000 to 178725; or as 1 to  $178\frac{1}{4}$ . But in the 4th prop. of this book we found, that if both Earth and Moon were revolv'd about their common centre of gravity, the mean distance of the one from the other would be nearly  $60\frac{1}{2}$  mean semidiameters of the Earth. And

(1) How you can find the duplicate proportion - the  
- 1000 to 178725 is the duplicate proportion.

the force, by which the Moon may be kept revolving in its orbit about the Earth in rest at the distance  $PT$  of  $60\frac{1}{2}$  semidiameters of the Earth, is to the force by which it may be revolv'd in the same time at the distance of 60 semidiameters, as  $60\frac{1}{2}$  to 60; and this force is to the force of gravity with us, very nearly as 1 to  $60 \times 60$ . Therefore the mean force  $ML$  is to the force of gravity on the surface of our Earth, as  $1 \times 60\frac{1}{2}$  to  $60 \times 60 \times 60 \times 178\frac{2}{3}$ , or as 1 to 638092, 6. whence by the proportion of the lines  $TM$ ,  $ML$ , the force  $TM$  is also given; and these are the forces with which the Sun disturbs the motions of the Moon. Q E I.

# PROPOSITION XXVI. PROBLEM VII.

To find the horary increment of the area, which the Moon, by a radius drawn to the Earth, describes in a circular orbit.

We have above shew'd that the area, which the Moon describes by a radius drawn to the Earth, is proportional to the time of description; excepting in (so far as) the Moon's motion is disturb'd by the action of the Sun. And here we propose to investigate the inequality of the moment, or horary increment of that area, or motion so disturb'd. To render the calculus more easy, we shall suppose the orbit of the Moon to be circular, and neglect all inequalities, but that only which is now under consideration. And because of the immense distance of the Sun, we shall further suppose, that the lines  $SP$  and  $ST$ , are parallel. By this means, the force  $LM$  Pl. 10. Fig. 4. will be always reduc'd to its mean quantity  $TP$ , as well as the force  $TM$ , to its mean quantity  $3 PK$ . These forces, (by cor. 2. of the laws of motion) compose the force  $TL$ ; and this force by letting fall the

perpendicular  $LE$  upon the radius  $TP$ , is resolv'd into the forces  $TE$ ,  $EL$ ; of which the force  $TE$ , acting constantly in the direction of the radius  $TP$ , neither accelerates or retards the description of the area  $TPC$ , made by that radius  $TP$ ; but  $EL$  acting on the radius  $TP$  in a perpendicular direction, accelerates or retards the description of the area in proportion as it accelerates or retards the Moon. That acceleration of the Moon, in its passage from the quadrature  $C$ , to the conjunction  $A$ , is in every moment of time, as the generating accelerative force  $EL$ , that is, as  $\frac{3 PK \times TK}{TP}$  Let the time

be represented by the mean motion of the Moon, or (which comes to the same thing) by the angle  $CTP$ , or even by the arc  $CP$ . At right angles upon  $CT$ , erect  $CG$  equal to  $CT$ . And supposing the quadrantal arc  $AC$  to be divided into an infinite number of equal parts  $Pp$  &c. these parts may represent the like infinite number of the equal parts of time. Let fall  $pk$  perpendicular on  $CT$ ; and draw  $TG$  meeting with  $KP$ ,  $kp$  produc'd, in  $F$  and  $f$ ; then will  $FK$  be equal to  $TK$ , and  $Kk$  be to  $PK$  as  $Pp$  to  $Tp$ , that is, in a giv'n proportion; and therefore  $FK \times Kk$ , or the area  $FKkf$ , will be as  $\frac{3 PK \times TK}{TP}$ , that is as  $EL$ ; and

compounding, the whole area  $GCKF$  will be as the sum of all the forces  $EL$  impress'd upon the Moon in the whole time  $CP$ ; and therefore also as the velocity generated by that sum, that is, as the acceleration of the description of the area  $CTP$ , or as the increment of the moment thereof. The force by which the Moon may in its periodic time  $CADB$  of  $27^d. 7^h. 43'$ , be retain'd revolving about the Earth in rest at the distance  $TP$ , would cause a body, falling in the time  $CT$ , to describe the length  $\frac{1}{2} CT$ , and at the same time to acquire a velocity equal to that with which the Moon is

is mov'd in its orbit. This appears from cor. 9. prop. 4. book 1. But since  $Kd$ , drawn perpendicular on  $TP$ , is *but* a third part of  $EL$ , and *equal* to the half of  $TP$ , or  $ML$ , in the octants, the force  $EL$  in the octants, *where* *double* it is greatest, will exceed the force  $ML$ , in the proportion of 3 to 2; and therefore will be to that force by which the Moon in its periodic time may be retain'd revolving about the Earth at rest, as 100 to  $\frac{2}{3} \times 17872\frac{1}{2}$ , or 11915; and in the time  $CT$  will generate a velocity equal to  $\frac{100}{11915}$  parts of the velocity of the Moon; but in the time  $CPA$ , will generate a greater velocity in the proportion of  $CA$  to  $CT$  or  $TP$ . Let the greatest force  $EL$  in the octants be represented by the area  $FK \times Kk$ , or by the rectangle  $\frac{1}{2} TP \times Pp$ , which is equal thereto. And the velocity which that *a ello* greatest force can generate in any time  $CP$ , will be to the velocity which any other lesser force  $EL$  can *minor* generate in the same time, as the rectangle  $\frac{1}{2} TP \times CP$  to the area  $KCGF$ ; but the velocities generated in the whole time  $CPA$ , will be one to the other as the rectangle  $\frac{1}{2} TP \times CA$  to the triangle  $TCG$ ; or as the quadrantal arc  $CA$  to the radius  $TP$ . And therefore (by prop. 9. book 5. elem.) the latter velocity generated in the whole time, will be  $\frac{100}{11915}$  parts of the velocity of the Moon. To this velocity of the Moon, which is proportional to the mean moment of the area (supposing this mean moment to be represented by the number 11915) we add and subtract the half of the other velocity; the sum 11915 + 50, or 11965 will represent the greatest moment of the area in the syzygy  $A$ ; and the difference 11915 - 50, or 11865, the least moment thereof in the quadratures. *el minor* Therefore the areas, which in equal times, are described in the syzygies and quadratures, are, one to the other, as 11965 to 11865. And if to the least moment 11865, we add a moment which shall be *to*



*but 29.5*  
*conjunction*  
*cuadrado*
to 100, the difference of the two former moments as the trapezium  $FKCG$  to the triangle  $TCG$ , or which comes to the same thing, as the square of the sine  $PK$  to the square of the radius  $TP$ , (that is, as  $Pd$  to  $TP$ ) the sum will represent the moment of the area, when the Moon is in any intermediate place  $P$ .

But these things take place, only in the hypothesis that the Sun and the Earth are at rest, and that the synodical revolution of the Moon is finished in  $27^d. 7^h. 43'$ . But since the Moon's synodical period is really  $29^d. 12^h. 44'$ , the increments of the moments must be enlarged, in the same proportion as the time is, that is, in the proportion of 1080853 to 1000000. Upon which account, the whole increment, which was  $\frac{100}{21913}$  parts of the mean moment, will now become  $\frac{100}{11023}$  parts thereof. And therefore the moment of the area, in the quadrature of the Moon, will be to the moment thereof in the syzygy, as 11023—50 to 11023—50; or as 10973 to 11073; and to the moment thereof when the Moon is in any intermediate place  $P$ , as 10973 to 10973— $Pd$ ; that is, supposing  $TP=100$ .

The area therefore, which the Moon, by a radius drawn to the Earth, describes in the several little equal parts of time, is nearly as the sum of the number 219,46, and the versed sine of the double distance of the Moon from the nearest quadrature, considered in a circle which hath unity for its radius. Thus it is, when the variation in the octants is in its mean quantity. But if the variation there is greater or less, that versed sine must be augmented or diminished in the same proportion.

PRO-

PROPOSITION XXVII. PROBLEM VIII.

*From the horary motion of the Moon, to find its distance from the Earth.*

The area which the Moon, by a radius drawn to the Earth, describes in every moment of time, is as the horary motion of the Moon, and the square of the distance of the Moon from the Earth conjunctly. And therefore the distance of the Moon from the Earth is in a proportion compounded of the subduplicate proportion of the area directly, and the subduplicate proportion of the horary motion inversely. *Q. E. I.*

COR. 1. Hence the apparent diameter of the Moon is given. For it is reciprocally as the distance of the Moon from the Earth. Let astronomers try how accurately this rule agrees with the phænomena. *not constant*

COR. 2. Hence also the orbit of the Moon may be more exactly defin'd from the phænomena than hitherto could be done. *perhaps constant*

PROPOSITION XXVIII. PROBLEM IX.

*To find the Diameters of the orbit, in which, without eccentricity, the Moon would move.* *sub* *describ'd*

The curvature of the orbit which a body describes, if attracted in lines perpendicular to the orbit, is as the force of attraction directly, and the square of the velocity inversely. <I estimate the curvatures of lines, compared one with another, according to the evanescent proportion of the sines or tangents of their angles of contact to equal radij, supposing those radij to be infinitely diminished.> But the attraction of the Moon towards the Earth in the syzygies, is the excess of its gravity

gravity towards the Earth above the force of the Sun  $2PK$  (see *Fig.* prop. 25.) by which force, the accelerative gravity of the Moon towards the Sun exceeds the accelerative gravity of the Earth towards the Sun, or is exceeded by it. But in the quadratures that attraction is the sum of the gravity of the Moon towards the Earth, and the Sun's force  $KT$ , by which the Moon is attracted towards the

*ponendo* Earth. And these attractions, putting  $N$  for  $\frac{AT-CT}{2}$ , are

nearly as  $\frac{178725}{AT^2} + \frac{2000}{CT \times N}$  and  $\frac{178725}{CT^2} + \frac{1000}{AT \times N}$ ,

*Ad hunc* or as  $178725 N \times CT^2 - 2000 AT^2 \times CT$ , and  $178725 N \times AT^2 - 1000 CT^2 \times AT$ . For if the accelerative gravity of the Moon towards the Earth be represented by the number 178725, the mean force  $ML$ , which in the quadratures is  $PT$  or  $TK$ , and draws the Moon towards the Earth, will be 1000; and the mean force  $TM$ , in the syzygies will be 3000; from which, if we subtract the mean force  $ML$ , there will remain 2000, the force by which the Moon in the syzygies is drawn from the Earth, and which we above called  $2PK$ . But the velocity of the Moon in the syzygies  $A$  and  $B$ , is to its velocity in the quadratures  $C$  and  $D$ , as  $CT$  to  $AT$ , and the moment of the area, which the Moon by a radius drawn to the Earth describes in the syzygies, to the moment of that area described in the quadratures conjunctly; that is, as 11073  $CT$  to 10973  $AT$ . Take this ratio twice inversely, and the former ratio once directly, and the curvature of the orb of the Moon in the syzygies will be to the curvature thereof in the quadratures, as  $120406729 \times 178725 AT^2 \times CT^2 \times N - 120406729 \times 2000 AT^4 \times CT$ , to  $122611329 \times 178725 AT^2 \times CT^2 \times N + 122611329 \times 1000 CT^4 \times AT$ , that is, as 2151969  $AT \times CT \times N - 24081 AT^3$  to 2191371  $AT \times CT \times N + 12261 CT^3$ .

Because

Because the figure of the Moon's orbit is unknown, let <sup>discovered</sup> us, in its <sup>large</sup>stead, assume the ellipse  $DBCA$ , Pl. 10. Fig. 5. <sup>assume</sup> in the centre of which we suppose the Earth to be situated, and the greater axe  $de$  to lie between the quadratures, as the lesser  $AB$  between the syzygies. But since the plane of this ellipse is revolved about the Earth by an angular motion, and the orbit, whose curvature we now examine should be described in a plane <sup>privat</sup>(void of) such motion; we are to consider the figure which the Moon, while it is revolved in that ellipse, describes in this plane, that is to say the figure  $Cpa$ , the several points  $p$  of which are found by assuming any point  $P$  in the ellipse, which may represent the place of the Moon, and drawing  $Tp$  equal to  $TP$ , in such manner that the angle  $PTp$  may be equal to the apparent motion of the Sun from the time of the last quadrature in  $C$ ; or (which comes to the same thing) that the angle  $CTp$  may be to the angle  $CTP$ , as the time of the synodic revolution of the Moon to the time of the periodic revolution thereof, or as  $29^d. 12^h. 44'$ , to  $27^d. 7^h. 43'$ . If therefore in this proportion we take the angle  $CTa$  to the right angle  $CTA$ , and make  $Ta$  of equal length with  $TA$ ; we shall have  $a$  the lower, and  $C$  the upper apsis of this orbit. But by computation I find, that the difference betwixt the curvature of this orbit  $Cpa$  at the vertex  $a$ , and the curvature of a circle described about the centre  $T$ , with the interval  $TA$ , is to the difference betwixt the curvature of the ellipse at the vertex  $A$ , and the curvature of the same circle, in the duplicate proportion of the angle  $CTP$  to the angle  $CTp$ ; and that the curvature of the ellipse in  $A$ , is to the curvature of that circle, in the duplicate proportion of  $TA$  to  $TC$ ; and the curvature of that circle to the curvature of a circle described about the centre  $T$  with the interval  $TC$ , as  $TC$  to  $TA$ ; but that the curvature of this last arch is to the curvature of the ellipse in  $C$ , in the duplicate

uplicate proportion of  $TA$  to  $TC$ ; and that the difference betwixt the curvature of the ellipse in the vertex  $C$ , and the curvature of this last circle, is to the difference betwixt the curvature of the figure  $Tpa$ , at the vertex  $C$ , and the curvature of this same last circle, in the duplicate proportion of the angle  $CTp$  to the angle  $CTP$ . All which proportions are easily drawn from the sines of the angles of contact, and of the differences of those angles. But by comparing those proportions together, we find the curvature of the figure  $Cpa$  at  $a$ , to be to its curvature at  $C$ , as  $AT^3 \times \frac{16824}{100000} CT^2$   $AT$  to  $CT^3$   $- \frac{16824}{100000} AT^2$   
 ya -  $\times CT$ . Where the number  $\frac{16824}{100000}$  represents the difference of the squares of the angles  $CTP$  and  $CTp$ , applied to the square of the lesser angle  $CTP$ ; or (which is all one) the difference of the squares of the times  $27^d. 7^h. 43'$ , and  $29^d. 12^h. 44'$  applied to the square of the time  $27^d. 7^h. 43'$ .

ya que Since therefore  $a$  represents the syzygy of the Moon, and  $C$  its quadrature, the proportion now found must be the same with that proportion of the curvature of the Moon's orb in the syzygies, to the curvature thereof in the quadratures, which we found above. Therefore, in order to find the proportion of  $CT$  to  $AT$ , Let us multiply the extremes and the means, and the terms which come out applied to  $AT \times CT$ , become  $2062,79 CT^4 - 2151969 N \times CT^3 + 368676 N \times AT \times CT^2 + 36342 AT^2 \times CT^2 - 362047 N \times AT^2 \times CT - 2191371 N \times AT^3 - 4051,4 AT^4 = 0$ . Now if for the half sum  $N$  of the terms  $AT$  and  $CT$ , we put  $1$ , and  $x$  for their half difference; then  $CT$  will be  $= 1 - x$ , and  $AT = 1 + x$ . And substituting those values in the equation, after resolving thereof, we shall find  $x = 0,00719$ ; and from thence the semidiameter  $CT = 1,00719$ , and the semidiameter  $AT = 0,99281$ , which numbers are nearly as  $70\frac{1}{4}$ , and  $69\frac{1}{4}$ . Therefore the Moon's distance from the Earth

Earth in the syzygies, is to its distance in the quadratures (setting aside the consideration of eccentricity) *double de la distance* as  $69\frac{1}{4}$  to  $70\frac{1}{4}$ ; or in round numbers as 69 to 70.

## PROPOSITION XXIX. PROBLEM X.

*To find the variation of the Moon.*

This inequality is owing partly to the elliptic figure of *ditto* the Moon's orbit, partly to the inequality of the moments of the area which the Moon by a radius drawn to the Earth describes. If the Moon  $P$  revolved in the ellipse  $DBCA$ , about the Earth quiescent in the centre of the ellipse, and by the radius  $TP$ , drawn to the Earth, described the area  $CTP$ , proportional to the time of description; and the greatest semidiameter  $CT$  of the ellipse was to the least  $TA$  as 70 to 69; the tangent of the angle  $CTP$  would be to the tangent of the angle of the mean motion computed from the quadrature  $C$ , as the semidiameter  $TA$  of the ellipse, to its semidiameter  $TC$ , or as 69 to 70. But the description of the area  $CTP$ , as the Moon advances from the quadrature to the syzygy, ought to be in such manner accelerated, that the moment of the area in the Moon's syzygy, may be to the moment thereof in its quadrature, as 11073 to 10973; and that the excess of the moment in any intermediate place  $P$ , above the moment in the quadrature, may be as the square of the sine of the angle  $CTP$ . Which we may effect with accuracy enough, if we diminish the tangent of the angle  $CTP$ , *quida* in the subduplicate proportion of the number 10973 to the number 11073, that is, in proportion of the number 68,6877 to the number 69. Upon which account the tangent of the angle  $CTP$ , will now be to the tangent of the mean motion, as 68,6877 to 70; and the angle  $CTP$ , in the octants, *estimation* where the *double* mean

mean motion is  $45^\circ$ , will be found  $44^\circ. 27'. 28''$ . which subtracted from  $45^\circ$ . the angle of the mean motion, leaves the greatest variation  $32'. 32''$ . Thus it would be, if the Moon in passing from the quadrature to the syzygy, described an angle  $CTA$  of 90 degrees only. But because of the motion of the Earth, by which the Sun is apparently transferr'd in *consequentia*, the Moon, before it overtakes the Sun, describes an angle  $CTA$ , greater than a right angle, in the proportion of the time of the synodic revolution of the Moon, to the time of its periodic revolution, that is, in the proportion of  $29^d. 12^h. 44'$ . to  $27^d. 7^h. 43'$ . Whence it comes to pass, that all the angles about the centre  $T$ , are dilated in the same proportion, and the greatest variation, which otherwise would be *but*  $32'. 32''$ , now augmented in the said proportion becomes  $35'. 10''$ .

And this is its magnitude in the mean distance of the Sun from the Earth, neglecting the differences, which may arise from the curvature of the *orbis magnus*, and the stronger action of the Sun upon the Moon when horn'd and new, than when gibbous and full. In other distances, of the Sun from the Earth, the greatest variation is in a proportion compounded of the duplicate proportion of the time of the synodic revolution of the Moon (the time of the year being given) directly, and the triplicate proportion of the distance of the Sun from the Earth, inversely. And therefore, in the apogee of the Sun, the greatest variation is  $33'. 14''$ , and in its perigee,  $37'. 11''$ , if the eccentricity of the Sun is to the transverse semidiameter of the *orbis magnus*, as  $16 \frac{1}{6}$  to 1000.

Hitherto we have investigated the variation in an orb not eccentric, in which, to wit, the Moon in its octants is always in its mean distance from the Earth. If the Moon, on account of its eccentricity, is more or less removed from the Earth, than if placed in this orb, the variation may be something greater, or something







s, than according to this rule. But I leave the defect to the determination of astronomers de  
the phenomena.

PROPOSITION XXX. PROBLEM XI.

*find the horary motion of the nodes of the Moon in a circular orbit, Pl. 11. Fig. 1.*

Let  $S$  represent the Sun,  $T$  the Earth,  $P$  the Moon,  $Pn$  the orbit of the Moon,  $Npn$  the orthographic projection of the orbit upon the plane of the ecliptic;  $n$  the nodes;  $nTNm$ , the line of the nodes produced indefinitely;  $PI$ ,  $PK$  perpendiculars upon the lines  $ST$ ,  $Qq$ ;  $Pp$  a perpendicular upon the plane of the ecliptic;  $A$ ,  $B$  the Moon's syzygies in the plane of the ecliptic;  $AZ$  a perpendicular let fall upon  $Nn$ , the line of the nodes;  $Q$ ,  $q$  the quadratures of the Moon in the plane of the ecliptic, and  $pK$ , a perpendicular on the line  $Qq$  lying between the quadratures. The force of the Sun to disturb the motion of the Moon (by prop. 25.) is twofold, one proportional to the line  $LM$ , the other to the line  $MT$ , in the scheme of that proposition. And the Moon by the former force is drawn towards the Earth, by the latter towards the Sun, in a direction parallel to the right line  $ST$  joining the Earth and the Sun. The former force  $LM$  acts in the direction of the plane of the Moon's orbit, and therefore makes no change upon the situation thereof, and is upon that account to be neglected. The latter force  $MT$ , by which the plane of the Moon's orbit is disturbed, is the same with the force  $3PK$  or  $3IT$ . And this force (by prop. 25.) is to the force, by which the Moon may, in its periodic time, be uniformly revolved in a circle about the Earth at rest, as  $3IT$  to the radius of the circle multiplied by the  
VOL. II. T number

number 178,725, or as  $IT$  to the radius thereof multiplied by 59,575. But in this calculus, and all that follows I consider all the lines drawn from the Moon to the Sun, as parallel to the line which joins the Earth and the Sun, because what inclination there is, almost as much diminishes all effects in some cases, as it augments them in others, and we are now enquiring after the mean motions of the nodes, neglecting such niceties as are of no moment, and would only serve to render the calculus more perplexed.

Now suppose  $PM$  to represent an arc which the Moon describes in the least moment of time, and  $ML$  a little line, the half of which the Moon, by the impulse of the said force  $3 IT$ , would describe in the same time. And joining  $PL$ ,  $MP$ , let them be produced to  $m$  and  $l$ , where they cut the plane of the ecliptic, and upon  $Tm$  let fall the perpendicular  $PH$ . Now since the right line  $ML$  is parallel to the plane of the ecliptic, and therefore can never meet with the right line  $ml$  which lies in that plane, and yet both those right lines lie in one common plane  $LM Pml$ , they will be parallel, and upon that account the triangles  $LMP$ ,  $lmp$  will be similar. And seeing  $MPm$  lies in the plane of the orbit, in which the Moon did move while in the place  $P$ ; the point  $m$  will fall upon the line  $Nn$ , which passes through the nodes  $N$ ,  $n$ , of that orbit. And because the force by which the half of the little line  $LM$  is generated, if the whole had been together, and at once impressed in the point  $P$ , would have generated that whole line, and caused the Moon to move in the arc whose chord is  $LP$ ; that is to say, would have transferred the Moon from the plane  $MPmT$  into the plane  $LPIT$ ; therefore the angular motion of the nodes generated by that force, will be equal to the angle  $mTl$ . But  $ml$  is to  $mP$ , as  $ML$  to  $MP$ ; and since  $MP$ , because of the time given, is also given,  $ml$  will be as the rectangle

angle  $ML \times mP$ , that is, as the rectangle  $IT \times mP$ . And, if  $Tml$  is a right angle, the angle  $mTl$  will be as  $\frac{ml}{Tm}$  and therefore as  $\frac{IT \times Pm}{Tm}$ , that is, (because  $Tm$  and  $mP$ ,  $TP$  and  $PH$  are proportional) as  $\frac{IT \times PH}{TP}$ ; and therefore, because  $TP$  is given, as  $IT \times PH$ . But if the angle  $Tml$  or  $STN$  is oblique, the angle  $mTl$  will be yet less, in proportion of the sine of the angle  $STN$  to the radius, or  $AZ$  to  $AT$ . And therefore the velocity of the nodes, is as  $IT \times PH \times AZ$ , or as the solid content of the sines of the three angles,  $TPl$ ,  $PTN$ , and  $STN$ .

If these are right angles, as happens when the nodes are in the quadratures, and the Moon in the syzygy, the little line  $ml$  will be removed to an infinite distance, and the angle  $mTl$  will become equal to the angle  $mPl$ . But in this case the angle  $mPl$  is to the angle  $PTM$ , which the Moon in the same time by its apparent motion describes about the Earth, as 1 to 59,575. For the angle  $mPl$  is equal to the angle  $LPm$ , that is, to the angle of the Moon's deflexion from a rectilinear path, which angle, if the gravity of the Moon should have then ceased, the said force of the Sun  $3IT$  would by it self have generated in that given time; and the angle  $PTM$  is equal to the angle of the Moon's deflexion from a rectilinear path, which angle, if the force of the Sun  $3IT$  should have then ceased, the force alone by which the Moon is retained in its orbit would have generated in the same time. And these forces (as we have above shew'd) are, the one to the other, as 1 to 59,575. Since therefore, the mean horary motion of the Moon (in respect of the fixt Stars) is  $32'. 56''. 27'''$ .  $12\frac{1}{2}^{iv}$ , the horary motion of the node in this case will be  $33''$ .  $10'''$ .  $33^{iv}$ .  $12^v$ . But in other cases, the horary motion will be to  $33''$ .  $10'''$ .

10<sup>m</sup>. 33<sup>iv</sup>. 12<sup>v</sup>. as the <sup>solid</sup> content of the sines of the three angles  $TPI$ ,  $PTN$  and  $STN$  (or of the distances of the Moon from the quadrature, of the Moon from the node, and of the node from the Sun) to the cube of the radius. And as often as the sine of any angle is changed from positive to negative, and from negative to positive, so often must the regressive be changed into a progressive, and the progressive into a regressive motion. Whence it comes to pass, that the nodes are progressive, as often as the Moon happens to be placed between either quadrature, and the node nearest to that quadrature. In other cases, they are regressive, and by the excess of the regrefs above the progress, they are monthly transferred in antecedentia.

COR. 1. Hence if from  $P$  and  $M$ , the extreme points of a least arc  $PM$ , Pl. II. Fig. 2. on the line  $Qq$  joining the quadratures we let fall the perpendiculars  $PK$ ,  $Mk$ , and produce the same till they cut the line of the nodes  $Nn$ , in  $D$  and  $d$ ; the horary motion of the nodes will be as the area  $MPDd$ , and the square of the time  $AZ$  conjunctly. For let  $PK$ ,  $PH$  and  $AZ$  be the three said lines, viz.  $PK$  the sine of the distance of the Moon from the quadrature,  $PH$  the sine of the distance of the Moon from the node, and  $AZ$  the sine of the distance of the node from the Sun: and the velocity of the node will be as the solid content of  $PK \times PH \times AZ$ . But  $PT$  is to  $PK$ , as  $PM$  to  $Kk$ ; and therefore, because  $PT$  and  $PM$  are given,  $Kk$  will be as  $PK$ . Likewise  $AT$  is to  $PD$ , as  $AZ$  to  $PH$ , and therefore  $PH$  is as the rectangle  $PD \times AZ$ , and by compounding those proportions,  $PK \times PH$  is as the solid content  $Kk \times PD \times AZ$ , and  $PK \times PH \times AZ$ , as  $Kk \times PD \times AZ^2$ . that is, as the area  $PDdM$  and  $AZ^2$  conjunctly. Q. E. D.

COR.

COR. 2. In any given position of the nodes, their mean horary motion is half their horary motion in the Moon's syzygies; and therefore is to  $16''$ .  $35'''$ .  $16^{iv}$ .  $36^v$ , as the square of the sine of the distance of the nodes from the syzygies to the square of the radius, or as  $AZ^2$ , to  $AT^2$ . For if the Moon, by an uniform motion describes the semicircle  $QAq$ , the sum of all the areas  $PDdM$  during the time of the Moon's passage from  $Q$  to  $M$ , will make up the area  $QMdE$ , terminating at the tangent  $QE$  of the circle. And by the time that the Moon has arrived at the point  $n$ , that sum will (make up) the whole area  $EQAn$  described by the line  $PD$ ; but when the Moon proceeds from  $n$  to  $q$ , the line  $PD$  will fall without the circle, and will describe the area  $nqe$ , terminating at the tangent  $qe$  of the circle; which area, because the nodes were before regressive, but are now progressive, must be subducted from the former area, and being it self equal to the area  $QEN$ , will leave the semicircle  $NQAn$ . While therefore the Moon describes a semicircle, the sum of all the areas  $PDdM$  will be the area of that semicircle; and while the Moon describes a complete circle, the sum of those areas will be the area of the whole circle. But the area  $PDdM$ , when the Moon is in the syzygies is the rectangle of the arc  $PM$  into the radius  $PT$ ; and the sum of all the areas, every one equal to this area, in the time that the Moon describes a complete circle is the rectangle of the whole circumference into the radius of the circle; and this rectangle, being double the area of the circle, will be double the quantity of the former sum. If therefore the nodes went on with that velocity uniformly continued, which they acquire in the Moon's syzygies, they would describe a space double of that which they describe in fact; and therefore the mean motion, by which, if uniformly continued, they would describe the same space with that which they do in fact describe

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by an unequal motion, is *but* one half of that motion which they are possessed of in the Moon's syzygies. Wherefore since their greatest horary motion, if the nodes are in the quadratures, is  $33''$ .  $10'''$ .  $33^{\text{iv}}$ .  $12^{\text{v}}$ , their mean horary motion in this case will be  $16''$ .  $35'''$ .  $16^{\text{iv}}$ .  $36^{\text{v}}$ . And seeing the horary motion of the nodes is every where as  $AZ^2$  and the area  $PDdM$  conjunctly, and therefore in the Moon's syzygies, the horary motion of the nodes is as  $AZ^2$  and the area  $PDdM$  conjunctly, that is, (because the area  $PDdM$  described in the syzygies is given) as  $AZ^2$ ; therefore the mean motion also will be as  $AZ^2$ , and therefore when the nodes are without the quadratures, this motion will be. to  $16''$ .  $35'''$ .  $16^{\text{iv}}$ .  $36^{\text{v}}$ . as  $AZ^2$  to  $AT^2$ . Q. E. D.

## PROPOSITION XXXI. PROBLEM XII.

*To find the horary motion of the nodes of the Moon in an elliptic orbit, Pl. 12. Fig. 1.*

Let  $Qpmaq$  represent an ellipse, described with the greater axe  $Qq$ , and the lesser axe  $ab$ ;  $QAqB$  a circle circumscribed;  $T$  the Earth in the common centre of both;  $S$  the Sun;  $p$  the Moon moving in this ellipse; and  $pm$  an arc which it describes in the least moment of time;  $N$  and  $n$  the nodes joined by the line  $Nn$ ;  $pK$  and  $mk$  perpendiculars upon the axe  $Qq$ , produced both ways till they meet the circle in  $P$  and  $M$ , and the line of the nodes in  $D$  and  $d$ . And if the Moon, by a radius drawn to the Earth, describes an area proportional to the time of description, the horary motion of the node in the ellipse will be as the area  $pDdm$ , and  $AZ^2$  conjunctly.

For let  $PF$  touch the circle in  $P$ , and produced meet  $TN$  in  $F$ ; and  $pf$  touch the ellipse in  $p$ , and produced meet the same  $TN$  in  $f$ , and both tangents concur







in the axe  $TQ$  at  $T$ . And let  $ML$  represent the space which the Moon, by the impulse of the abovementioned force  $3IT$  or  $3PK$ , would describe with a transverse motion, in the mean time while, revolving in the circle it describes the arc  $PM$ ; and  $ml$  denote the space, which the Moon revolving in the ellipse would describe in the same time by the impulse of the same force  $3IT$  or  $3PK$ ; and let  $LP$  and  $lp$  be produced till they meet the plane of the ecliptic in  $G$  and  $g$ , and  $FG$  and  $fg$  be joined, of which  $FG$  produced may cut  $pf$ ,  $pg$ , and  $TQ$  in  $c$ ,  $e$  and  $R$  respectively; and  $fg$  produced may cut  $TQ$  in  $r$ . Because the force  $3IT$  or  $3PK$  in the circle, is to the force  $3IT$  or  $3pK$  in the ellipse, as  $PK$  to  $pK$ , or as  $AT$  to  $aT$ ; the space  $ML$ , generated by the former force, will be to the space  $ml$  generated by the latter, as  $PK$  to  $pK$ , that is, because of the similar figures  $PTKp$ , and  $FTRc$ , as  $FR$  to  $cR$ . But (because of the similar triangles  $PLM$ ,  $PGF$ )  $ML$  is to  $FG$ , as  $PL$  to  $PG$ , that is (on account of the parallels  $Lk$ ,  $PK$ ,  $GR$ ) as  $pl$  to  $pe$ , that is, (because of the similar triangles  $plm$ ,  $cpe$ ) as  $lm$  to  $ce$ ; and inversely as  $LM$  is to  $lm$ , or as  $FR$  is to  $cR$ , so is  $FG$  to  $ce$ . And therefore if  $fg$  was to  $ce$ , as  $fy$  to  $cT$ , that is as  $fr$  to  $cR$ , (that is as  $fr$  to  $FR$  and  $FR$  to  $cR$  conjunctly, that is, as  $fT$  to  $FT$ , and  $FG$  to  $ce$  conjunctly) because the ratio of  $FG$  to  $ce$ , expung'd on both sides, leaves the ratios  $fg$  to  $FG$  and  $fT$  to  $FT$ ,  $fg$  would be to  $FG$ , as  $fT$  to  $FT$ ; and therefore the angles which  $FG$  and  $fg$  would subtend at the Earth  $T$  would be equal each to other. But these angles, (by what we have shew'd in the preceding proposition) are the motions of the nodes, while the Moon describes, in the circle the arc  $PM$ , in the ellipse the arc  $pm$ : And therefore the motions of the nodes in the circle, and in the ellipse, would be equal to each other. Thus I say it would be if  $fg$  was to  $ce$ , as  $fT$  to  $cT$ , that

is, if  $fg$  was equal to  $\frac{ce \times fr}{cr}$ . But because of the similar triangles  $fgp$ ,  $cep$ ,  $fg$  is to  $ce$  as  $fp$  to  $cp$ ; and therefore  $fg$  is equal to  $\frac{ce \times fp}{cp}$ ; and therefore the angle which  $fg$  subtends in fact, is to the former angle which  $FG$  subtends, that is to say, the motion of the nodes in the ellipse is to the motion of the same in the circle, as this  $fg$  or  $\frac{ce \times fp}{cp}$ , to the former  $fg$

or  $\frac{ce \times fr}{cr}$ , that is as  $fp \times cr$  to  $fr \times cp$ , or as  $fp$  to  $fr$ , and  $cr$  to  $cp$ , that is, if  $ph$  parallel to  $TN$  meet  $FP$  in  $h$ , as  $Fh$  to  $Fr$  and  $Fr$  to  $FP$ ; that is, as  $Fh$  to  $FP$  or  $Dp$  to  $DP$ , and therefore as the area  $Dpmd$  to the area  $DPMd$ . And therefore seeing (by corol. 1. prop. 30.) the latter area and  $AZ^2$  conjunctly are proportional to the horary motion of the nodes in the circle, the former area and  $AZ^2$  conjunctly will be proportional to the horary motion of the nodes in the ellipse. Q. E. D.

COR. Since therefore in any given position of the nodes, the sum of all the areas  $pDdm$ , in the time while the Moon is carried from the quadrature to any place  $m$ , is the area  $mpQEd$  terminated at the tangent of the ellipse  $QE$ ; and the sum of all those areas, in one entire revolution, is the area of the whole ellipse: the mean motion of the nodes in the ellipse will be to the mean motion of the nodes in the circle, as the ellipse to the circle; that is, as  $Ta$  to  $TA$  or 69 to 70. And therefore since (by corol. 2. prop. 30.) the mean horary motion of the nodes in the circle is to  $16''. 35'''. 16^{iv}. 36^v$ . as  $AZ^2$  to  $AT^2$ , if we take the angle  $16''. 21'''. 3^{iv}. 30^v$ . to the angle  $16''. 35'''. 16^{iv}. 36^v$ . as 69 to 70, the mean horary motion of the nodes in the ellipse will be to  $16''. 21'''. 3^{iv}. 30^v$ . as  $AZ^2$  to  $AT^2$ ; that is, as the square of the sine of the distance of the node from the Sun to the square of the radius.

But

But the Moon, by a radius drawn to the Earth, describes the area in the syzygies with a greater velocity than it does that in the quadratures, and upon that account the time is contracted in the syzygies, and prolonged in the quadratures; and together with the time the motion of the nodes is likewise augmented or diminished. But the moment of the area in the quadrature of the Moon, was to the moment thereof in the syzygies as 10973 to 11073; and therefore the mean moment in the octants is to the excess in the syzygies, and to the defect in the quadratures, as 11023, the half sum of those numbers, to their half difference 50. Wherefore since the time of the Moon's motion in the several little equal parts of its orbit, is reciprocally as its velocity; the mean time in the octants will be to the excess of the time in the quadratures, and to the defect of the time in the syzygies, arising from this cause, nearly as 11023 to 50. But reckoning from the quadratures to the syzygies, I find that the excess of the moments of the area, in the several places, above the least moment in the quadratures, is nearly as the square of the sine of the Moon's distance from the quadratures; and therefore the difference betwixt the moment in any place, and the mean moment in the octants, is as the difference betwixt the square of the sine of the Moon's distance from the quadratures, and the square of the sine of 45 degrees, or half the square of the radius; and the increment of the time in the several places between the octants and quadratures, and the decrement thereof between the octants and syzygies is in the same proportion. But the motion of the nodes while the Moon describes the several little equal parts of its orbit, is accelerated or retarded in the duplicate proportion of the time. For that motion while the Moon describes  $PM$ , is (*ceteris paribus*) as  $ML$ , and  $ML$  is in the duplicate proportion of the time. Wherefore the motion of the nodes in the syzygies,

in

in the time while the Moon describes giv'n little parts of its orbit, is diminish'd in the duplicate proportion of the number 11073 to the number 11023; and the decrement is to the remaining motion as 100 to 10973; but to the whole motion as 100 to 11073 nearly. But the decrement in the places between the octants and syzygies, and the increment in the places between the octants and quadratures, is to this decrement, nearly as the whole motion in these places to the whole motion in the syzygies, and the difference betwixt the square of the sine of the Moon's distance from the quadrature, and the half square of the radius, to the half square of the radius conjunctly. Wherefore, if the nodes are in the quadratures, and we take two places, one on one side, one on the other, equally distant from the octant and other two distant by the same interval, one from the syzygy, the other from the quadrature, and from the decrements of the motions in the two places between the syzygy and octant, we subtract the increments of the motions in the two other places between the octant and the quadrature; the remaining decrement will be equal to the decrement in the syzygy: as will easily appear by computation. And therefore the mean decrement, which ought to be subducted from the mean motion of the nodes, is the fourth part of the decrement in the syzygy. The whole horary motion of the nodes in the syzygies (when the Moon by a radius drawn to the Earth, was suppos'd to describe an area proportional to the time) was  $32''. 42'''. 7^{iv}$ . And we have shew'd, that the decrement of the motion of the nodes, in the time while the Moon, now moving with greater velocity, describes the same space, was to this motion as 100 to 11073; and therefore this decrement is  $17'. 43^{iv}. 11^v$ . The fourth part of which  $4''' . 25^{iv} . 48^v$ . subtracted from the mean horary motion above found  $16'' . 21''' . 3^{iv} . 30^v$ . leaves  $16'' . 16''' . 37^{iv} . 42^v$ . their correct mean horary motion.

If

If the nodes are without the quadratures, and two places are consider'd, one on one side, one on the other equally distant from the syzygies; the sum of the motions of the nodes when the Moon is in those places, will be to the sum of their motions, when the Moon is in the same places and the nodes in the quadratures, as  $AZ^2$ , to  $AT^2$ . And the decrements of the motions, arising from the causes but now explained, will be mutually as the motions themselves, and therefore the remaining motions will be mutually betwixt themselves as  $AZ^2$ . to  $AT^2$ . And the mean motions will be as the remaining motions. And therefore in any giv'n position of the nodes, their correct mean horary motion is to  $16''$ .  $16'''$ .  $37^{iv}$ .  $42^v$ . as  $AZ^2$ , to  $AT^2$ . that is, as the square of the sine of the distance of the nodes from the syzygies to the square of the radius.

## PROPOSITION XXXIII. PROBLEM XIII.

*To find the mean motion of the nodes of the Moon. Pl. 12. Fig. 2.*

The yearly mean motion is the sum of all the mean horary motions, throughout the course of the year. Suppose that the node is in  $N$ , and that after ev'ry hour is elaps'd, it is drawn back again to its former place; so that, notwithstanding its proper motion, it may constantly remain in the same situation, with respect to the fixt Stars; while in the mean time the Sun  $S$ , by the motion of the Earth, is seen to leave the node and to proceed till it compleats its apparent annual course by an uniform motion. Let  $Aa$  represent a given least arc, which the right line  $TS$  always drawn to the Sun, by its intersection with the circle  $NAn$ , describes in the least given moment of time; and the mean

mean horary motion (from what we have above shew'd) will be as  $AZ^2$ , that is (because  $AZ$  and  $ZT$  are proportional) as the rectangle of  $AZ$  into  $ZT$ , that is, as the area  $AZTA$ . And the sum of all the mean horary motions from the beginning will be as the sum of all the areas  $ATZA$ , that is as the area  $NAZ$ . But the greatest  $AZTA$  is equal to the rectangle of the arc  $AA$  into the radius of the circle; and therefore the sum of all these rectangles in the whole circle, will be to the like sum of all the greatest rectangles, as the area of the whole circle to the rectangle of the whole circumference into the radius, that is, as 1 to 2. But the horary motion corresponding to that greatest rectangle, was  $16''$ .  $16'''$ .  $37^{iv}$ .  $42^v$ . and this motion in the complete course of the sidereal year  $365^d$ .  $6^h$ .  $9'$ . amounts to  $39^\circ$ .  $38'$ .  $7''$ .  $50'''$ . and therefore the half thereof  $19^\circ$ .  $49'$ .  $3''$ .  $55'''$ . is the mean motion of the nodes corresponding to the whole circle. And the motion of the nodes, in the time while the Sun is carry'd from  $N$  to  $A$  is to  $19^\circ$ .  $49'$ .  $3''$ .  $55'''$ . as the area  $NAZ$  to the whole circle.

Thus it would be if the node was after every hour drawn back again to its former place, that so, after a compleat revolution, the Sun at the year's end would be found again in the same node which it had left when the year begun. But because of the motion of the node in the mean time, the Sun must needs meet the node sooner, and now it remains that we compute the abbreviation of the time. Since then the Sun, in the course of the year, travels 360 degrees, and the node in the same time by its greatest motion would be carried  $39^\circ$ .  $38'$ .  $7''$ .  $50'''$ , or 39,6355 degrees; and the mean motion of the node in any place  $N$ , is to its mean motion in its quadratures, as  $AZ^2$  to  $AT^2$ : the motion of the Sun will be to the motion of the node in  $N$ , as  $360 AT^2$ , to 39,6355  $AZ^2$ ; that is, as 9,0827646  $AT^2$  to  $AZ^2$ . Wherefore if we suppose the circumference

ference  $NAn$  of the whole circle to be divided into little equal parts, such as  $Aa$ , the time in which the Sun would describe the little arc  $Aa$ , if the circle was quiescent, will be to the time of which it would describe the same arc, supposing the circle together with the nodes to be revolv'd about the centre  $T$ , reciprocally as  $9,0827646 AT^2$  to  $9,0827646 AT^2 + AZ^2$ . For the time is reciprocally as the velocity with which the little arc is describ'd, and this velocity is the sum of the velocities of both Sun and node. If therefore the sector  $NTA$  represent the time in which the Sun by it self, without the motion of the node, would describe the arc  $NA$ , and the indefinitely small part  $ATa$  of the sector represent the little moment of the time, in which it would describe the least arc  $Aa$ ; and (letting fall  $aT$  perpendicular upon  $Nn$ ) *depends* if in  $AZ$  we take  $dZ$ , of such length, that the rectangle of  $dZ$  into  $ZT$ , may be to the least part  $ATa$  of the sector, as  $AZ^2$  to  $9,0827646 AT^2 + AZ^2$ , that is to say, that  $dZ$  may be to  $\frac{1}{2}AZ$ , as  $AT^2$  to  $9,0827646 AT^2 + AZ^2$ ; the rectangle of  $dZ$  into  $ZT$  will represent the decrement of the time arising from the motion of the node, while the arc  $Aa$  is describ'd. And if the curve  $NdGn$  is the locus where the point  $d$  is always found, the curvilinear area  $NdZ$  will be as the whole decrement of time while the whole arc  $NA$  is describ'd. And therefore, the excess of the sector  $NAT$  above the area  $NdZ$  will be as the whole time. But because the motion of the node in a less time, is less in proportion of the time, the area  $AaYZ$  must also be diminish'd in the same proportion. Which may be done by taking in  $AZ$  the line  $eZ$  of such length, that it may be to the length of  $AZ$ , as  $AZ^2$  to  $9,0827646 AT^2 + AZ^2$ . For so the rectangle of  $eZ$  into  $ZT$ , will be to the area  $AZTa$ , as the decrement of the time in which the arc  $Aa$  is describ'd, to the whole time in which it would



would have been describ'd, if the node had been quiescent. And therefore that rectangle will be as the decrement of the motion of the node. And if the curve  $NeFn$  is the locus of the point  $e$ , the whole area  $NeZ$ , which is the sum of all the decrements of that motion, will be as the whole decrement thereof during the time in which the arc  $AN$  is describ'd; and the remaining area  $NAe$  will be as the remaining motion, which is the true motion of the node, during the time in which the whole arc  $NA$  is describ'd by the joint motions of both Sun and node. Now the area of the semicircle is to the area of the figure  $NeFn$  found by the method of infinite series, nearly as 793 to 60. But the motion corresponding or proportional to the whole circle was  $19^{\circ} 49'. 3''. 55'''$ . and therefore the motion corresponding to double the figure  $NeFn$  is  $1^{\circ} 29'. 58''. 2'''$ . which taken from the former motion leaves  $18^{\circ} 19'. 5''. 53'''$ . the whole motion of the node with respect to the fixed Stars in the interval between two of its conjunctions with the Sun; and this motion subducted from the annual motion of the Sun  $360^{\circ}$ . leaves  $341^{\circ} 40'. 54''. 7'''$ . the motion of the Sun in the interval between the same conjunctions. But as this motion is to the annual motion  $360^{\circ}$ . so is the motion of the node but just now found  $18^{\circ} 19'. 5''. 53'''$ . to its annual motion which will therefore be  $19^{\circ} 18'. 1''. 23'''$ . And this is the mean motion of the nodes in the sidereal year. By astronomical tables it is  $19^{\circ} 21'. 21''. 50'''$ . The difference is less than  $\frac{1}{30}$  part of the whole motion, and seems to arise from the eccentricity of the Moon's orbit, and its inclination to the plane of the ecliptic. By the eccentricity of this orbit, the motion of the nodes is too much accelerated, and on the other hand, by the inclination of the orbit, the motion of the nodes is something retarded, and reduc'd to its just velocity.

PRO-

## PROPOSITION XXXIII. PROBLEM XIV.

To find the true motion of the nodes of the Moon. Pl. 12. Fig. 3.

In the time which is as the area  $NTA-NdZ$  (in the preceding Fig.) that motion is as the area  $NAe$ , and is thence giv'n. But because the calculus is too difficult it will be better to use the following construction of the problem. About the centre  $C$ , with any interval  $CD$ , describe the circle  $BEFD$ , produce  $DC$  to  $A$ , so as  $AB$  may be to  $AC$ , as the mean motion to half the mean true motion when the nodes are in their quadratures (that is, as  $19^{\circ}. 18'. 1''. 23''$ . to  $19^{\circ}. 49'. 3''. 55'''$ . and therefore  $BC$  to  $AC$ , as the difference of those motions  $0^{\circ}. 31'. 2''. 32'''$ . to the latter motion  $19^{\circ}. 49'. 3''. 55'''$ . that is, as 1 to  $38\frac{3}{4}$ ). Then through the point  $D$ , draw the indefinite line  $Gg$ , touching the circle in  $D$   $BCE$ , or  $BCF$ , equal to the distance of the Sun from the place of the nodes when the nodes are in their quadratures; and drawing  $AE$  perpendicular  $DG$  in  $G$ , we take  $AE$  to be to the whole motion of the nodes when they are between its syzygies (that is, as the length of the tangent  $DG$  to the whole circumference  $BED$ ; and add this last as  $DAG$  may be us'd) to the mean motion of the nodes, while they are passing from the quadratures to the syzygies, and subtract it from their mean motion, while they are passing from the syzygies to the quadratures; we shall have their true motion. For the true motion so found will nearly agree with the true motion which comes out from assuming the time as the area  $NTA-NdZ$ , and the motion of the node as the area  $NAe$ ,

<sup>consequence</sup> as whoever will please to examine and make the computations will find. And this is the semi-menstrual equation of the motion of the nodes. But there is also a menstrual equation, but which is by no means necessary for finding of the Moon's latitude. For since the variation of the inclination of the Moon's orbit to the plane of the ecliptic is liable to a twofold inequality: the one semi-menstrual, the other menstrual: the menstrual inequality *of this variation*, and the menstrual equation of the nodes, so moderate and correct each other, that in computing the latitude of the Moon both may be neglected.

COR. From this and the preceding prop. it appears that the nodes are quiescent in their syzygies, but regressive in their quadratures, by an hourly motion of  $16''. 19'''. 26''''$ . And that the equation of the motion of the nodes in the octants is  $1^{\circ}. 30'$ . all which exactly agree with the phenomena of the heavens.

concrete

also

## SCHOLIUM.

Mr. *Machin* Astron. Prof. Gresh. and Dr. *Henry Pemberton* separately found out the motion of the nodes by a different method. Mention has been made of this method in another place. Their several papers, both of which I have seen, contained two propositions, and exactly agreed with each other in both of them. Mr. *Machin's* paper coming first to my hands, I shall here insert it.

OF





f the motion of the Moon's nodes.

PROPOSITION I.

*mean motion of the Sun from the node, is defined by a geometric mean proportional, between the mean motion of the Sun, and that mean motion with which the Sun recedes with the greatest swiftneſs from the node in the quadratures.*

Let  $T$  (Pl. 13. Fig. 1.) be the Earth's place,  $Vn$  the line of the Moon's nodes at any given time,  $KTM$  a perpendicular thereto,  $TA$  a right line revolving about the centre with the ſame angular velocity with which the Sun and the node recede from one another, in ſuch ſort that the angle between the quieſcent right line  $Nn$ , and the revolving line  $TA$ , may be always equal to the diſtance of the places of the Sun and node. Now if any right line  $TK$  be divided into parts,  $TS$  and  $SK$ , and thoſe parts be taken as the mean horary motion of the Sun to the mean horary motion of the node in the quadratures, and there be taken the right line  $TH$ , a mean proportional between the part  $TS$  and the whole  $TK$ , this right line will be proportional to the Sun's mean motion from the node.

For let there be deſcribed the circle  $NK_nM$  from the centre  $T$  and with the radius  $TK$ , and about the ſame centre, with the ſemi-axes  $TH$  and  $TN$ , let there be deſcribed an ellipſis  $NH_nL$ . And in the time in which the Sun recedes from the node through the arc  $Na$ , if there be drawn the right line  $Tba$ , the area of the ſector  $NTa$  will be the exponent of the ſum of the motions of the Sun and node in the ſame time. Let therefore the extremely ſmall arc  $aA$  be that which the right line  $Tba$ , revolving according to the aboveſaid law, will uniformly deſcribe in a given par-

“ ticle of time, and the extremely small sector  $TAa$  will  
 “ be as the sum of the velocities with which the Sun  
 “ and node are carried two different ways in that time.  
 “ Now the Sun’s velocity is almost uniform, its inequa-  
 “ lity being so small as scarcely to produce the least in-  
 “ equality in the mean motion of the nodes. The other  
 “ part of this sum, namely the mean quantity of the ve-  
 “ locity of the node, is increased in the recess from the  
 “ syzygies in a duplicate ratio of the sine of its distance  
 “ from the Sun (by corol. prop. 31. of this book) and  
 “ being greatest in its quadratures with the Sun in  $K$ ,  
 “ is in the same ratio to the Sun’s velocity as  $SK$  to  $TS$ ,  
 “ that is, as (the difference of the squares of  $TK$  and  
 “  $TH$ , or) the rectangle  $KHM$  to  $TH^2$ . But the  
 “ ellipsis  $NBH$  divides the sector  $ATa$ , the exponent  
 “ of the sums of these two velocities, into two parts  
 “  $ABba$  and  $BTb$ , proportional to the velocities. For  
 “ produce  $BT$  to the circle in  $\beta$ , and from the point  
 “  $B$  let fall upon the greater axis the perpendicular  $BG$ ,  
 “ which being produced both ways may meet the circle  
 “ in the points  $F$  and  $f$ ; and because the space  $ABba$   
 “ is to the sector  $TBb$  as the rectangle  $AB\beta$  to  $BT^2$ ,  
 “ (that rectangle being equal to the difference of the  
 “ squares of  $TA$  and  $TB$ , because the right line  $A\beta$   
 “ is equally cut in  $T$ , and unequally in  $B$ ;) therefore  
 “ when the space  $ABba$  is the greatest of all in  $K$ ,  
 “ this ratio will be the same as the ratio of the rectangle  
 “  $KHM$  to  $HT^2$ . But the greatest mean velocity of  
 “ the node was shewn above to be in that very ratio to  
 “ the velocity of the Sun; and therefore in the quadra-  
 “ tures the sector  $ATa$  is divided into parts propor-  
 “ tional to the velocities. And because the rectangle  $KHM$   
 “ is to  $HT^2$ , as  $FBf$  to  $BG^2$ , and the rectangle  $AB\beta$   
 “ is equal to the rectangle  $FBf$ ; therefore the little a-  
 “ rea  $ABba$ , where it is greatest, is to the remaining  
 “ sector  $TBb$ , as the rectangle  $AB\beta$  to  $BG^2$ . But the  
 “ ratio of these little areas always was as the rectangle  
 “  $AB\beta$

“  $AB\beta$  to  $BT$ , and therefore the little area  $ABb$  in  
 “ the place  $A$  is less than its correspondent little area in  
 “ the quadratures, in the duplicate ratio of  $BG$  to  $BT$ ,  
 “ that is, in the duplicate ratio of the sine of the Sun’s  
 “ distance from the node. And therefore the sum of all  
 “ the little areas  $ABb$ , to wit, the space  $ABN$  will <sup>taber</sup>  
 “ be as the motion of the node in the time in which  
 “ the Sun hath been going over the arc  $NA$  since he <sup>el</sup>  
 “ left the node. And the remaining space, namely the <sup>inquinado-  
I tea  
in aladament</sup>  
 “ elliptic sector  $NTB$ , will be as the Sun’s mean moti-  
 “ on in the same time. And because the mean annual  
 “ motion of the node is that motion which it performs <sup>quent</sup>  
 “ in the time that the Sun completes one period of its  
 “ course, the mean motion of the node from the Sun  
 “ will be to the mean motion of the Sun it self, as the  
 “ area of the circle to the area of the ellipsis; that is as  
 “ the right line  $TK$  to the right line  $TH$ , which is a  
 “ comean proportional between  $TK$  and  $TS$ ; or which <sup>— ?</sup>  
 “ mes to the same, as the mean proportional  $TH$  to the  
 “ right line  $TS$ .

## PROPOSITION II.

*The mean motion of the Moon’s nodes being given, to find their true motion.*

“ Let the angle  $A$  be the distance of the Sun from  
 “ the mean place of the node, or the Sun’s mean motion  
 “ from the node. Then if we take the angle  $B$ , whose <sup>enya</sup>  
 “ tangent is to the tangent of the angle  $A$ , as  $TH$  to  
 “  $TK$ , that is, in the subduplicate ratio of the mean ho-  
 “ rary motion of the Sun to the mean horary motion  
 “ of the Sun from the node, when the node is in the  
 “ quadrature, that angle  $B$  will be the distance of the  
 “ Sun from the node’s true place. For join  $FT$ , and by  
 “ the demonstration of the last proportion, the angle  
 “  $FTN$  will be the distance of the Sun from the mean

U 2

“ place



“ place of the node, and the angle  $ATN$  the distance  
 “ from the true place, and the tangents of these angles  
 “ are between themselves as  $TK$  to  $TH$ .

“ COR. Hence the angle  $FTA$  is the equation of  
 “ the Moon's nodes, and the sine of this angle where  
 “ it is greatest in the octants, is to the radius as  $KH$   
 “ to  $TK - TH$ . But the sine of this equation in a-  
 “ ny other place  $A$  is to the greatest sine, as the sine  
 “ of the sums of the angles  $FTN - ATN$  to the radius;  
 “ that is, nearly as the sine of double the distance of  
 “ the Sun from the mean place of the node (namely  
 “  $2 FTN$ ) to the radius.

*equatione  
 21 also*

### SCHOLIUM.

“ If the mean horary motion of the nodes in the qua-  
 “ dratures be  $16''$ .  $16'''$ .  $37^{iv}$ .  $42^v$ . that is in a whole  
 “ sidereal year  $39^\circ$ .  $38'$ .  $7''$ .  $50'''$ .  $TH$  will be to  
 “  $TK$  in the sub-duplicate ratio of the number  
 “  $9,0827646$  to the number  $10,827646$ , that is, as  
 “  $18,6524761$  to  $19,6524761$ . And therefore  $TH$   
 “ is to  $HK$  as  $18,6524761$  to  $1$ , that is, as the moti-  
 “ on of the Sun in a sidereal year to the mean motion  
 “ of the node  $19^\circ$ .  $18'$ .  $1''$ .  $23\frac{2}{3}'''$ .

“ But if the mean motion of the Moon's nodes in  
 “ 20 Julian years is  $386^\circ$ .  $50'$ .  $15''$ . as is collected from  
 “ the observations made use of in the theory of the  
 “ Moon, the mean motion of the nodes in one sidereal  
 “ year will be  $19^\circ$ .  $20'$ .  $31''$ .  $58'''$ . And  $TH$  will be  
 “ to  $HK$  as  $360^\circ$ . to  $19^\circ$ .  $20'$ .  $31''$ .  $58'''$ . that is,  
 “ as  $18,61214$  to  $1$ , and from hence the mean horary  
 “ motion of the nodes in the quadratures will come out  
 “  $16''$ .  $18'''$ .  $48^{iv}$ . And the greatest equation of the  
 “ nodes in the octants will be  $1^\circ$ .  $29'$ .  $57''$ .

PRO:

PROPOSITION XXXIV. PROBLEM XV.

To find the horary variation of the inclination of the Moon's orbit to the plane of the ecliptic.

Let  $A$  and  $a$ , (Pl. 13. Fig. 2.) represent the syzygies;  $Q$  and  $q$  the quadratures;  $N$  and  $n$  the nodes;  $P$  the place of the Moon in its orbit;  $p$  the orthographic projection of that place upon the plane of the ecliptic; and  $mTl$  the momentaneous motion of the nodes as above. If upon  $Tm$  we let fall the perpendicular  $PG$ , and joining  $pG$  we produce it till it meet  $Tl$  in  $g$ , and join also  $Pg$ ; the angle  $PGp$  will be the inclination of the Moon's orbit to the plane of the ecliptic when the Moon is in  $P$ ; and the angle  $Pgp$  will be the inclination of the same after a small moment of time is elaps'd; and therefore the angle  $G Pg$  will be the momentaneous variation of the inclination. But this angle  $G Pg$  is to the angle  $GTg$ , as  $TG$  to  $PG$  and  $Pp$  to  $PG$  conjunctly. And therefore if for the moment of time we assume an hour; since the angle  $GTg$  (by prop. 30.) is to the angle  $33''. 10'''. 33^{iv}$ . as  $IT \times PG \times AZ$ , to  $AT^3$ , the angle  $G Pg$  (or the horary variation of the inclination) will be to the angle  $33''. 10''', 33^{iv}$ . as  $IT \times AZ \times TG \times \frac{Pp}{PG}$  to  $AT^3$ . *transcurrido* **Q. E. I.**

And thus it would be if the Moon was uniformly revolv'd in a circular orbit. But if the orbit is elliptical, the mean motion of the nodes will be diminish'd in proportion of the lesser axis to the greater, as we have shewn above. And the variation of the inclination will be also diminish'd in the same proportion. *et ubi*

COR. I. Upon  $Nn$  erect the perpendicular  $TF$ , and let  $pM$  be the horary motion of the Moon in the plane of

of the ecliptic; upon  $QT$  let fall the perpendiculars  $pK$ ,  $Mk$ , and produce them till they meet  $TF$  in  $H$  and  $b$ ; then  $IT$  will be to  $AT$ , as  $Kk$  to  $Mp$ ; and  $TG$  to  $Hp$  as  $TZ$  to  $AT$ ; and therefore  $IT \times TG$  will be equal to  $\frac{Kk \times Hp \times TZ}{Mp}$ , that is, equal to the

area  $HpMb$  multiplied into the ratio  $\frac{TZ}{Mp}$ : and therefore the horary variation of the inclination will be to  $33''. 10'''$ .  $33^{iv}$ . as the area  $HpMb$  multiply'd into  $AZ \times \frac{TZ}{Mp} \times \frac{Pp}{PG}$  to  $AT^3$ .

COR. 2. And therefore, if the Earth and nodes were after every hour drawn back from their new, and instantly restor'd to their old places, so as their situation might continue given for a whole periodic month together; the whole variation of the inclination during that month would be to  $33''. 10'''$ .  $33^{iv}$ , as the aggregate of all the areas  $HpMb$ , generated in the time of one revolution of the point  $p$ , (with due regard in summing to their proper signs  $-$  and  $+$ ). multiply'd into  $AZ \times TZ \times \frac{Pp}{PG}$  to  $Mp \times AT^3$ , that is, as the whole cir-

cle  $QAqa$  multiply'd into  $AZ \times TZ \times \frac{Pp}{PG}$  to  $Mp \times AT^3$ , that is, as the circumference  $QAqa$  multiply'd into  $AZ \times TZ \times \frac{Pp}{PG}$  to  $2 Mp \times AT^2$ .

COR. 3. And therefore, in a giv'n position of the nodes, the mean horary variation, from which, if uniformly continu'd through the whole month, that menstrual variation might be generated, is to  $33''. 10'''$ .  $33^{iv}$ . as  $AZ \times TZ \times \frac{Pp}{PG}$  to  $2 AT^2$ , or as  $Pp \times \frac{AZ \times TZ}{\frac{1}{2} AT}$  to  $PG \times 4 AT$ , that is (because  $Pp$  is to  $PG$ , as the fine

fine of the aforefaid inclination to the radius ; and *antedich*  
 $\frac{AZ \times TZ}{\frac{1}{2} AT}$  to 4  $AT$ , as the fine of double the angle  
 $AT^n$  to four times the radius) as the fine of the same  
 inclination multiply'd into the fine of double the dis-  
 tance of the nodes from the Sun, to four times the  
 square of the radius.

COR. 4. Seeing the horary variation of the inclina-  
 tion, when the nodes are in the quadratures, is (by this  
 prop.) to the angle  $33''. 10''' . 33^{iv}$ , as  $IT \times AZ \times TG \times$   
 $\frac{PP}{PG}$  to  $AT^3$ , that is, as  $\frac{IT \times TG}{\frac{1}{2} AT} \times \frac{PP}{PG}$ , to 2  $AT$ ,  
 that is, as the fine of double the distance of the Moon  
 from the quadratures multiply'd into  $\frac{PP}{PG}$  to twice the  
 radius : the sum of all the horary variations during the  
 time that the Moon, in this situation of the nodes,  
 passes from the quadrature to the syzygy (that is in  
 the space of  $177\frac{1}{2}$  hours) will be to the sum of as ma-  
 ny angles  $33''. 10''' . 33^{iv}$ . or  $5878''$ , as the sum of all  
 the fines of double the distance of the Moon from the  
 quadratures multiply'd into  $\frac{PP}{PG}$ , to the sum of as ma-  
 ny diameters ; that is, as the diameter multiplied into  
 $\frac{PP}{PG}$  to the circumference ; that is, if the inclination be  
 $5^\circ . 1'$ , as  $7 \times \frac{874}{10000}$  to 22, or as 278 to 10000. And  
 therefore the whole variation, compos'd out of the sum  
 of all the horary variations in the forefaid time, is  $163''$ .  
 or  $2' . 43''$ . *antedich*

## PROPOSITION XXXV. PROBLEM XVI.

To a given time to find the inclination of the Moon's orbit to the plane of the ecliptic.

Let  $AD$  (Pl. 14. Fig. 1.) be the line of the greatest inclination, and  $AB$  the line of the least. Bisect  $BD$  in  $C$ ; and round the centre  $C$ , with the interval  $BC$ , describe the circle  $BGD$ . In  $AC$  take  $CE$  in the same proportion to  $EB$  as  $EB$  to twice  $BA$ . And if to the time giv'n we set off the angle  $AEG$  equal to double the distance of the nodes from the quadratures, and upon  $AD$  let fall the perpendicular  $GH$ ;  $AH$  will be the sine of the inclination requir'd.

For  $GE^2$  is equal to  $GH^2 + HE^2 = BHD + HE^2 = HBD + HE^2 - BH^2 = HBD + BE^2 - 2BH \times BE = BE^2 + 2EC \times BH = 2EC \times AB + 2EC \times BH = 2EC \times AH$ . Wherefore since  $2EC$  is giv'n,  $GE^2$  will be as  $AH$ . Now let  $AEG$  represent double the distance of the nodes from the quadratures, in a given moment of time after, and the arc  $Gg$ , on account of the giv'n angle  $GEG$ , will be as the distance  $GE$ . But  $Hh$  is to  $Gg$ , as  $GH$  to  $GC$ , and therefore  $Hh$  is as the rectangle  $GH \times Gg$ , or  $GH \times GE$ , that is, as  $\frac{GH}{GE} \times GE^2$  or  $\frac{GH}{GE} \times AH$ ; that is, as

$AH$  and the sine of the angle  $AEG$  conjunctly. If therefore in any one case,  $AH$  be the sine of inclination, it will increase by the same increments as the sine of inclination doth, by cor. 3. of the preceding prop. and therefore will always continue equal to that sine. But when the point  $G$  falls upon either point  $B$  or  $D$ ,  $AH$  is equal to this sine, and therefore remains always equal thereto. Q. E. D.

In this demonstration I have suppos'd, that the angle





gle  $BEG$  representing double the distance of the nodes from the quadratures, increaseth uniformly. For I cannot descend to ev'ry minute circumstance of inequality. Now suppose that  $BEG$  is a right angle, and that  $Gg$  is in this case the horary increment of double the distance of the nodes from the Sun; then by cor. 3. of the last prop. the horary variation of the inclination in the same case, will be to  $33''. 10'''$ .  $33^{iv}$ . as the rectangle of  $AH$  the sine of the inclination into the sine of the right angle  $BEG$ , double the distance of the nodes from the Sun, to four times the square of the radius; that is, as  $AH$  the sine of the mean inclination to four times the radius, that is, seeing the mean inclination is about  $5^\circ. 8\frac{1}{2}'$ , as its sine  $896$  to  $40000$ , the quadruple of the radius, or as  $224$  to  $10000$ . But the whole variation, corresponding to  $BD$  the difference of the sines, is to this horary variation, as the diameter  $BD$  to the arc  $Gg$ , that is, conjunctly as the diameter  $BD$  to the semi-circumference  $BGD$ , and as the time of  $2079\frac{7}{8}$  hours, in which the node proceeds from the quadratures to the syzygies, to one hour, that is, as  $7$  to  $11$  and  $2079\frac{7}{8}$  to  $1$ . Wherefore compounding all these proportions, we shall have the whole variation  $BD$  to  $33''. 10'''$ .  $33^{iv}$ . as  $224 \times 7 \times 2079\frac{7}{8}$  to  $110000$ , that is, as  $29645$  to  $1000$ ; and from thence that variation  $BD$  will come out  $16'. 23\frac{1}{2}''$ . whole

And this is the greatest variation of the inclination, abstracting from the situation of the Moon in its orbit. For if the nodes are in the syzygies, the inclination suffers no change from the various positions of the Moon. But if the nodes are in the quadratures, the inclination is less when the Moon is in the syzygies than when it is in the quadratures, by a difference of  $2'. 43''$ . as we shew'd in cor. 4. of the preceding prop. and the whole mean variation  $BD$ , diminish'd by  $1'. 21\frac{1}{2}''$ . the half of this excess, becomes  $15'. 2''$  when the subtended



the Moon is in the quadratures; and increas'd by the same, becomes  $17'. 45''$ . when the Moon is in the syzygies. If therefore the Moon be in the syzygies, the whole variation in the passage of the nodes from the quadratures to the syzygies will be  $17'. 45''$ . And therefore if the inclination be  $5^\circ. 17'. 20''$ . when the nodes are in the syzygies, it will be  $4^\circ. 59'. 35''$ . when the nodes are in the quadratures and the Moon in the syzygies. The truth of all which is confirm'd by observations.

Now if the inclination of the orbit should be requir'd, when the Moon is in the syzygies, and the nodes any where between them and the quadratures; let  $AB$  be to  $AD$ , as the sine of  $4^\circ. 59'. 35''$ . to the sine of  $5^\circ. 17'. 20''$ . and take the angle  $AEG$ , equal to double the distance of the nodes from the quadratures; and  $AH$  will be the sine of the inclination desir'd. To this inclination of the orbit the inclination of the same is equal, when the Moon is  $90^\circ$ . distant from the nodes. In other situations of the Moon, this menstrual inequality to which the variation of the inclination is obnoxious in the calculus of the Moon's latitude, is balanc'd and in a manner (took off,) by the menstrual inequality of the motion of the nodes (as we said before) and therefore may be neglected in the computation of the said latitude.

#### SCHOLIUM.

By these computations of the lunar motions, I was willing to shew that by the theory of gravity the motions of the Moon could be calculated from their physical causes. By the same theory I moreover found, that the annual equation of the mean motion of the Moon arises from the various dilatation which the orbit of the Moon suffers from the action of the Sun, accord-

According to cor. 6. prop. 66. book 1. The force of this action is greater in the perigeon Sun, and dilates the Moon's orbit; in the apogee Sun it is less, and permits the orbit to be again contracted. The Moon moves slower in the dilated, and faster in the contracted orbit; and the annual equation, by which this inequality is regulated, vanishes in the apogee and perigee of the Sun. In the mean distance of the Sun from the Earth it arises to about  $11'. 50''$ . In other distances of the Sun, it is proportional to the equation of the Sun's centre, and is added to the mean motion of the Moon, while the Earth is passing from its aphelion to its perihelion, and subducted while the Earth is in the opposite semicircle. Taking for the radius of the *orbis magnus*, 1000, and  $16\frac{7}{8}$  for the Earth's eccentricity, this equation when of the greatest magnitude, by the theory of gravity (comes out)  $11'. 49''$ . But the eccentricity of the Earth seems to be something greater, and with the eccentricity this equation will be augmented in the same proportion. Suppose the eccentricity  $16\frac{1}{2}$ , and the greatest equation will be  $11'. 51''$ . more rapid

Further, I found that the apogee and nodes of the Moon move faster in the perihelion of the Earth, where the force of the Sun's action is greater, than in the aphelion thereof, and that in the reciprocal triplicate proportion of the Earth's distance from the Sun. And hence arise annual equations of those motions proportional to the equation of the Sun's centre. Now the motion of the Sun is in the reciprocal duplicate proportion of the Earth's distance from the Sun, and the greatest equation of the centre, which this inequality generates, is  $1^\circ. 56'. 20''$ . corresponding to the abovemention'd eccentricity of the the Sun  $16\frac{1}{2}$ . But if the motion of the Sun had been in the reciprocal triplicate proportion of the distance, this inequality would have generated the greatest equation  $2^\circ. 54'. 30''$ . more rapid

30". And therefore the greatest equations which the inequalities of the motions of the Moon's apogee and nodes do generate, are to  $2^{\circ} \ 54' \ 30''$ . as the mean diurnal motion of the Moon's apogee and the mean diurnal motion of its nodes are to the mean diurnal motion of the Sun. Whence the greatest equation of the mean motion of the apogee (comes out)  $19' \ 43''$ . and the greatest equation of the mean motion of the nodes  $9' \ 24''$ . The former equation is added, and the latter subducted, while the Earth is passing from its perihelion to its aphelion, and contrariwise when the Earth is in the opposite semicircle.

By the theory of gravity I likewise found, that the action of the Sun upon the Moon is something greater when the transverse diameter of the Moon's orbit passeth through the Sun, than when the same is perpendicular upon the line which joins the Earth and the Sun: And therefore the Moon's orbit is something larger in the former than in the latter case. And hence arises another equation of the Moon's mean motion, depending upon the situation of the Moon's apogee in respect of the Sun; which is in its greatest quantity, when the Moon's apogee is in the octants of the Sun, and vanishes when the apogee arrives at the quadratures or syzygies. And it is added to the mean motion, while the Moon's apogee is passing from the quadrature of the Sun to the syzygy, and subducted while the apogee is passing from the syzygy to the quadrature. This equation, which I shall call the semi-annual, when greatest in the octants of the apogee, arises to about  $3' \ 45''$ . so far as I could collect from the phenomena. And this is its quantity in the mean distance of the Sun from the Earth. But it is increased and diminished in the reciprocal triplicate proportion of the Sun's distance, and therefore is nearly  $3' \ 34''$ . when that distance is greatest, and  $3' \ 56''$ . when least. But when the Moon's apogee is without the octants, it becomes

*less*

*less.*

less, and is to its greatest quantity, as the sine of double the distance of the Moon's apogee from the nearest syzygy, or quadrature to the radius.

By the same theory of gravity, the action of the Sun upon the Moon is something greater, when the line of the Moon's nodes passes through the Sun, than when it is at right angles with the line which joins the Sun and the Earth. And hence arises another equation of the Moon's mean motion, which I shall call the second semi-annual, and this is greatest when the nodes are in the octants of the Sun, and vanishes when they are in the syzygies or quadratures; and in other positions of the nodes is proportional to the sine of double the distance of either node from the nearest syzygy or quadrature. And it is added to the mean motion of the Moon, if the Sun is *in antecedentiâ* to the node which is nearest to him, and subducted if *in consequentiâ*; and in the octants, where it is of the greatest magnitude, it arises to  $47''$ . in the mean distance of the Sun from the Earth, as I find from the theory of gravity. In other distances of the Sun this equation, greatest in the octants of the nodes, is reciprocally as the cube of the Sun's distance from the Earth, and therefore in the Sun's perigee it comes to about  $49''$ , and in its apogee to about  $45''$ .

By the same theory of gravity, the Moon's apogee goes forward at the greatest rate, when it is either in conjunction with or in opposition to the Sun, but in its quadratures with the Sun it goes backward. And the eccentricity comes, in the former case, to its greatest quantity, in the latter to its least, by cor. 7. 8. and 9. prop. 66. book 1. And those inequalities by the corollaries we have nam'd, are very great, and generate the principal, which I call the semi-annual, equation of the apogee. And this semi-annual equation in its greatest quantity comes to about  $12^{\circ} 18''$ . as nearly

ly as I could collect from the phenomena. Our countryman *Horrox* was the first <sup>one</sup> who advanced the theory of the Moon's moving in an ellipse about the Earth placed in its lower focus. Dr. *Halley* improved the notion, by putting the centre of the ellipse in an epicycle whose centre is uniformly revolved about the Earth. And from the motion in this epicycle the mentioned inequalities in the progress and regress of the apogee, and in the quantity of eccentricity do arise. Suppose the mean distance of the Moon from the Earth to be divided into 100000 parts, and let *T* (Pl. 14. Fig. 2.) represent the Earth, and *TC* the Moon's mean eccentricity of 5505 such parts. Produce *TC* to *B*, so as *CB* may be the sine of the greatest semi-annual equation  $12^{\circ}. 18'$ , to the radius *TC*; and the circle *BDA* described about the centre *C*, with the interval *CB*, will be the epicycle spoke<sup>?</sup> of, in which the centre of the Moon's orbit is placed, and revolved according to the order of the letters *BDA*. (Set off) the angle *BCD* equal to twice the annual argument, or twice the distance of the Sun's true place from the place of the Moon's apogee once equated,<sup>?</sup> and *CTD* will be the semi-annual equation of the Moon's apogee, and *TD* the eccentricity of its orbit, tending to the place of the apogee now twice equated. But having the Moon's mean motion, the place of its apogee, and its eccentricity, as well as the longer axe of its orbit 200000; from these data the true place of the Moon in its orbit, together with its distance from the Earth, may be determined by the methods commonly known.

In the perihelion of the Earth where the force of the Sun is greatest, the centre of the Moon's orbit moves faster about the centre *C*, than in the aphelion, and that in the reciprocal triplicate proportion of the Sun's distance from the Earth. But because the equation of the Sun's centre is included in the annual argument, the centre of the Moon's orbit moves faster in

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its epicycle  $BDA$ , in the reciprocal duplicate proportion of the Sun's distance from the Earth. Therefore that it may move yet faster in the reciprocal simple proportion of the distance; suppose that from  $D$  the centre of the orbit a right line  $DE$  is drawn, tending towards the Moon's apogee once equated, that is, parallel to  $TC$ ; and (set off) the angle  $EDF$  equal to the excess of the foreſaid annual argument above the distance of the Moon's apogee from the Sun's perigee in *conſequentia*; or, which comes to the ſame thing, take the angle  $CDF$  equal to the complement of the Sun's true anomaly to  $360^\circ$ . And let  $DF$  be to  $DC$ , as twice the eccentricity of the *orbis magnus* to the Sun's mean distance from the Earth and the Sun's mean diurnal motion from the Moon's apogee to the Sun's mean diurnal motion from its own apogee conjunctly, that is, as  $33\frac{2}{3}$  to 1000, and  $52'. 27''. 16'''$ . to  $59'. 8''. 10'''$ . conjunctly; or as 3 to 100. And imagine the centre of the Moon's orbit, placed in the point  $F$ , to be revolved in an epicycle whose centre is  $D$ , and radius  $DF$ , while the point  $D$  moves in the circumference of the circle  $DABD$ . For by this means the centre of the Moon's orbit comes to deſcribe a certain curve line, about the centre  $C$ , with a velocity which will be almoſt reciprocally as the cube of the Sun's distance from the Earth, as it ought to be.

The calculus of this motion is difficult, but may be render'd more eaſy by the following approximation. Aſſuming as above the Moon's mean distance from the Earth of 100000 parts, and the eccentricity  $TC$  of 5505 ſuch parts, the line  $CB$  or  $CD$  will be found  $1172\frac{3}{4}$ , and  $DF$   $35\frac{1}{2}$  of thoſe parts. And this line  $DF$  at the distance  $TC$  ſubtends the angle at the Earth, which the removal of the centre of the orbit from the place  $D$  to the place  $F$  generates in the motion of this centre; and double this line  $DF$  in a parallel poſition, at the distance of the upper focus of the Moon's orbit

orbit from the Earth, subtends at the Earth the same angle as  $DF$  did before, which that removal generates in the motion of this upper focus; but at the distance of the Moon from the Earth this double line  $2 DF$  at the upper focus, in a parallel position to the first line  $DF$ , subtends an angle at the Moon which the said removal generates in the motion of the Moon, which angle may be therefore called the second equation of the Moon's centre. And this equation, in the mean distance of the Moon from the Earth, is nearly as the sine of the angle which that line  $DF$  contains with the line drawn from the point  $F$  to the Moon, and when in its greatest quantity amounts to  $2'. 25''$ . But the angle which the line  $DF$  contains with the line drawn from the point  $F$  to the Moon, is found either by subtracting the angle  $EDF$  from the mean anomaly of the Moon, or by adding the distance of the Moon from the Sun, to the distance of the Moon's apogee from the apogees of the Sun. And as the radius to the sine of the angle thus found, so is  $2'. 25''$ . to the second equation of the centre; to be added, if the forementioned sum be less than a semicircle, to be subtracted if greater. And from the Moon's place in its orbit thus corrected, its longitude may be found in the syzygies of the luminaries.

The atmosphere of the Earth to the height of 35 or 40 miles refracts the Sun's light. This refraction scatters and spreads the light over the Earth's shadow; and the dissipated light near the limits of the shadow dilates the shadow. Upon which accounts, to the diameter of the shadow, as it comes out by the parallax, I add 1 or  $1\frac{1}{2}$  minute in lunar eclipses.

But the theory of the Moon ought to be examined and proved from the phenomena, first in the syzygies; then in the quadratures; and last of all in the octants; and whoso pleases to undertake the work, will find it not amiss to assume the following mean motions of the Sun

Sun and Moon, at the royal observatory of *Greenwich* to the last day of *December* at noon, anno 1700, O. S. viz. The mean motion of the Sun  $\text{vs } 20^{\circ}. 43'. 40''$ . and of its apogee  $\text{S } 7^{\circ}. 44'. 30''$ . the mean motion of the Moon  $\text{M } 15^{\circ}. 21'. 00''$ ; of its apogee,  $\text{K } 8^{\circ}. 20'. 00''$ . and of its ascending node,  $\text{N } 27^{\circ}. 24'. 20''$ ; and the difference of meridians betwixt the observatory at *Greenwich* and the royal observatory at *Paris*,  $0^{\text{h}}. 9'. 20''$ . but the mean motion of the Moon and of its apogee, are not (yet) obtained with sufficient accuracy.

PROPOSITION XXXVI. PROBLEM XVII.  
*To find the force of the Sun to move the Sea.*

The Sun's force  $ML$  or  $PT$  to disturb the motions of the Moon, was, (by prop. 25.) in the Moon's quadratures, to the force of gravity with us, as 1 to 638092,6. And the force  $TM-LM$ , or  $2PK$  in the Moon's syzygies, is double that quantity. But descending to the surface of the Earth, these forces are diminished in proportion of the distances from the centre of the Earth, that is, in the proportion of  $60\frac{1}{2}$  to 1; and therefore the former force on the Earth's surface is to the force of gravity, as 1 to 38604600. And by this force the Sea is depressed in such places as are 90 degrees distant from the Sun. But by the other force which is twice as great, the Sea is rais'd; not only in the places directly under the Sun, but in those also which are directly opposed to it. And the sum of these forces is to the force of gravity, as 1 to 12868200. And because the same force excites the same motion, whether it depresses the waters in those places which are 90 degrees distant from the Sun; or raises them in the places which are directly under, and directly opposed to the Sun; the foresaid sum will be



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have  
the total force of the Sun to disturb the Sea, and will have the same effect as if the whole was employed in raising the Sea in the places directly under and directly oppos'd to the Sun, and did not act at all in the places which are 90 degrees removed from the Sun.

And this is the force of the Sun to disturb the Sea in any given place, where the Sun is at the same time both vertical, and in its mean distance from the Earth. In other positions of the Sun, its force to raise the Sea is as the versed sine of double its altitude above the horizon of the place directly, and the cube of the distance from the Earth reciprocally.

COR. Since the centrifugal force of the parts of the Earth, arising from the Earth's diurnal motion, which is to the force of gravity as 1 to 289, raises the waters under the equator to a height exceeding that under the poles by 85472 *Paris* feet, as above in prop. 19. the force of the Sun which we have now shewed to be to the force of gravity, as 1 to 12868200, and therefore is to that centrifugal force as 289 to 12868200, or as 1 to 44527, will be able to raise the waters in the places directly under and directly oppos'd to the Sun, to a height exceeding that in the places which are 90 degrees removed from the Sun, only by one *Paris* foot and  $113\frac{1}{3}$  inches. For this measure is to the measure of 85472 feet, as 1 to 44527.

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### PROPOSITION XXXVII. PROBLEM XVIII. *To find the force of the Moon to move the Sea.*

The force of the Moon to move the Sea is to be deduced from its proportion to the force of the Sun, and this proportion is to be collected from the proportion of the motions of the Sea, which are the effects of those forces. Before the mouth of the river *Avon*, three miles below *Bristol*, the height of the ascent of the  
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the water, in the vernal and autumnal syzygies of the luminaries, (by the observations of *Samuel Sturmy*) amounts to about 45 feet, but in the quadratures to 25 only. The former of those heights arises from the sum of the foreſaid forces, the latter from their difference. If therefore S and L are ſuppoſed to repreſent reſpectively the forces of the Sun and Moon, while they are in the equator, as well as in their mean diſtances from the Earth, we ſhall have  $L+S$  to  $L-S$  as 45 to 25, or as 9 to 5. ultima

At *Phymouth* (by the obſervations of *Samuel Colepreſs*) the tide in its mean height riſes to about 16 feet, and in the ſpring and autumn the height thereof in the ſyzygies may exceed that in the quadratures by more than 7 or 8 feet. Suppoſe the greateſt difference of thoſe heights to be 9 feet, and  $L+S$  will be to  $L-S$  as  $20\frac{1}{2}$  to  $11\frac{1}{2}$ , or as 41 to 23; a proportion that agrees well enough with the former. But becauſe of the great tide at *Briſtol*, we are rather to depend upon the obſervations of *Sturmy*, and therefore till we procure ſomething that is more certain, we ſhall uſe the proportion of 9 to 5. media

But becauſe of the reciprocal motions of the waters, the greateſt tides do not happen at the times of the ſyzygies of the luminaries, but as we have ſaid before, are the third in order after the ſyzygies; (or reckoning from the ſyzygies) follow next after the third appulſe of the Moon to the meridian of the place after the ſyzygies; or rather (as *Sturmy* obſerves) are the third after the day of the new or full Moon, or rather nearly after the twelfth hour from the new or full Moon, and therefore fall nearly upon the forty third hour after the new or full of the Moon. But in this port they fall out about the ſeventh hour after the appulſe of the Moon to the meridian of the place; and therefore follow next after the appulſe of the Moon to the meridian, when the Moon is diſtant from the Sun, or from oppoſition baitant

with the Sun by about 18 or 19 degrees *in consequentia*.

So the summer and winter seasons come not to their height in the solstices themselves, but when the Sun is advanced beyond the solstices by about a <sup>second</sup> tenth part of its whole course, that is, by about 36 or 37 degrees.

In like manner the greatest tide is raised after the ap-pulse of the Moon to the meridian of the place, when the Moon has passed by the Sun, *or the opposition there-of*, by about a tenth part of the whole motion from *one greatest tide to the next following greatest tide*. Suppose that distance about  $18\frac{1}{2}$  degrees. And the Sun's

force in this distance of the Moon from the syzygies and quadratures, will be of less moment to augment and diminish that part of the motion of the Sea which proceeds from the motion of the Moon, than in the syzygies and quadratures themselves, in the proportion of the radius to the co-sine of double this distance, or of an angle of 37 degrees, that is, in proportion of 10000000 to 7986355. And therefore in the preceding analogy, in place of S we must put 0,7986355 S.

But further, the force of the Moon in the quadratures must be diminished, on account of its declination from the equator. For the Moon in those quadratures, or rather in  $18\frac{1}{2}$  degrees past the quadratures, declines from the equator by about  $22^{\circ} 13'$ . And the force of either luminary to move the Sea is diminished as it declines from the equator, nearly in the duplicate proportion of the co-sine of the declination. And therefore the force of the Moon in those quadratures is only 0,8570327 L; whence we have  $L + 0,7986355 S$ , to  $0,8570327 L - 0,7986355 S$ , as 2 to 5.

Further yet, the diameters of the orbit, in which the Moon should move, setting aside the consideration of eccentricity, are one to the other, as 69 to 70. And therefore the Moon's distance from the Earth in the syzygies, is to its distance in the quadratures, *ceteris paribus*,

*paribus*, as 69 to 70. And its distances, when  $18\frac{1}{2}$  degrees advanced beyond the syzygies, where the greatest tide was excited, and when  $18\frac{1}{2}$  degrees passed by the quadratures, where the least tide was produced, are to its mean distance as 69,098747 and 69,897345 to  $69\frac{1}{2}$ . But the force of the Moon to move the Sea is in the reciprocal triplicate proportion of its distance. And therefore its forces, in the greatest and least of those distances, are to its force in its mean distance, as 0,9830427 and 1,017522 to 1. From whence we have  $1,017522 L \times 0,7986355 S$  to  $0,9830427 \times 0,8570327 L - 0,7986355 S$  as 9 to 5. And  $S$  to  $L$ , as 1 to 4,4815. Wherefore since the force of the Sun is to the force of gravity as 1 to 12868200, the Moon's force will be to the force of gravity, as 1 to 2871400.

COR. 1. Since the waters excited by the Sun's force rise to the height of a foot and  $11\frac{1}{3}$  inches, the Moon's force will raise the same to the height of 8 feet and  $7\frac{1}{2}$  inches; and the joint forces of both will raise the same to the height of  $10\frac{1}{2}$  feet; and when the Moon is in its perigee, to the height of  $12\frac{1}{2}$  feet, and more, especially when the wind sets the same way as the tide. And a force of that quantity is abundantly sufficient to excite all the motions of the Sea, and agrees well with the proportion of those motions. For in such Seas as lye free and open from east to west, as in the *Pacific* Sea, and in those tracts of the *Atlantic* and *Ethiopic* Seas which lye without the tropics, the waters commonly rise to 6, 9, 12, or 15 feet. But in the *Pacific* Sea, which is of a greater depth as well as of a larger extent, the tides are said to be greater than in the *Atlantic* and *Ethiopic* Seas. For to have a full tide raised, an extent of Sea from east to west is required of no less than 90 degrees. In the *Ethiopic* Sea, the waters rise to a less height within the tropics than in the temperate zones,

because of the narrowness of the Sea between *Africa* and the southern parts of *America*. In the middle of the open Sea the waters cannot rise without falling, together and at the same time, upon both the eastern and western shoars; when notwithstanding in our narrow Seas, they ought to fall on those shores by alternate turns. Upon which account, there is commonly but a small flood and ebb in such islands, as lie far distant from the continent. On the contrary in some ports, where to fill and empty the bays alternately, the waters are with great violence forced in and out through shallow chanel, the flood and ebb must be greater than ordinary, as at *Plymouth* and *Chepstow-Bridge* in *England*, at the mountains of *St. Michael*, and the town of *Auranches* in *Normandy*, and at *Cambaia* and *Pegu* in the *East-Indies*. In these places the Sea is hurried in and out with such violence, as sometimes to lay the shoars under water, sometimes to leave them dry, for many miles. Nor is this force of the influx and efflux to be broke, till it has raised and depressed the waters to 30, 40, or 50 feet and above. And a like account is to be given of long and shallow chanel or streights, such as the *Magellanic* streights and those chanel which environ *England*. The tide in such ports and streights, by the violence of the influx and efflux, is augmented above measure. But on such shoars as ly towards the deep and open Sea, with a steep descent, where the waters may freely rise and fall without that precipitation of influx and efflux, the proportion of the tides agrees with the forces of the Sun and Moon.

COR. 2. Since the Moon's force to move the Sea is to the force of gravity, as 1 to 2871400, it is evident that this force is far less than to appear sensibly in statical or hydrostatical experiments, or even in those of pendulums. It is in the tides only that this force shews it self by any sensible effect.

COR

COR. 3. Because the force of the Moon to move the Sea is to the like force of the Sun as 4,4815 to 1; and those forces (by cor. 14. prop. 66. book 1.) are as the densities of the bodies of the Sun and Moon and the cubes of their apparent diameters conjunctly; the density of the Moon will be to the density of the Sun as 4,4815 to 1 directly, and the cube of the Moon's diameter to the cube of the Sun's diameter inversely; that is, (seeing the mean apparent diameters of the Moon and Sun are  $31'. 16\frac{1}{2}''$ . and  $32'. 12''$ .) as 4891 to 1000. But the density of the Sun was to the density of the Earth, as 1000 to 4000; and therefore the density of the Moon is to the density of the Earth as 4891 to 4000, or as 11 to 9. Therefore the body of the Moon is more dense and more earthly, than the Earth it self.

COR. 4. And since the true diameter of the Moon, (from the observations of astronomers) is to the true diameter of the Earth, as 100 to 365, the mass of matter in the Moon will be to the mass of matter in the Earth as 1 to 39,788.

COR. 5. And the accelerative gravity on the surface of the Moon will be about three times less than the accelerative gravity on the surface of the Earth.

COR. 6. And the distance of the Moon's centre from the centre of the Earth will be to the distance of the Moon's centre from the common centre of gravity of the Earth and Moon, as 40,788 to 39,788.

COR. 7. And the mean distance of the centre of the Moon from the centre of the Earth will be (in the Moon's octants) nearly  $60\frac{2}{3}$  of the greatest semidiameters of the Earth. For the greatest semidiameter of the Earth was 19658600 *Paris* feet, and the mean distance of the centres of the Earth and Moon, consisting of  $60\frac{2}{3}$  such semidiameters, is equal to 1187379440 feet. And this distance (by the preceeding cor.) is to the distance of the Moon's centre

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from

from the common centre of gravity of the Earth and Moon, as 40,788 to 39,788; which latter distance therefore is 1158268534 feet. And since the Moon, in respect of the fixt Stars, performs its revolution in 27<sup>d</sup>. 7<sup>h</sup>. 43<sup>4</sup>/<sub>5</sub>. the versed-sine of that angle which the Moon in a minute of time describes is 12752341, to the radius 1000,000000,000000. And as the radius is to this versed-sine, so are 1158268534 feet to 14,7706353 feet. The Moon therefore falling towards the Earth, by that force which retains it in its orbit, would in one minute of time describe 14,7706353 feet. And if we augment this force in the proportion of  $178\frac{2}{4}$  to  $177\frac{2}{4}$ , we shall have the total force of gravity at the orbit of the Moon, by cor. prop. 3. And the Moon falling by this force, in one minute of time would describe 14,8538067 feet. And at the 60<sup>th</sup> part of the distance of the Moon from the Earth's centre. That is, at the distance of 197896573 feet from the centre of the Earth, a body falling by its weight, would, in one second of time, likewise describe 14,8538067 feet. And therefore at the distance of 19615800, which compose one mean semi-diameter of the Earth, a heavy body would describe in falling 15,11175, or 15 feet, 1 inch and  $4\frac{1}{11}$  lines in the same time. This will be the descent of bodies in the latitude of 45 degrees. And by the foregoing table to be found under prop. 20. the descent in the latitude of *Paris* will be a little greater by an excess of about  $\frac{2}{3}$  parts of a line. Therefore by this computation heavy bodies in the latitude of *Paris* falling in *vacuo* will describe 15 *Paris* feet, 1 inch,  $4\frac{2}{3}$  lines very nearly in one second of time. And if the gravity be diminished by taking away a quantity equal to the centrifugal force arising in that latitude from the Earth's diurnal motion; heavy bodies falling there will describe in one second of time 15 feet, 1 inch, and  $1\frac{1}{2}$  line. And with this velocity heavy bodies

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do really fall in the latitude of *Paris*, as we have shewn above in prop. 4. and 19.

COR. 8. The mean distance of the centres of the Earth and Moon in the syzygies of the Moon is equal to 60 of the greatest semidiameters of the Earth, subducting only about one 30<sup>th</sup> part of a semidiameter. *inst. rays* And in the Moon's quadratures the mean distance of the same centres is  $60\frac{1}{2}$  such semidiameters of the Earth. For these two distances are to the mean distance of the Moon in the octants, as 69 and 70 to  $69\frac{1}{2}$ , by prop. 28.

COR. 9. The mean distance of the centres of the Earth and Moon in the syzygies of the Moon is 60 mean semidiameters of the Earth, and a 10<sup>th</sup> part of one semidiameter; and in the Moon's quadratures the mean distance of the same centres is 61 mean semidiameters of the Earth, subducting one 30<sup>th</sup> part of one semidiameter.

COR. 10. In the Moon's syzygies its mean horizontal parallax in the latitudes of 0, 30, 38, 45, 52, 60, 90 degrees, is 57'. 20". 57'. 16". 57'. 14". 57'. 12". 57'. 10". 57'. 8". 57'. 4". respectively.

In these computations I don't consider the magnetic attraction of the Earth whose quantity is very small and unknown. If this quantity should ever be found out, and the measures of degrees upon the meridian, the lengths of isochronous pendulums in different parallels, the laws of the motions of the Sea, and the Moon's parallax, with the apparent diameters of the Sun and Moon, should be more exactly determined from phænomena; we should then be unable to bring *inexact* this calculation to a greater accuracy.

PRO-



## PROPOSITION XXXVII. PROBLEM XIX.

*To find the figure of the Moon's body.*

If the Moon's body were fluid like our Sea, the force of the Earth to raise that fluid, in the nearest and remotest parts, would be to the force of the Moon, by which our Sea is raised in the places under and opposite to the Moon, as the accelerative gravity of the Moon towards the Earth, to the accelerative gravity of the Earth towards the Moon, and the diameter of the Moon to the diameter of the Earth conjunctly, that is, as 39,788 to 1, and 100 to 365 conjunctly, or as 1081 to 100. Wherefore, since our Sea, by the force of the Moon, is raised to  $8\frac{1}{2}$  feet; the lunar fluid would be raised by the force of the Earth to 93 feet. And upon this account, the figure of the Moon would be a spheroid, whose greatest diameter produced would pass through the centre of the Earth, and exceed the diameters perpendicular thereto, by 186 feet. Such a figure therefore the Moon affects, and must have put on from the beginning. *Q. E. I.*

COR. Hence it is, that the same face of the Moon always respects the Earth; nor can the body of the Moon possibly rest in any other position, but would return always by a libratory motion to this situation. But those librations however must be exceeding slow, because of the weakness of the forces which excite them; so that the face of the Moon which should be always obverted to the Earth, may for the reason assigned in prop. 17. be turned towards the other focus of the Moon's orbit, without being immediately drawn back, and converted again towards the Earth.

L E M.

L E M M A I.

If  $APEp$  (Pl. 14. Fig. 3.) represent the Earth uniformly dense, mark'd with the centre  $C$ , the poles  $P, p$ , and the equator  $AE$ ; and if about the centre  $C$ , with the radius  $CP$ , we suppose the sphere  $Pape$  to be described, and  $QR$  to denote the plane on which a right line, drawn from the centre of the Sun to the centre of the Earth, insists at right angles, and farther suppose, that the several particles of the whole exterior Earth  $Pape$ , without the height of the said sphere, endeavour to recede towards this side and that side from the plane  $QR$ , every particle by a force proportional to its distance from that plane; I say in the first place, that the whole force and efficacy of all the particles, that are situate in  $AE$  the circle of the equator, and disposed uniformly without the globe, encompassing the same after the manner of a ring, to wheel the Earth about its centre, is to the whole force and efficacy of as many particles, in that point  $A$  of the equator which is at the greatest distance from the plane  $QR$ , to wheel the Earth about its centre with a like circular motion, as 1 to 2. And that circular motion will be performed about an axis lying in the common section of the equator and the plane  $QR$ .

For let there be described from the centre  $K$ , with the diameter  $IL$ , the semicircle  $INLK$ . Suppose the semicircumference  $INL$  to be divided into innumerable

able equal parts, and from the several parts  $N$  to the diameter  $IL$  let fall the sines  $NM$ . Then the sums of the squares of all the sines  $NM$  will be equal to the sums of the squares of the sines  $KM$ , and both sums together will be equal to the sums of the squares of as many semidiameters  $KN$ ; and therefore the sum of the squares of all the sines  $NM$  will be but half so great as the sum of the squares of as many semidiameters  $KN$ .

Suppose now the circumference of the circle  $AE$  to be divided into the like number of little equal parts, and from every such part  $F$  a perpendicular  $FG$  to be let fall upon the plane  $QR$ , as well as the perpendicular  $AH$  from the point  $A$ . Then the force by which the particle  $F$  recedes from the plane  $QR$ , will (by supposition) be as that perpendicular  $FG$ , and this force multiplied by the distance  $CG$  will represent the power of the particle  $F$  to turn the Earth round its centre. And therefore the power of a particle in the place  $F$ , will be to the power of a particle in the place  $A$ , as  $FG \times GC$  to  $AH \times HC$ ; that is, as  $FC^2$  to  $AC^2$ : and therefore the whole power of all the particles  $F$ , in their proper places  $F$ , will be to the power of the like number of particles in the place  $A$ , as the sum of all the  $FC^2$  to the sum of all the  $AC^2$ , that is, (by what we have demonstrated before) as 1 to 2.  $\mathcal{Q}$ .  $E. D.$

And because the action of those particles is exerted in the direction of lines perpendicularly receding from the plane  $QR$ , and that equally from each side of this plane, they will wheel about the circumference of the circle of the equator, together with the adherent body of the Earth, round an axe, which lies as well in the plane  $QR$ , as in that of the equator.

L E M.

LEMMA II.

The same things still supposed, I say in the second place, that the total force or power of all the particles situated (every where) about the sphere to turn the Earth about the said axe, is to the whole force of the like number of particles, uniformly dispos'd round the whole circumference of the equator  $AE$  in the fashion of a ring, to turn the whole Earth about with the like circular motion, as 2 to 5. Pl. 14. Fig. 4.

For, let  $IK$  be any lesser circle parallel to the equator  $AE$ , and let  $L, l$  be any two equal particles in this circle, situated without the sphere  $Pape$ . And if upon the plane  $QR$ , which is at right angles with a radius drawn to the Sun, we let fall the perpendiculars  $LM, lm$ ; the total forces by which these particles recede from the plane  $QR$ , will be proportional to the perpendiculars  $LM, lm$ . Let the right line  $Ll$  be drawn parallel to the plane  $Pape$ , and bisect the same in  $X$ ; and thro' the point  $X$  draw  $Nn$ , parallel to the plane  $QR$ , and meeting the perpendiculars  $LM, lm$  in  $N$  and  $n$ ; and upon the plane  $QR$  let fall the perpendicular  $XT$ . And the contrary forces of the particles  $L$  and  $l$ , to wheel about the Earth contrarywise, are as  $LM \times MC$ , and  $lm \times mC$ , that is, as  $LN \times MC + NM \times MC$ , and  $ln \times mC - nm \times mC$ ; or  $LN \times MC + NM \times MC$ , and  $LN \times mC - NM \times mC$ , and  $LN \times Mm - NM \times MC + mC$ , the difference of the two, is the force of both taken together to turn the Earth round. The affirmative part of this difference  $LN \times Mm$ , or  $2LN \times NX$ , is to  $2AH \times HC$ , the force of two particles of the same size situated

ated in  $A$ , as  $LX^2$  to  $AC^2$ . And the negative part  $NM \times MC - mC$ , or  $2XY \times CY$ , is to  $2AH \times HC$ , the force of the same two particles situated in  $A$ , as  $CX^2$  to  $AC^2$ . And therefore the difference of the parts, that is, the force of the two particles  $L$  and  $I$ , taken together, to wheel the Earth about, is to the force of two particles, equal to the former and situated in the place  $A$ , to turn in like manner the Earth round, as  $LX^2 - CX^2$  to  $AC^2$ . But if the circumference  $IK$  of the circle  $IK$  is supposed to be divided into an infinite number of little equal parts  $L$ , all the  $LX^2$  will be to the like number of  $IX^2$ , as 1 to 2 (by lem. 1.) and to the same number of  $AC^2$ , as  $IX^2$  to  $2AC^2$ ; and the same number of  $CX^2$ , to as many  $AC^2$ , as  $2CX^2$  to  $2AC^2$ . Wherefore the united forces of all the particles in the circumference of the circle  $IK$ , are to the joint forces of as many particles in the place  $A$ , as  $IX^2 - 2CX^2$  to  $2AC^2$ ; and therefore (by lem. 1.) to the united forces of as many particles in the circumference of the circle  $AE$ , as  $IX^2 - 2CX^2$  to  $AC^2$ .

Now if  $Pp$  the diameter of the sphere is conceiv'd to be divided into an infinite number of equal parts, upon which a like number of circles  $IK$  are supposed to insist, the matter in the circumference of every circle  $IK$  will be as  $IX^2$ . And therefore the force of that matter to turn the Earth about will be as  $IX^2$  into  $IX^2 - 2CX^2$ . And the force of the same matter, if it was situated in the circumference of the circle  $AE$ , would be as  $IX^2$  into  $AC^2$ . And therefore the force of all the particles of the whole matter, situated without the sphere in the circumferences of all the circles, is to the force of the like number of particles situated in the circumference of the greatest circle  $AE$ , as all the  $IX^2$  into  $IX^2 - 2CX^2$  to as many  $IX^2$  into  $AC^2$ , that is, as all the  $AC^2 - CX^2$  into  $AC^2 - 3CX^2$  to as many  $AC^2 - CX^2$  into  $AC$ .

$AC^2$ , that is, as all the  $AC^4 - 4AC^2 \times CX^2 + 3CX^4$  to as many  $AC^4 - AC^2 \times CX^2$ , that is, as the whole fluent quantity whose fluxion is  $AC^4 - 4AC^2 \times CX^2 + 3CX^4$ , to the whole fluent quantity whose fluxion is  $AC^4 - AC^2 \times CX^2$ ; and therefore by the method of fluxions, as  $AC^4 \times CX - \frac{4}{3}AC^2 \times CX^3 + \frac{3}{5}CX^5$  to  $AC^4 \times CX - \frac{1}{3}AC^2 \times CX^3$ ; that is, if for  $CX$  we write the whole  $Cp$ , or  $AC$ , as  $\frac{4}{15}AC^5$  to  $\frac{2}{3}AC^5$ , that is, as 2 to 5. Q. E. D.

### LEMMA III.

*to be proved*

The same things still supposed, I say in the third place, that the motion of the whole Earth about the axe abovenamed, arising from the motions of all the particles, will be to the motion of the foresaid ring about the same axe, in a proportion compounded of the proportion of the matter in the Earth to the matter in the ring; and the proportion of three squares of the quadrantal arc of any circle, to two squares of its diameter, that is, in the proportion of the matter to the matter, and of the number 925275, to the number 1000000.

For the motion of a cylinder, revolv'd about its quiescent axe, is to the motion of the inscrib'd sphere revolv'd together with it, as any four equal squares to three circles inscrib'd in three of those squares: And the motion of this cylinder is to the motion of an exceeding thin ring, surrounding both sphere and cylinder in their common contact, as double the matter in the cylinder to triple the matter in the ring: And this motion of the ring, uniformly continued about

about the axe of the cylinder, is to the uniform motion of the same about its <sup>own</sup> diameter perform'd in the same periodic time, as the circumference of a circle to double its diameter.

## HYPOTHESIS II.

*If the other parts of the Earth were <sup>quitada</sup> (took away), and the remaining ring was carried alope about the Sun in the orbit of the Earth by <sup>the</sup> annual motion, while by the diurnal motion it was in the mean time revolved about its own axe, inclined to the plane of the ecliptic by an angle of  $23\frac{1}{2}$  degrees; the motion of the equinoctial points would be the same, whether the ring were fluid, or whether it consisted of a hard and rigid matter.*

### PROPOSITION XXXIX. PROBLEM XX.

*To find the precession of the equinoxes.*

The middle horary motion of the Moon's nodes, in a circular orbit when the nodes are in the quadratures, was  $16''$ .  $35'''$ .  $16^{iv}$ .  $36^v$ . the half of which  $8''$ .  $17'''$ .  $38^{iv}$ .  $18^v$ . (for the reasons above explain'd) is the mean horary motion of the nodes in such an orbit, which motion in a whole sidereal year becomes  $20^\circ$ .  $11'$ .  $46''$ . Because therefore the nodes of the Moon in such an orbit would be yearly transfer'd  $20^\circ$ .  $11'$ .  $46''$ . in *antecedentia*; and if there were more Moons, the motion of the nodes of every one, <sup>(by cor. 16. prop. 66. book 1.)</sup> would be as its periodic time; if upon the surface of the Earth, a Moon was revolv'd in the time of a sidereal day, the annual motion of the nodes of this Moon would be to  $20^\circ$ .  $11'$ .  $46''$ .

as







3<sup>h</sup>. 56'. the sidereal day, to 27<sup>d</sup>. 7<sup>h</sup>. 43'. the pe-  
 lic time of our Moon, that is, as 1436 to 39343.  
 and the same thing would happen to the nodes of a  
 ring of Moons encompassing the Earth, whether these *circumduct*  
 moons did not mutually touch each the other, or whe- *hage*  
 ther they were molten and form'd into a continued *fundida*  
 ring, or whether that ring should become rigid and in-  
 flexible.

Let us then suppose that this ring is in quantity of  
 matter equal to the whole exterior Earth *PapApEpE*,  
 which lies without the sphere *Pape* (see *Fig. Lem.*  
 2.) and because this sphere is to that exterior Earth,  
 as  $aC^2$  to  $AC^2 - aC^2$ , that is, (seeing *PC* or  
*aC* the least semidiameter of the Earth is to *AC* the *maximus*  
 greatest semidiameter of the same as 229 to 230) as  
 52441 to 459; if this ring encompass'd the Earth *circumduct*  
 round the equator, and both together were revolv'd  
 about the diameter of the ring, the motion of the ring  
 (by *lem. 3.*) would be to the motion of the inner *interior*  
 sphere, as 459 to 52441 and 1000000 to 925275  
 conjunctly, that is, as 4590 to 485223; and therefore  
 the motion of the ring would be to the sum of the  
 motions of both ring and sphere, as 4590 to 489813.  
 Wherefore if the ring adheres to the sphere, and com-  
 municates its motion to the sphere, by which its nodes  
 or equinoctial points recede: the motion remaining in  
 the ring will be to its former motion, as 4590 to  
 489813, upon which account the motion of the equi-  
 noctial points will be diminish'd in the same proportion.  
 Wherefore the annual motion of the equinoctial points  
 of the body, compos'd of both ring and sphere, will  
 be to the motion, 20°. 11'. 46". as 1436 to 39343  
 and 4590 to 489813 conjunctly, that is as 100 to  
 292369. But the forces by which the nodes of a  
 number of Moons (as we explained above) and there-  
 fore by which the equinoctial points of the ring re-  
 ce-de (that is the forces 3 *IT* in *Fig. prop. 30*) are in the  
 Y several

extensive  
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above

several particles as the distances of those particles from the plane  $QR$ ; and by these forces the particles recede from that plane: and therefore (by lem. 2.) if the matter of the ring was spread all over the surface of the sphere, after the fashion of the figure  $PapAPEpE$ , in order to make up that exterior part of the Earth, the total force or power of all the particles to wheel about the Earth round any diameter of the equator, and therefore to move the equinoctial points, would become less than before, in the proportion of 2 to 5. Wherefore the annual regrefs of the equinoxes now would be to  $20^{\circ}. 11'. 46''$ . as 10 to 73092: that is, would be  $9''. 56'''$ .  $50^{iv}$ .

But because the plane of the equator is inclin'd to that of the ecliptic, this motion is to be diminish'd in the proportion of the sine 91706, (which is the co-sine of  $23 \frac{1}{2}$  deg.) to the radius 100000. And the remaining motion will now be  $9''. 7'''$ .  $20^{iv}$ . which is the annual precession of the equinoxes, arising from the force of the Sun.

But the force of the Moon to move the sea was to the force of the Sun nearly as 4,4815 to 1. And the force of the Moon to move the equinoxes is to that of the Sun in the same proportion. Whence the annual precession of the equinoxes, proceeding from the force of the Moon, comes out  $40''. 52'''$ .  $52^{iv}$ . and the total annual precession, arising from the united forces of both, will be  $50''. 00'''$ .  $12^{iv}$ . the quantity of which motion agrees with the phaenomena. For the precession of the equinoxes, by astronomical observations, is about  $50''$ . yearly.

If the height of the Earth at the equator exceeds its height at the poles by more than  $17 \frac{1}{2}$  miles, the matter thereof will be more rare near the surface, than at the center; and the precession of the equinoxes will be augmented by the excess of height, and diminished by the greater rarity.

And now we have described the system of the Sun, the Earth, Moon and Planets, it (remains) that we (add) something about the Comets. *restemur* *agregamos*

## L E M M A IV.

*That the Comets are higher than the Moon, and in the regions of the Planets.*

As the Comets were placed by astronomers above the Moon because they were found to have no diurnal parallax; so their annual parallax is a convincing proof of their descending into the regions of the Planets. For all the Comets which move in a direct course according to the order of the signs, about the end of their appearance become more than ordinarily slow or retrograde, if the Earth is between them and the Sun: and more than ordinarily swift, if the Earth is approaching to a heliocentric opposition with them. Whereas, on the other hand, those which move against the order of the signs, towards the end of their appearance, appear swifter than they ought to be, if the Earth is between them and the Sun; and slower, and perhaps retrograde, if the Earth is in the other side of its orbit. And these appearances proceed chiefly from the diverse situations which the Earth acquires in the course of its motion, after the same manner as it happens to the Planets, which appear sometimes retrograde, sometimes more slowly, and sometimes more swiftly, progressive, according as the motion of the Earth falls in with that of the Planet, or is directed the contrary way. If the Earth move the same way with the Comet, but, by an angular motion about the Sun, so much swifter that right lines drawn from the Earth to the Comet converge towards the parts beyond the Comet; the Comet seen from the Earth because of its slower motion will appear

*regla-orden*  
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*contra*  
*mayor velocidad*  
*necesitan*  
*mayor lentitud*  
*principalmente*  
*lentamente*  
*rapidamente*  
*desciende*  
*mayor lentitud*  
*del otro lado*  
*mayor lentitud*

Y 2

appear

pear retrograde; and even, if the Earth is slower than  
 the Comet, the motion of the Earth being subducted,  
 the motion of the Comet will at least appear retarded.  
 But if the Earth tends the contrary way to that of the  
 Comet, the motion of the Comet will from thence ap-  
 pear accelerated. And from this apparent acceleration,  
 or retardation, or regressive motion, the distance of the  
 Comet may be inferr'd in this manner. Let  $\angle Q A$ ,  
 $\angle Q B$ ,  $\angle Q C$  (Pl. 15. Fig. 1.) be three observed lon-  
 gitudes of the Comet about the time of its first ap-  
 pearing, and  $\angle Q F$  its last observed longitude before  
 its disappearing. Draw the right line  $A B C$ , whose  
 parts  $A B$ ,  $B C$ , intercepted between the right lines  
 $Q A$  and  $Q B$ ,  $Q B$  and  $Q C$ , may be one to the other,  
 as the two times between the three first observations.  
 Produce  $A C$  to  $G$ , so as  $A G$  may be to  $A B$  as the  
 time between the first and last observation to the time  
 between the first and second; and join  $Q G$ . Now if  
 the Comet did move uniformly in a right line, and the  
 Earth either stood still, or was likewise carried for-  
 wards in a right line by an uniform motion: the angle  
 $\angle Q G$  would be the longitude of the Comet at the  
 time of the last observation. The angle therefore  
 $\angle F Q G$ , which is the difference of the longitude, pro-  
 ceeds from the inequality of the motions of the Comet  
 and the Earth. And this angle, if the Earth and Co-  
 met move contraryways, is added to the angle  $\angle Q G$ ,  
 and accelerates the apparent motion of the Comet. But  
 if the Comet move the same way with the Earth, it is  
 subtracted, and either retards the motion of the Co-  
 met, or perhaps renders it retrograde, as we have but  
 now explained. This angle therefore, proceeding chiefly  
 from the motion of the Earth, is justly to be esteem'd  
 the parallax of the Comet; neglecting, (to wit,) some  
 little increment or decrement that may arise from the  
 unequal motion of the Comet in its orbit. And from  
 this parallax we thus deduce the distance of the Comet.

Let

Let  $S$ , (Pl. 15. Fig. 2.) represent the Sun,  $a c T$  the *orbis magnus*,  $a$  the Earth's place in the first observation,  $c$  the place of the Earth in the third observation,  $T$  the place of the Earth in the last observation, and  $T \Upsilon$  a right line drawn to the beginning of Aries. (Set off) the angle  $\Upsilon T V$ , equal to the angle  $\Upsilon Q F$ , that is, equal to the longitude of the Comet at the time when the Earth is in  $T$ ; join  $a c$ , and produce it to  $g$ , so as  $a g$  may be to  $a c$ , as  $A G$  to  $A C$ ; and  $g$  will be the place at which the Earth would have arrived in the time of the last observation, if it had continued to move uniformly in the right line  $a c$ . Wherefore if we draw  $g \Upsilon$ , parallel to  $T \Upsilon$ , and make the angle  $\Upsilon g V$ , equal to the angle  $\Upsilon Q G$ , this angle  $\Upsilon g V$  will be equal to the longitude of the Comet seen from the place  $g$ , and the angle  $T V g$  will be the parallax which arises from the Earth's being transferr'd from the place  $g$  into the place  $T$ ; and therefore  $V$  will be the place of the Comet in the plane of the ecliptic. And this place  $V$  is commonly lower than the orb of Jupiter.

The same thing may be deduced from the incurvation of the way of the Comets. For these bodies move almost in great circles, while their velocity is great, but about the end of their course, when that part of their apparent motion which arises from the parallax bears a greater proportion to their whole apparent motion, they commonly deviate from those circles, and when the Earth goes to one side, they deviate to the other. And this deflexion, because of its corresponding with the motion of the earth, must arise chiefly from the parallax. And the quantity thereof is so considerable, as, by my computation, to place the disappearing Comets a good deal lower than Jupiter. Whence it follows that when they approach nearer to us in their perigees and perihelions, they often descend below the orbs of Mars and the inferior Planets.

The near approach of the Comets is further confirmed from the light of their heads. For the light of a celestial body, illuminated by the Sun and receding to remote parts, is diminished in the quadruplicate proportion of the distance; to wit, in one duplicate proportion, on account of the increase of the distance from the Sun, and in another duplicate proportion, on account of the decrease of the apparent diameter. Wherefore if both the quantity of light and the apparent diameter of a Comet are given, its distance will be also given, by taking the distance of the Comet to the distance of a Planet, in the direct proportion of their diameters and the reciprocal subduplicate proportion of their lights. Thus in the Comet of the year 1682, Mr. *Flamsteed* observed with a telescope of 16 feet, and measured with a micrometer, the least diameter of its head, 2'. 00. But the nucleus, or star in the middle of the head, scarcely amounted to the tenth part of this measure; and therefore its diameter was only 11" or 12". But in the light and splendor of its head, it surpass'd that of the Comet in the year 1680. and might be compared with the Stars of the first or second magnitude. Let us suppose that Saturn with its ring was about four times more lucid; and because the light of the ring was almost equal to the light of the globe within, and the apparent diameter of the globe is about 21". and therefore the united light of both globe and ring would be equal to the light of a globe whose diameter is 30". it follows that the distance of the Comet was to the distance of Saturn, as 1 to  $\sqrt{4}$  inversly and 12" to 30 directly; that is, as 24 to 30, or 4 to 5. Again the Comet in the month of April 1665, as *Hevelius* informs us, excelled almost all the fixt Stars in splendor, and even Saturn it self, as being of a much more vivid colour. For this Comet was more lucid than that other which had appeared about the end of the preceding year and had been compared to the Stars of the first magnitude.

The

The diameter of its head was about 6'. but the nucleus, compared with the Planets by means of a telescope, was plainly less than Jupiter; and sometimes judged less, sometimes judged equal to the globe of Saturn within the ring. Since then the diameters of the heads of the Comets seldom exceed 8' or 12'. and the diameter of the nucleus or central star is but about a tenth or, perhaps fifteenth part of the diameter of the head; it appears that these stars are generally of about the same apparent magnitude with the Planets. But in regard their light may be often compared with the light of Saturn, yea and sometimes exceeds it; it is evident, that all Comets in their perihelions, must either be placed below, or not far above Saturn. And they are much mistaken, who remove them almost as far as the fixt Stars. For if it was so, the Comets could receive no more light from our Sun, than our Planets do from the fixt Stars.

So far we have gone, without considering the obscuration which Comets suffer from that plenty of thick smook, which encompasseth their heads, and through which the heads always shew dull, as through a cloud. For by how much the more a body is obscured by this smook, by so much the more near it must be allowed to come to the Sun, that it may vye with the Planets in the quantity of light which it reflects. Whence it is probable that the Comets descend far below the orb of Saturn, as we proved before from their parallax. But above all the thing is evinced from their tails, which must be owing either to the Sun's light reflected by a smoke arising from them, and dispersing it self through the æther, or to the light of their own heads. In the former case, we must shorten the distance of the Comets, lest we be obliged to allow that the smook arising from their heads, is propagated through such a vast extent of space and with such a velocity and expansion, as will seem altogether incredible.



color  
 dible. In the latter case, the whole light of both head and tail is to be ascribed to the central nucleus. But then if we suppose all this light to be united and condens'd within the disc of the nucleus, certainly the nucleus will by far exceed Jupiter it self in splendor, especially when it emits a very large and lucid tail. If therefore, under a less apparent diameter, it reflects more light, it must be much more illuminated by the Sun, and therefore much nearer to it. And the same argument will bring down the heads of Comets sometimes within the orb of Venus, viz. when being hid under the Sun's rays, they emit such huge and splendid tails, like beams of fire, as sometimes they do. For if all that light was supposed to be gathered together into one Star, it would sometimes exceed not one Venus only, but a great many such united into one.

grande  
tailor  
finest minute  
must be  
as can be  
 Lastly, the same thing is infer'd from the light of the heads, which increases in the recess of the Comets from the Earth towards the Sun; and decreases in their return from the Sun towards the Earth. For so the Comet of the year 1665 (by the observations of *Hevelius*) from the time that it was first seen, was always losing of its apparent motion, and therefore had already passed its perigee; but yet the splendor of its head was daily increasing, till being hid under the Sun's rays, the Comet ceas'd to appear. The Comet of the year 1683 (by the observations of the same *Hevelius*) about the end of *July*, when it first appeared, moved at a very slow rate, advancing only about 40 or 45 minutes in its orb in a day's time. But from that time its diurnal motion was continually upon the increase, till *September* 4, when it arose to about 5 degrees. And therefore in all this interval of time, the Comet was approaching to the Earth. Which is likewise proved from the diameter of its head, measured with a micrometer. For *August* 6. *Hevelius* found it only 6'. 05" including the *coma*, which *Sept.* 2. he observed to be 9'. 07".

9'.07". and therefore its head appeared far less about the beginning, than towards the end of the motion: tho' about the beginning, because nearer to the Sun, it appeared far more lucid than towards the end, as the same *Hervelius* declares. Wherefore in all this interval of time, on account of its recess from the Sun, it decreas'd in splendor, notwithstanding its access towards the Earth. The Comet of the year 1618 about the middle of December, and that of the year 1680, about the end of the same month, did both move with their greatest velocity, and were therefore then in their perigees. But the greatest splendor of their heads was seen two weeks before, when they had just got clear of the Sun's rays; and the greatest splendor of their tails, a little more early, when yet nearer to the Sun. The head of the former Comet (according to the observations of *Cysartus*) December 1. appeared greater than the Stars of the first magnitude, and December 16. (then in the perigee) it was but little diminished in magnitude, but in the splendor and brightness of its light, a great deal. January 7, *Kepler* being uncertain about the head left off observing. December 12. the head of the latter Comet was seen and observ'd by Mr. *Flamsteed*, when but 9 degrees distant from the Sun; which is scarcely to be done in a Star of the third magnitude. December 15 and 17. it appeared as a Star of the third magnitude, its lustre being diminished by the brightness of the clouds near the setting Sun. December 26. when it mov'd with the greatest velocity, being almost in its perigee, it was less than the mouth of *Pegasus*, a Star of the third magnitude. Jan. 3. it appeared as a Star of the fourth. Jan. 9. as one of the fifth. Jan. 13. it was hid by the splendor of the Moon then in her increase. January 25. it was scarcely equal to the Stars of the seventh magnitude. If we compare equal intervals of time, on one side and on the other, from the perigee, we shall find that the head of the Comet, which at

both

both intervals of time, was <sup>also by in</sup> far, <sup>not alone</sup> but yet equally, re-  
 mov'd from the Earth, and should have therefore shone  
 with equal splendor, appear'd brightest on the side of  
 the perigee towards the Sun; and disappeared on the  
 other. Therefore from the great difference of light  
 in the one situation and in the other, we conclude the great  
 vicinity of the Sun and Comet in the former. For the  
 light of Comets uses to be regular, and to appear  
 greatest when the heads move fastest, and are therefore  
 in their perigees; <sup>rather</sup> excepting in so far as it is increased by  
 their nearness to the Sun.

COR. 1. Therefore the Comets shine by the Sun's  
 light, which they reflect.

COR. 2. From what has been said, we may likewise  
 understand, why Comets are so frequently seen in that  
 hemisphere in which the Sun is, and so seldom in the  
 other. If they were visible in the regions far above  
 Saturn, they would appear more frequently in the parts  
 opposite to the Sun. For such as were in those parts  
 would be nearer to the Earth, whereas the presence of  
 the Sun must obscure and hide those that appear  
 in the hemisphere in which he is. Yet looking over  
 the history of Comets, I find that four or five times  
 more have been seen in the hemisphere towards the Sun,  
 than in the opposite hemisphere; besides, without doubt,  
 not a few, which have been hid by the light of the  
 Sun. For Comets descending into our parts neither  
 emit tails nor are so well illuminated by the Sun as to  
 discover themselves to our (naked eyes), until they are  
 come nearer to us than Jupiter. But the far greater  
 part of that spherical space, which is describ'd about  
 the Sun with so small an interval, lies on that side of  
 the Earth which regards the Sun; and the Comets in  
 that greater part are commonly more strongly il-  
 luminated, as being for the most part nearer to the  
 Sun.

COR.

COR. 3. Hence also it is evident, that the celestial spaces are void of resistance. For though the Comets are carried in oblique paths, and sometimes contrary to the course of the Planets, yet they move every way with the greatest freedom, and preserve their motions for an exceeding long time, even where contrary to the course of the Planets. I am out in my judgment, if they are not a sort of Planets, revolving in orbits returning into themselves with a perpetual motion. For as to what some writers contend, that they are no other than meteors, led into this opinion by the perpetual changes that happen to their heads, it seems to have no foundation. For the heads of Comets are encompassed with huge atmospheres, and the lowermost parts of these atmospheres must be the densest. And therefore it is in the clouds only, not in the bodies of the Comets themselves, that these changes are seen. Thus the Earth, if it was view'd from the Planets, would, without all doubt, shine by the light of its clouds, and the solid body would scarcely appear through the surrounding clouds. Thus also the belts of Jupiter are form'd in the clouds of that Planet, for they change their position one to another, and the solid body of Jupiter is hardly to be seen through them. And much more must the bodies of Comets be hid under their atmospheres, which are both deeper and thicker.

## PROPOSITION XL. THEOREM XX.

*That the Comets move in some of the conic sections, having their foci in the center of the Sun; and by radij drawn to the Sun describe areas proportional to the times.*

This proposition appears from cor. 1. prop. 13. book 1. compared with prop. 8. 12. and 13. book 3.

COR. 1. Hence if Comets are revolv'd in orbits returning into themselves, those orbits will be ellipses; and their periodic times be to the periodic times of the Planets in the sesquiplicate proportion of their principal axes. And therefore the Comets, which for the most part of their course are higher than the Planets, and upon that account describe orbits with greater axes, will require a longer time to finish their revolutions. Thus if the axe of a Comet's orbit was four times greater than the axe of the orbit of Saturn, the time of the revolution of the Comet would be to the time of the revolution of Saturn, that is, to 30 years, as  $4\sqrt{4}$  (or 8) to 1, and would therefore be 240 years.

COR. 2. But their orbits will be so near to parabolas, that parabolas may be us'd for them without sensible error.

COR. 3. And therefore by cor. 7. prop. 16. book 1. the velocity of every Comet will always be to the velocity of any Planet, suppos'd to be revolv'd at the same distance in a circle about the Sun, nearly in the subduplicate proportion of double the distance of the Planet from the centre of the Sun, to the distance of the Comet from the Sun's centre very nearly. Let us suppose the radius of the *orbis magnus*, or the greatest semidiameter of the ellipse which the Earth describes,

to

to consist of 100000000 parts; and then the Earth by its mean diurnal motion will describe 1720212 of those parts, and 71675 $\frac{1}{2}$  by its horary motion. And therefore the Comet, at the same mean distance of the Earth from the Sun, with a velocity which is to the velocity of the Earth as  $\sqrt{2}$  to 1, would by its diurnal motion describe 2432747 parts, and 101364 $\frac{1}{2}$  parts by its horary motion. But at greater or less distances both the diurnal and horary motion will be to this diurnal and horary motion in the reciprocal subduplicate proportion of the distances, and is therefore given.

COR. 4. Wherefore, if the *latus rectum* of the parabola is quadruple of the radius of the *orbis magnus*, and the square of that radius is suppos'd to consist of 100000000 parts: the area which the Comet will daily describe by a radius drawn to the Sun will be 1216373 $\frac{1}{2}$  parts; and the horary area will be 50682 $\frac{1}{4}$  parts. But if the *latus rectum* is greater or less in any proportion, the diurnal and horary area will be less or greater, in the subduplicate of the same proportion reciprocally.

# L E M M A V.

To find a curve line of the parabolic kind, which shall pass through any given number of points. Pl. 15. Fig. 3. class - 21 - piece

Let those points be *A, B, C, D, E, F, &c.* and from the same to any right line *HN*, given in position, let fall as many perpendiculars *AH, BI, CK, DL, EM, FN, &c.*

Case 1. If *HI, IK, KL, &c.* the intervals of the points *H, I, K, L, M, N, &c.* are equal, take *b, 2b, 3b, 4b,*

*4b, 5b, &c.* the first differences of the perpendiculars  $AH, BI, CK, &c.$  their second differences  $c, 2c, 3c, 4c, &c.$  their third,  $d, 2d, 3d, &c.$  that is to say, so as  $AH - BI$  may be  $= b$ ,  $BI - CK = 2b$ ,  $CK - DL = 3b$ ,  $DL - EM = 4b$ ,  $EM - FN = 5b$ , &c. then  $b - 2b = c$ , &c. and so on to the last difference, which is here  $f$ . Then erecting any perpendicular  $RS$ , which may be considered as an ordinate of the curve required; in order to find the length of this ordinate, suppose the intervals  $HI, IK, KL, LM, &c.$  to be units, and let  $AH = a$ ,  $HS = p$ ,  $\frac{1}{2}p$  into  $IS = q$ ,  $\frac{1}{3}q$  into  $SK = r$ ,  $\frac{1}{4}r$  into  $SL = s$ ,  $\frac{1}{5}s$  into  $SM = t$ ; proceeding, to wit, to  $ME$ , the last perpendicular but one, and prefixing negative signs before the terms  $HS, IS, &c.$  which lye from  $S$  towards  $A$ ; and affirmative signs before the terms  $SK, SL, &c.$  which lie on the other side of the point  $S$ . And observing well the signs,  $RS$  will be  $= a - bp + cq - dr + es - ft, + &c.$

*Case 2.* But if  $HI, IK, &c.$  the intervals of the points  $H, I, K, L, &c.$  are unequal, take  $b, 2b, 3b, 4b, 5b, &c.$  the first differences of the perpendiculars  $AH, BI, CK, &c.$  divided by the intervals between those perpendiculars;  $c, 2c, 3c, 4c, &c.$  their second differences divided by the intervals between every two;  $d, 2d, 3d, &c.$  their third differences, divided by the interval between every three;  $e, 2e, &c.$  their fourth differences, divided by the intervals between every four; and so forth; that is, in such manner, that  $b$  may be  $= \frac{AH - BI}{HI}$ ,  $2b = \frac{BI - CK}{IK}$ ,  $3b = \frac{CK - DL}{KL}$ , &c.

then  $c = \frac{b - 2b}{HK}$ ,  $2c = \frac{2b - 3b}{IL}$ ,  $3c = \frac{3b - 4b}{KM}$ , &c.

then  $d = \frac{c - 2c}{HL}$ ,  $2d = \frac{2c - 3c}{IM}$ , &c. And those differences being found, let  $AH$  be  $= a$ ,  $HS = p$ ,  
 $p$  into

$p$  into  $IS=q$ ,  $q$  into  $+SK=r$ ,  $r$  into  $+SL=s$ ,  $s$  into  $+SM=t$ ; proceeding, to wit, to  $ME$ , the last perpendicular but one; and the ordinate  $RS$  will be  $=a+bp+cq+dr+es+ft, +\&c.$

COR. Hence the areas of all curves may be nearly found. For if some number of points of the curve to be squar'd are found, and a parabola be suppos'd to be drawn through those points; the area of this parabola will be nearly the same with the area of the curvilinear figure propos'd to be squar'd. But the parabola can be always squar'd geometrically by methods vulgarly known.

## LEMMA VI.

*Certain observed places of a Comet being given, to find the place of the same to any intermediate given time.*

Let  $HI, IK, KL, LM$  (in the preceding Fig.) represent the times between the observations;  $HA, IB, KC, LD, ME$ , five observ'd longitudes of the Comet, and  $HS$  the given time between the first observation and the longitude required. Then if a regular curve  $ABCDE$  is suppos'd to be drawn through the points  $A, B, C, D, E$ , and the ordinate  $RS$  is found out by the preceding lemma,  $RS$  will be the longitude required.

After the same method, from five observ'd latitudes we may find the latitude to a given time.

If the differences of the observed longitudes are small, suppose of 4 or 5 degrees, three or four observations will be sufficient to find a new longitude and latitude. But if the differences are greater, as of 10 or 20 degrees, five observationsought to be used.

*necessary*

LEMMA



## L E M M A VII.

*Through a given point P, (Pl. 15. Fig. 4.) to draw a right line BC, whose parts PB, PC, cut off by two right lines AB, AC, given in position, may be, one to the other, in a given proportion.*

From the given point P, suppose any right line PD to be drawn to either of the right lines given as AB, and produce the same towards AC the other given right line, as far as E, so as PE may be to PD in the given proportion. Let EC be parallel to AD. Draw CPB, and PC will be to PB, as PE to PD. Q. E. F.

## L E M M A VIII.

*Let ABC (Pl. 16. Fig. 1.) be a parabola, having its focus in S. By the chord AC bisected in I (cut off) the segment ABCI, whose diameter is  $I\mu$ , and vertex  $\mu$ . In  $I\mu$  produced take  $\mu O$  equal to one half of  $I\mu$ . Join OS, and produce it to  $\xi$ , so as  $S\xi$  may be equal to 2 SO. Now, supposing a Comet to revolve in the arc CBA, draw  $\xi B$ , cutting AC in E; I say, the point E will (cut off) from the chord AC the segment AE, <sup>case</sup> nearly proportional to the time.*

For, if we join EO, cutting the parabolic arc ABC in T, and draw  $\mu X$  touching the same arc in the vertex  $\mu$ , and meeting EO in X, the curvilinear area  
AEX $\mu$ A





$AEX\mu A$  will be to the curvilinear area  $ACT\mu A$ , as  $AE$  to  $AC$ . And therefore since the triangle  $ASE$  is to the triangle  $ASC$  in the same proportion, the whole area  $ASEX\mu A$  will be to the whole area  $ASCT\mu A$ , as  $AE$  to  $AC$ . But because  $\xi O$  is to  $SO$  as 3 to 1, and  $EO$  to  $XO$  in the same proportion,  $SX$  will be parallel to  $EB$ : and therefore joining  $BX$ , the triangle  $SEB$  will be equal to the triangle  $XEB$ . Wherefore if to the area  $ASEX\mu A$  we add the triangle  $EXB$ , and from the sum subduct the triangle  $SEB$ , there will remain the area  $ASBX\mu A$  equal to the area  $ASEX\mu A$ , and therefore in proportion to the area  $ASCT\mu A$  as  $AE$  to  $AC$ . But the area  $ASBT\mu A$  is nearly equal to the area  $ASBX\mu A$ , and this area  $ASBT\mu A$  is to the area  $ASCT\mu A$ , as the time of description of the arc  $AB$  to the time of description of the whole arc  $AC$ . And therefore  $AE$  is to  $AC$  nearly in the proportion of the times. *Q. E. D.*

COR. When the point  $B$  falls upon the vertex  $\mu$  of the parabola,  $AE$  is to  $AC$  accurately in the proportion of the times.

# SCHOLIUM.

If we join  $\mu\xi$  cutting  $AC$  in  $\delta$ , and in it take  $\xi n$  in proportion to  $\mu B$ , as  $27MI$  to  $16M\mu$ , and draw  $Bn$ : this  $Bn$  will cut the chord  $AC$  in the proportion of the times, more accurately than before. But the point  $n$  is to be taken beyond, or on this side the point  $\xi$ , according as the point  $B$  is more or less distant from the principal vertex of the parabola than the point  $\mu$ . *See the last*

Z

LEMMA



same in the height  $S\mu$ , in the reciprocal subduplicate proportion of  $SP$  to  $S\mu$ , that is, in the proportion of  $S\mu$  to  $SN$ ; the length describ'd with this velocity will be to the length in the same time describ'd in the tangent, as  $S\mu$  to  $SN$ . Wherefore since  $AC$ , and the length describ'd with this new velocity, are in the same proportion to the length describ'd in the tangent, they must be equal betwixt themselves. Q. E. D.

COR. Therefore a Comet, with that velocity which it hath in the height  $S\mu - \frac{1}{3} I\mu$ , would, in the same time, describe the chord  $AC$  nearly.

# L E M M A XI.

If a Comet (void of) all motion was let fall from the height  $SN$ , or  $S\mu - \frac{1}{3} I\mu$ , towards the Sun; and was still impell'd to the Sun by the same force, uniformly continued, by which it was impell'd at first; the same in one half of that time in which it might describe the arc  $AC$  in its own orbit, would in descending, describe a space equal to the length  $I\mu$ .

For in the same time that the Comet would require to describe the parabolic arc  $AC$ , it would (by the last lemma) with that velocity which it hath in the height  $SP$ , describe the chord  $AC$ ; and therefore (by cor. 7. prop. 16. book 1.) if it was in the same time suppos'd to revolve by the force of its own gravity, in a circle whose semidiameter was  $SP$ , it would describe an arc of that circle, the length of which would be to the chord of the parabolic arc  $AC$ , in the subduplicate proportion of 1 to 2. Wherefore if with that weight, which in the height  $SP$  it hath towards the Sun, it should fall from that height towards the Sun, it would

Z 2

(by

(by cor. 9. prop. 4. book I.) in half the said time describe a space equal to the square of half the said chord apply'd to quadruple the height  $SP$ , that is, it would

describe the space  $\frac{AI^2}{4SP}$ . But since the weight of the

Comet towards the Sun in the height  $SN$ , is to the weight of the same towards the Sun in the height  $SP$ , as  $SP$  to  $S\mu$ : the Comet, by the weight which it hath in the height  $SN$ , in falling from that height towards the Sun, would in the same time describe the

space  $\frac{AI^2}{4S\mu}$ , that is, a space equal to the length  $I\mu$  or  $\mu M$ . Q. E. D.

# PROPOSITION XLI. PROBLEM XXI.

*From three observations given to determine the orbit of a Comet moving in a parabola.*

This being a problem of very great difficulty, I try'd many methods of resolving it; and several of those problems, the composition whereof I have giv'n in the first book, tended to this purpose. But afterwards I contriv'd the following solution, which is something more simple.

Select three observations distant one from another by intervals of time nearly equal. But let that interval of time in which the Comet moves more slowly, be somewhat greater than the other; so, to wit, that the difference of the times may be to the sum of the times, as the sum of the times to about 600 days; or that the point  $E$  (Pl. 16 Fig. 1.) may fall upon  $M$  nearly, and may err therefrom, rather towards  $I$  than towards  $A$ . If such direct observations are not at hand, a new place of the Comet must be found by lem. 6.

Let

Let  $S$  (Pl. 16. Fig. 2.) represent the Sun;  $T, t, \tau$ , three places of the Earth in the *orbis magnus*;  $T A, t B, \tau C$ , three observ'd longitudes of the Comet;  $V$  the time between the first observation and the second;  $W$  the time between the second and the third;  $X$  the length, which, in the whole time,  $V + W$ , the Comet might describe with that velocity which it hath in the mean distance of the Earth from the Sun: which length is to be found by cor. 3. prop. 40. book 3. and  $t V$  a perpendicular upon the chord  $T \tau$ . In the mean observed longitude  $t B$ , take at pleasure the point  $B$ , for the place of the Comet <sup>a place</sup> in the plane of the ecliptic; and from thence towards <sup>desde alli</sup> the Sun  $S$ , draw the line  $B E$ , which may be to the perpendicular  $t V$ , as the content under  $S B$  and  $S t^2$  to <sup>contenido?</sup> the cube of the hypotenuse of the right angl'd triangle <sup>capacidad</sup> whose sides are  $S B$  and the tangent of the latitude of the Comet; in the second observation to the radius  $t B$ . And through the point  $E$ , (by lemma 7.) draw the right line  $A E C$ , whose parts  $A E$  and  $E C$ , terminating in the right lines  $T A$  and  $\tau C$ , may be, one to the other, as the times  $V$  and  $W$ : then  $A$  and  $C$  will be nearly the places of the Comet in the plane of the ecliptic in the first and third observations, if  $B$  was its place rightly assum'd in the second.

Upon  $A C$ , bisected in  $I$ , erect the perpendicular  $I i$ . Through  $B$  draw the obscure line  $B i$  parallel to  $A C$ . Join the obscure line  $S i$ , cutting  $A C$  in  $\lambda$ , and compleat the parallelogram  $i \lambda \mu$ . Take  $I \sigma$  equal to  $3 I \lambda$ , and through the Sun  $S$ , draw the obscure line  $\sigma \xi$  equal to  $3 S \sigma + 3 i \lambda$ . Then, cancelling the letters  $A, E, C, I$ , <sup>suprimiendo</sup> from the point  $B$  towards the point  $\xi$ , draw the new obscure line  $B E$ , which may be to the former  $B E$  in the duplicate proportion of the distance  $B S$  to the quantity  $S \mu + \frac{1}{3} i \lambda$ . And through the point  $E$ , draw again the right line  $A E C$  by the same rule as before, that is, so as its parts  $A E$  and  $E C$  may be one to the other as the times  $V$  and  $W$ , between



the observations. Thus  $A$  and  $C$  will be the places of the Comet more accurately.

Upon  $AC$ , bisected in  $I$ , erect the perpendiculars  $AM$ ,  $CN$ ,  $IO$ , of which  $AM$  and  $CN$  may be the tangents of the latitudes in the first and third observations, to the radij  $TA$  and  $\tau C$ . Join  $MN$ , cutting  $IO$  in  $O$ . Draw the rectangular parallelogram  $iI\lambda\mu$ , as before. In  $IA$  produc'd, take  $ID$  equal to  $S\mu - \frac{2}{3}i\lambda$ . Then in  $MN$ , towards  $N$ , take  $MP$ , which may be to the above found length  $X$ , in the subduplicate proportion of the mean distance of the Earth from the Sun (or of the semidiameter of the *orbis magnus*) to the distance  $OD$ . If the point  $P$  fall upon the point  $N$ ;  $A$ ,  $B$ , and  $C$  will be three places of the Comet, through which its orbit is to be describ'd in the plane of the ecliptic. But if the point  $P$  falls not upon the point  $N$ ; in the right line  $AC$  take  $CG$  equal to  $NP$ , so as the points  $G$  and  $P$  may lie on the same side of the line  $NC$ .

By the same method, as the points  $E$ ,  $A$ ,  $C$ ,  $G$ , were found from the assum'd point  $B$ , from other points  $b$  and  $\beta$  assum'd at pleasure, find out the new points  $e$ ,  $a$ ,  $c$ ,  $g$ ; and  $\epsilon$ ,  $\alpha$ ,  $\kappa$ ,  $\gamma$ . Then through  $G$ ,  $g$ , and  $\gamma$ , draw the circumference of a circle  $Gg\gamma$ , cutting the right line  $\tau C$  in  $Z$ : and  $Z$  will be one place of the Comet in the plane of the ecliptic. And in  $AC$ ,  $ac$ ,  $\alpha\kappa$ , taking  $AF$ ,  $af$ ,  $\alpha\phi$  equal respectively to  $CG$ ,  $cg$ ,  $\kappa\gamma$ ; through the points  $F$ ,  $f$ , and  $\phi$ , draw the circumference of a circle  $Ff\phi$ , cutting the right line  $AT$  in  $X$ ; and the point  $X$  will be another place of the Comet in the plane of the ecliptic. And at the points  $X$  and  $Z$ , erecting the tangents of the latitudes of the Comet to the radij  $TX$ , and  $\tau Z$ , two places of the Comet in its own orbit will be determin'd. Lastly, if (by prop. 19. book 1.) to the focus  $S$ , a parabola is describ'd passing through those two places, this parabola will be the orbit of the Comet. *Q. E. I.*

The

The demonstration of this construction follows from the preceding lemmas: because the right line  $AC$  is cut in  $E$  in the proportion of the times by lem. 7, as it ought to be by lem. 8: and  $BE$ , by lem. 11, is a portion of the right line  $BS$  or  $B\xi$  in the plane of the ecliptic, intercepted between the arc  $ABC$  and the chord  $AEC$ ; and  $MP$ , (by cor. lem. 10.) is the length of the chord of that arc, which the Comet should describe in its proper orbit between the first and third observation, and therefore is equal to  $MN$ , providing  $B$  is a true place of the Comet in the plane of the ecliptic. — ?

But it will be convenient to assume the points  $B, b, \beta$ , not at random, but nearly true. If the angle  $AQt$ , at which the projection of the orbit in the plane of the ecliptic cuts the right line  $tB$ , is rudely known; at that angle with  $Bt$  draw the obscure line  $AC$ , which may be to  $\frac{2}{3}T\tau$  in the subduplicate proportion of  $SQ$  to  $St$ . And drawing the right line  $SEB$ , so as its part  $EB$  may be equal to the length  $Vt$ , the point  $B$  will be determin'd which we are to use for the first time. Then cancelling the right line  $AC$ , and drawing a new  $AC$  according to the preceding construction, and more-over, finding the length  $MP$ ; in  $tB$  take the point  $b$ , by this rule, that if  $TA$ , and  $\tau C$  intersect each other in  $T$ , the distance  $Tb$  may be to the distance  $TB$  in a proportion compounded of the proportion of  $MP$  to  $MN$  and the subduplicate proportion of  $SB$  to  $Sb$ . And by the same method you may find the third point  $\beta$ , if you please to repeat the operation the third time. But if this method is follow'd, two operations generally will be sufficient. For if the distance  $Bb$  happens to be very small; after the points  $F, f$ , and  $G, g$ , are found, draw the right lines  $Ff$  and  $Gg$ , and they will cut  $TA$  and  $\tau C$  in the points requir'd  $X$  and  $Z$ . *adema*

## EXAMPLE.

Let the Comet of the year 1680 be propos'd. The following table shews the motion thereof, as observ'd by *Flamsteed*, and calculated afterwards by him, from his observations, and corrected by *Dr. Halley* from the same observations.

	Time		Sun's Longitude.	Comet's	
	Appar.	True.		Longitude.	Lat. N.
	h.	h. m.	°.	°.	°.
1680 Dec. 12	4.46	4.46. 0	VS 1.51.23	VS 6.32.30	8.28. 0
21	6.32½	6.36.59	11.06.44	≈ 5.08.12	21.42.13
24	6.12	6.17.52	14.09.26	18.49.23	25.23. 5
26	5.14	5.20.44	16.09.22	28.24.13	27.00.52
29	7.55	8.03.02	19.19.43	X 13.10.41	28.09.58
30	8.02	8.10.26	20.21.09	17.38.20	28.11.53
1681 Jan. 5	5.51	6.01.38	26.22.18	Υ 8.48.53	26.15. 7
9	6.49	7.00.53	≈ 0.29.02	18.44.04	24.11.56
10	5.54	6.06.10	1.27.43	20.40.50	23.43.52
13	6.56	7.08.55	4.33.20	25.59.48	22.17.28
25	7.44	7.58.42	16.45.36	Σ 9.35. 0	17.56.30
30	8.07	8.21.53	21.49.58	13.19.51	16.42.18
Feb. 2	6.20	6.34.51	24.46.59	15.13.53	16.04. 1
5	6.50	7.04.41	27.49.51	16.59.06	15.27. 3

To these you may add some observations of mine.

	Ap. Time.	Comet's	
		Longitude.	Lat. North.
	h. m.	°.	°.
1681 Feb. 25	8.30	Σ 26.18.35	12.46.46
27	8.15	27.04.20	22.36.12
Mar. 1	11. 0	27.52.42	12.23.40
2	8. 0	28.12.48	12.19.38
5	11.30	29.18. 0	12.03.16
7	9.30	II 0.4. 0	11.57. 0
9	8.30	0.43. 4	11.45.52

These observations were made by a telescope of 7 feet, with a micrometer and threads plac'd in the focus of

Fig. I. p. 336.

$\mu$

$\gamma$

$\Lambda$

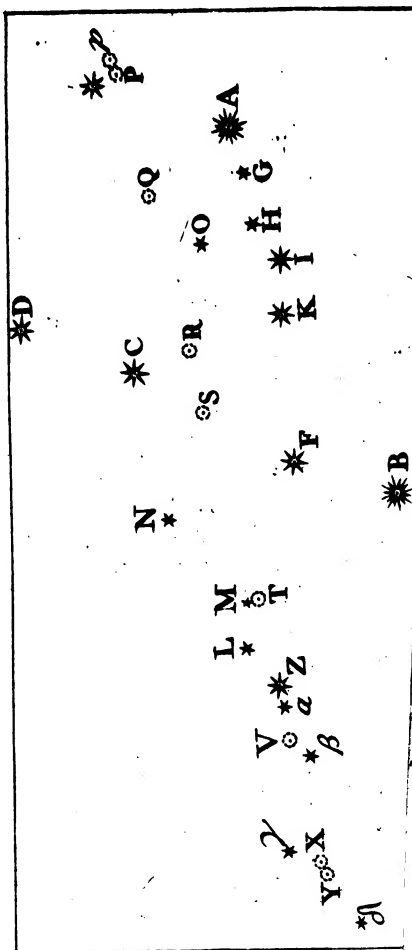
$O$

41.

$\mu$

$\tau$   
 $\nu$   
 $t$

*Plate XVII. Vol. II. Pag. 345.*



cope ; by which instruments we determin'd  
as both of the fixt Stars among themselves

Comet in respect of the fixt Stars. Let

7.) represent the Star of the fourth mag-

the left heel of *Perseus*, (*Bayer's*  $\alpha$ ) *B* the

Star of the third magnitude in the left foot

*C* a Star of the sixth magnitude (*Bayer's*  $\eta$ )

heel of the same foot, and *D, E, F, G, H, I,*

*N, O, Z, \alpha, \beta, \gamma, \delta*, other smaller Stars in

foot. And let *p, P, Q, R, S, T, V, X*, re-

the places of the Comet in the observations

set down ; and reckoning the distance *AB* of

parts, *AC* was  $52\frac{1}{4}$  of those parts, *BC*,  $58\frac{2}{3}$  ;

$57\frac{1}{2}$  ; *BD*,  $82\frac{1}{11}$  ; *CD*,  $23\frac{2}{3}$  ; *AE*,  $29\frac{4}{7}$  ;

$57\frac{1}{2}$  ; *DE*,  $49\frac{1}{12}$  ; *AI*,  $27\frac{1}{2}$  ; *BI*,  $52\frac{1}{6}$  ; *CI*,

;*DI*,  $53\frac{1}{11}$  ; *AK*,  $38\frac{2}{3}$  ; *BK*,  $43$  ; *CK*,  $31\frac{2}{9}$  ;

$29$  ; *FB*,  $23$  ; *FC*,  $36\frac{1}{4}$  ; *AH*,  $18\frac{2}{7}$  ; *DH*,  $50\frac{7}{8}$  ;

$46\frac{1}{2}$  ; *CN*,  $31\frac{1}{3}$  ; *BL*,  $45\frac{1}{12}$  ; *NL*,  $31\frac{1}{7}$ . *HO*

to *HI* as 7 to 6, and produc'd did pass be-

en the Stars *D* and *E*, so as the distance of the Star

from this right line was  $\frac{1}{6}$  *CD*. *LM* was to *LN*

2 to 9, and produc'd did pass through the Star *H*.

thus were the positions of the fixt Stars determin'd

respect of one another.

Mr. *Pound* has since observed a second time the posi-

tions of these fixed Stars amongst themselves, and col-

lected their longitudes and latitudes according to the

following table.

entre  
requiere  
taron  
me  
taron-taron  
puesta  
abajo  
calculan



The

The fixed Stars.	Their Longitudes.	Latitude North.	The fixed Stars.	Their Longitudes.	Latitude North.
	° ' "	° ' "		° ' "	° ' "
A	♄ 26.41.50	12. 8.36	L	♄ 29.33.34	12. 7.48
B	28.40.23	11.17.54	M	29.18.54	12. 7.20
C	27.58.30	12.40.25	N	28.48.29	12.31. 9
E	26.27.17	12.52. 7	Z	29.44.48	11.57.13
F	28.28.37	11.52.22	α	29.52. 3	11.55.48
G	26.56. 8	12. 4.58	β	II 0. 8.23	11.48.56
H	27.11.45	12. 2. 1	γ	0.40.10	11.55.18
I	27.25. 2	11.53.11	δ	1. 3.20	11.30.42
K	27.42. 7	11.53.26			

The positions of the Comet to these fix'd Stars were observ'd to be as follows.

Friday, Feb. 25. O. S. at  $8\frac{1}{2}^h$ , P. M. the distance of the Comet in  $p$  from the Star  $E$ , was less than  $\frac{1}{3} AE$ , and greater than  $\frac{1}{4} AE$ , and therefore nearly equal to  $\frac{1}{4} AE$ ; and the angle  $ApE$  was a little obtuse, but almost right. For from  $A$ , letting fall a perpendicular on  $pE$ , the distance of the Comet from that perpendicular was  $\frac{1}{4} pE$ .

The same night at  $9\frac{1}{2}^h$ , the distance of the Comet in  $P$  from the Star  $E$ , was greater than  $\frac{1}{4\frac{1}{2}} AE$ , and

less than  $\frac{1}{5\frac{1}{4}} AE$ , and therefore nearly equal to  $\frac{10}{4\frac{7}{8}}$  of  $AE$ , or  $\frac{2}{3} AE$ . But the distance of the Comet from the perpendicular let fall from the Star  $A$  upon the right line  $PE$ , was  $\frac{1}{4} PE$ .

Sunday, Feb. 27  $8\frac{1}{4}^h$  P. M. the distance of the Comet in  $Q$ , from the Star  $O$ , was equal to the distance of the Stars  $O$  and  $H$ ; and the right line  $QO$  produc'd pass'd between the Stars  $K$  and  $B$ . I could not, by reason of intervening clouds, determine the position of the Star to greater accuracy.

Tuesday, March 1.  $11^h$ . P. M. the Comet in  $R$ , lay exactly in a line between the Stars  $K$  and  $C$ , so as the

the part  $CR$  of the right line  $CRK$ , was a little greater than  $\frac{1}{3}CK$  and a little less than  $\frac{1}{3}CK + \frac{1}{8}CR$ , and therefore  $= \frac{1}{3}CK + \frac{1}{16}CR$ , or  $\frac{4}{15}CK$ .

Wednesday, March 2. 8<sup>h</sup>. P. M. the distance of the Comet in  $S$  from the Star  $C$ , was nearly  $\frac{1}{2}FC$ ; the distance of the Star  $F$  from the right line  $CS$  produc'd was  $\frac{1}{4}FC$ ; and the distance of the Star  $B$  from the same right line was five times greater than the distance of the Star  $F$ . And the right line  $NS$  produc'd pass'd between the Stars  $H$  and  $I$ , five or six times nearer to the Star  $H$  than to the Star  $I$ . mer color

Saturday, March 5. 11<sup>h</sup> $\frac{1}{2}$  P. M. when the Comet was in  $T$ , the right-line  $MT$  was equal to  $\frac{1}{2}ML$ , and the right-line  $LT$  produc'd pass'd between  $B$  and  $F$ , four or five times nearer to  $F$  than to  $B$ , cutting off from  $BF$  a fifth or sixth part thereof towards  $F$ : and  $MT$  produc'd pass'd on the (out-side) of the space  $BF$ , towards the Star  $B$ , four times nearer to the Star  $B$  than to the Star  $F$ .  $M$  was a very small Star scarcely to be seen by the telescope, but the Star  $L$  was greater, and of about the eighth magnitude. Subado

Monday, March 7. 9<sup>h</sup> $\frac{1}{2}$  P. M. The Comet being in  $V$ , the right line  $V\alpha$  produced did pass between  $B$  and  $F$ , cutting off, from  $BF$  towards  $F$ ,  $\frac{1}{5}$  of  $BF$ , and was to the right line  $V\beta$  at 5 to 4. And the distance of the Comet from the right line  $\alpha\beta$  was  $\frac{1}{2}V\beta$ . quint-  
exterior

Wednesday, March 9. 8<sup>h</sup> $\frac{1}{2}$  P. M. the Comet being in  $X$ , the right line  $\gamma X$  was equal to  $\frac{1}{4}\gamma\delta$ , and the perpendicular let fall from the Star  $\delta$  upon the right  $\gamma X$  was  $\frac{2}{5}$  of  $\gamma\delta$ . octave

The same night at 12<sup>h</sup>, the Comet being in  $\gamma$ , the right line  $\gamma T$  was equal to  $\frac{1}{3}$  of  $\gamma\delta$ , or a little less, as perhaps  $\frac{1}{6}$  of  $\gamma\delta$ , and a perpendicular let fall from the Star  $\delta$  on the right line  $\gamma T$  was equal to about  $\frac{1}{6}$  or  $\frac{1}{7}\gamma\delta$ . But the Comet being then extremely near the horizon was scarcely discernable, and therefore lunar



its place could not be determined with that certainty as in the foregoing observations.

From these observations, by constructions of figures and calculations, I deduced the longitudes and latitudes of the Comet: and Mr. Pound by correcting the places of the fixed Stars hath determined more correctly the places of the Comet, which correct places are set down above. Though my micrometer was none of the best, yet the errors in longitude and latitude (as derived from my observations) scarcely exceed one minute. The Comet (according to my observations) about the end of its motion, began to decline sensibly towards the north, from the parallel which it describ'd about the end of February.

Now in order to determine the orbit of the Comet out of the observations above describ'd; I selected those three which Flamsteed made, Dec. 21. Jan. 5. and Jan. 25. From which I found  $St$  of 9842, 1 parts, and  $Vt$  of 455, such as the semidiameter of the *orbis magnus* contains 10000. Then for the first observation, assuming  $tB$  of 5657 of those parts, I found  $SB$  9747,  $BE$  for the first time 412,  $S\mu$  9503,  $i\lambda$  413,  $BE$  for the second time 421,  $OD$  10186,  $X$  8528, 4;  $PM$  8450,  $MN$  8475,  $NP$  25. From whence, by the second operation, I collected the distance  $t\delta$  5640. And by this operation, I at last deduced the distances  $TX$  4775 and  $\tau Z$  11322. From which limiting the orbit, I found its descending node in  $\odot$  and ascending node in  $\vee$   $1^\circ 53'$ ; the inclination of its plane to the plane of the ecliptick  $61^\circ. 20'\frac{1}{3}$ ; the vertex thereof (or the perihelion of the Comet) distant from the Node  $8^\circ. 38'$ , and in  $\nearrow$   $27^\circ. 43'$ , with latitude  $7^\circ. 34'$  south; its *latus rectum* 236, 8; and the diurnal area describ'd by a radius drawn to the Sun 93585, supposing the square of the semidiameter of the *orbis magnus*, 100000000; that the Comet in this orbit mov'd directly according to the order of

the signs, and on Dec. 8<sup>d</sup>. 00<sup>h</sup>. 04<sup>'</sup> P. M. was in the vertex or perihelion of its orbit. All which I determin'd by scale and compass, and the chords of angles, taken from the table of natural sines, in a pretty large figure, in which, to wit, the radius of the *orbis magnus* (consisting of 10000 parts,) was equal to 16 $\frac{1}{2}$  inches of an *English Foot*. Bastant

Lastly, in order to discover whether the Comet did truly move in the orbit so determin'd, I investigated its places in this orbit partly by arithmetical operations, and partly by scale and compass, to the times of some of the observations, as may be seen in the following table. finalmente

The Comet's							
	Dist. from Sun.	Longitude computed.	Latitud. compu- red.	Longitud. observ'd.	Latitude observ'd.	Dif. Lo.	Dif. Lat.
Dec. 12	2792	VS 6°. 32'	8°. 18 $\frac{1}{2}$	VS 6°. 31 $\frac{1}{2}$	8°. 26	+1	- 7 $\frac{1}{2}$
29	8403	☿ 13. 13 $\frac{1}{2}$	28. 00	☿ 13. 11 $\frac{1}{2}$	28. 10 $\frac{1}{2}$	+2	- 10 $\frac{1}{2}$
Febr. 5	16669	☿ 17. 00	15. 29 $\frac{1}{2}$	☿ 16. 59 $\frac{1}{2}$	15. 27 $\frac{3}{4}$	+0	+ 2 $\frac{1}{4}$
Mar. 5	21737	29. 19 $\frac{1}{2}$	12. 4	29. 20 $\frac{1}{2}$	12. 3 $\frac{1}{2}$	-1	+ $\frac{1}{2}$

But afterwards Dr. *Halley* did determine the orbit to a greater accuracy by an arithmetical calculus, than could be done by linear descriptions; and retaining the place of the nodes in ☿ and ♄ 1° 53', and the inclination of the plane of the orbit to the ecliptic 61° 20 $\frac{1}{2}$ ', as well as the time of the Comets being in perihelio, Dec. 8<sup>d</sup>. 00. 04': he found the distance of the perihelion from the ascending node measur'd in the Comet's orbit 9°. 20', and the *latus rectum* of the parabola 2430 parts, supposing the mean distance of the Sun from the Earth to be 100000 parts. And from these *data*, by an accurate arithmetical calculus, he computed the places of the Comet to the times of the observations as follows. degreer

True

The Comet's					Errors in	
True Time.		Diff. from the Sun.	Longitude Computed.	Latitude Computed.	Long.	Lat.
Dec.	d. 12.	4. 46.	"	"	' 3. 5	' 2. 0
	21.	6. 37.	"	"	' 1. 42	' 1. 7
	24.	6. 18.	"	"	' 1. 3	' 0. 25
	26.	5. 20.	"	"	' 1. 28	' 0. 44
	29.	8. 3.	"	"	' 1. 59	' 0. 12
Jan.	30.	8. 10.	"	"	' 1. 45	' 0. 33
	5.	6. 1. $\frac{1}{2}$	"	"	' 0. 56	' 0. 8
	9.	7. 0.	"	"	' 0. 32	' 0. 58
	10.	6. 6.	"	"	' 0. 10	' 0. 18
	13.	7. 9.	"	"	' 0. 33	' 0. 2
Febr.	25.	7. 59.	"	"	' 1. 20	' 1. 25
	30.	8. 22.	"	"	' 2. 10	' 0. 11
	2.	6. 35.	"	"	' 2. 42	' 0. 14
	5.	7. 4. $\frac{1}{2}$	"	"	' 0. 41	' 2. 10
	25.	8. 41.	"	"	' 2. 49	' 1. 10
Mar.	5.	11. 39.	"	"	' 0. 35	' 2. 14

This Comet also appeared in the *November* before, and at *Coburg* in *Saxony* was observed by Mr. *Gottfried Kirch* on the 4<sup>th</sup> of that Month, on the 6<sup>th</sup> and 11<sup>th</sup> O. S; from its positions to the nearest fixed Stars observed with sufficient accuracy, sometimes with a two foot, and sometimes with a ten foot telescope; from

from the difference of longitudes of *Coburg* and *London*,  $11^{\circ}$ , and from the places of the fixed Stars observed by Mr. *Pound*, Dr. *Halley* has determined the places of the Comet as follows.

*Nov. 3d.*  $17^h. 2'$ , apparent time at *London*, the Comet was in  $\Omega$   $29$  deg.  $51'$ , with  $1$  deg.  $17'. 45''$ . latitude north.

*November 5.*  $15^h. 58'$  the Comet was in  $\Upsilon$   $3^{\circ}. 23'$ , with  $1^{\circ}. 6'$ . north lat.

*November 10.*  $16^h. 31'$ , the Comet was equally distant from two Stars in  $\mathcal{Q}$  which are  $\sigma$  and  $\tau$  in *Bayer*; but it had not quite touched the right line that joins them, but was very little distant from it. In *Flamsteed's* catalogue this Star  $\sigma$  was then in  $\Upsilon$   $14^{\circ}. 15'$ , with  $1$  deg.  $41'$ . lat. north nearly, and  $\tau$  in  $\Upsilon$   $17^{\circ}. 3\frac{1}{2}'$  with  $0$ . deg.  $34'$ . lat. south. And the middle point between those Stars was  $\Upsilon$   $15^{\circ}. 39\frac{1}{4}'$ , with  $0^{\circ}. 33\frac{1}{2}'$  lat. north. Let the distance of the Comet from that right line be about  $10'$  or  $12'$ ; and the difference of the longitude of the Comet and that middle point will be  $7'$ ; and the difference of the latitude nearly,  $7\frac{1}{2}'$ . And thence it follows, that the Comet was in  $\Upsilon$   $15^{\circ}. 32'$ , with about  $26'$  lat. north.

The first observation from the position of the Comet with respect to certain small fixed Stars had all the exactness that could be desired. The second also was accurate enough. In the third observation, which was the least accurate, there might be an error of 6 or 7 minutes, but hardly greater. The longitude of the Comet, as found in the first and most accurate observation, being computed in the aforesaid parabolic orbit, comes out  $\mathcal{Q}$   $29^{\circ}. 30'. 22''$ , its latitude north  $1^{\circ}. 25'. 7''$ , and its distance from the Sun 115546.

Moreover, Dr. *Halley* observing that a remarkable Comet had appeared four times at equal intervals of 575 years, that is, in the Month of *September* after *Julius Caesar* was killed, *An. Chr. 531* in the consulate of *Lam-*

*Lampadius* and *Orestes*, *An. Chr.* 1106 in the Month of *February*, and at the end of the year 1680; and that with a long and remarkable tail (except when it was seen after *Cæsar's death*, at which time, by reason of the inconvenient situation of the Earth, the tail was not so conspicuous:) set himself to find out an elliptic orbit whose greater axis should be 1382957 parts, the mean distance of the Earth from the Sun containing 10000 such; in which orbit a Comet might revolve in 575 years. And placing the ascending node in  $\odot$   $2^{\circ}$ ,  $2'$ ; the inclination of the plane of the orbit to the plane of the ecliptic in an angle of  $61^{\circ}$ .  $6'$ .  $48''$ ; the perihelion of the Comet in this plane in  $\nearrow$   $22^{\circ}$ .  $44'$ .  $25''$ ; the equal time of the perihelion *December* 7<sup>d</sup>. 23<sup>h</sup>. 9'; the distance of the perihelion from the ascending node in the plane of the ecliptic  $9^{\circ}$ .  $17'$ .  $35''$ ; and its conjugate axis 18481, 2; he computed the motions of the Comet in this ecliptic orbit. The places of the Comet, as deduced from the observations and as arising from computation made in this orbit, may be seen in the following table.

True

True time			Long. obs.			Lat. Nor. obs.			Long. comp.			Lat. calc.		
d	h	'	o	'	"	o	'	"	o	'	"	o	'	"
Nov.	3	16	47	Ω	29	51	0	1	17	45	Ω	29	51	22
	5	15	37	⊙	3	23	0	1	6	0	⊙	3	24	32
	10	16	18		15	32	0	0	27	0		15	33	2
	16	17	00	-	-	-	-	-	-	-	♂	8	16	45
	18	21	34	-	-	-	-	-	-	-	-	18	52	15
	20	17	0	-	-	-	-	-	-	-	-	28	10	36
	23	17	5	-	-	-	-	-	-	-	♂	13	22	42
Dec.	12	4	46	⊙	6	32	30	8	28	0	⊙	6	31	20
	21	6	37	⊙	5	8	12	21	42	13	⊙	5	6	14
	24	6	18		18	49	23	25	23	5		18	47	30
	26	5	21		28	24	13	27	0	52		28	21	42
	29	8	3	⊙	13	10	41	28	9	58	⊙	13	11	14
	30	8	10		17	38	0	28	11	53		17	38	27
	5	6	1½	⊙	8	48	53	26	15	7	⊙	8	48	51
Jan.	9	7	1		18	44	4	24	11	56		18	43	51
	10	6	6		20	40	50	23	43	32		20	40	23
	13	7	9		25	59	48	22	17	28		26	0	8
	25	7	59	⊙	9	35	0	17	56	30	⊙	9	34	11
	30	8	22		13	19	51	16	42	18		13	18	28
	2	6	35		15	13	53	16	4	1		15	11	59
	5	7	4½		16	59	6	15	27	3		16	59	17
Feb.	25	8	41		26	18	35	12	46	46		26	16	59
	1	11	10		27	52	42	12	23	40		27	51	47
Mar.	5	11	39		29	18	0	12	3	16		29	20	11
	9	8	38	♂	0	43	4	11	45	52	♂	0	42	43

To face Page 352

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amp.	Errors in			
	Long.		Lat.	
"	'	"	'	"
1. 32 N	+ 0. 22		- 0. 13	
6. 9	+ 1. 32		+ 0. 9	
1. 7	+ 1. 2		- 1. 53	
1. 7 S	- - - -		- - - -	
5. 54	- - - -		- - - -	
3. 35	- - - -		- - - -	
9. 0	- - - -		- - - -	
9. 6 N	- 1. 10		+ 1. 6	
4. 42	- 1. 58		+ 2. 29	
3. 35	- 1. 53		+ 0. 30	
2. 1	- 2. 31		+ 1. 9	
0. 38	+ 0. 33		+ 0. 40	
1. 37	+ 0. 7		- 0. 16	
4. 57	- 0. 2		- 0. 10	
2. 17	- 0. 13		+ 0. 21	
3. 25	- 0. 27		- 0. 7	
6. 32	+ 0. 20		- 0. 56	
6. 6	- 0. 49		- 0. 24	
0. 5	- 1. 23		- 2. 13	
2. 7	- 1. 54		- 1. 54	
7. 0	+ 0. 11		- 0. 3	
5. 22	- 1. 36		- 1. 24	
2. 28	- 0. 55		- 1. 12	
2. 50	+ 2. 11		- 0. 26	
5. 35	- 0. 21		- 0. 17	

The observations of this Comet from the beginning to the end agree as perfectly with the motion of the Comet in the orbit just now described, as the motions of the Planets do with the theories from whence they are calculated, and by this agreement plainly evince that it was one and the same Comet that appeared all that time; and also that the orbit of that Comet is here rightly defined.

In the foregoing table we have omitted the observations of Nov. 16, 18, 20 and 23 as not sufficiently accurate. For at those times several persons had observed the Comet. Nov. 17. O. S. Pontheus and his Companions at 6<sup>h</sup> in the morning at Rome (that is 5<sup>h</sup>. 10' at London) by threads directed to the fixt Stars, observ'd the Comet in  $\approx 8^{\circ}. 30'$ . with latitude,  $0^{\circ}. 40'$ . south. Their observations may be seen in a treatise, which Pontheus publish'd concerning this Comet. Cellius who was present, and communicated his observations in a Letter to Cassini, saw the Comet at the same hour in  $\approx 8^{\circ}. 30'$ . with latitude  $0^{\circ}. 30'$  south. It was likewise seen by Gallerius at the same hour at Avignon (that is at 5<sup>h</sup>. 42'. morning at London) in  $\approx 8^{\circ}$ . without latitude. But by the theory the Comet was at that time in  $\approx 8^{\circ}. 16'. 45''$ . and its latitude was  $0^{\circ}. 53'. 7''$ . south.

Nov. 18. at 6<sup>h</sup>. 30' in the morning at Rome (that is, at 5<sup>h</sup>. 40'. at London) Pontheus observ'd the Comet in  $\approx 13^{\circ}. 30'$ . with latitude  $1^{\circ}. 20'$ . south; and Cellius in  $\approx 13^{\circ}. 30'$ . with latitude  $1^{\circ}. 00'$ . south. But at 5<sup>h</sup> 30'. in the morning at Avignon Gallerius saw it in  $\approx 13^{\circ}. 00'$ . with latitude  $1^{\circ}. 00'$  south. In the university of La Fleche in France, at 5<sup>h</sup> in the morning (that is at 5<sup>h</sup>. 9'. at London) it was seen by P. Anjo, in the middle between two small Stars, one of which is the middle of the three which lye in a right-line in the southern hand of Virgo, Bayers  $\gamma$ , and the other is the outmost of the wing, Bayers  $\theta$ . Whence the

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Comet was then in  $\approx 12^{\circ}. 46'$  with latitude  $50'$  south. And I was informed by Dr. *Halley* that on the same day, at *Boston* in *New-England*, in the latitude of  $42 \frac{1}{2}$  deg. at  $5^h$  in the morning, (that is, at  $9^h. 44'$  in the morning at *London*,) the Comet was seen near  $\approx 14^{\circ}$ , with latitude  $1^{\circ}. 30'$  south.

*Nov. 19.* at  $4^h \frac{1}{2}$  at *Cambridge*, the Comet (by the observation of a young man) was distant from *Spica*  $\mathfrak{M}$  about  $2^{\circ}$  towards the north-west. Now the spike was at that time in  $\approx 19^{\circ}. 23'. 47''$  with latitude  $2^{\circ}. 1'. 59''$  south. The same day at  $5^h$  in the morning at *Boston* in *New-England*, the Comet was distant from *Spica*  $\mathfrak{M}$   $1^{\circ}$  with the difference of  $40'$  in latitude. The same day in the island of *Jamaica*, it was about  $1^{\circ}$  distant from *Spica*  $\mathfrak{M}$ . The same day Mr. *Arthur Storer* at the river *Patuxent* near *Hunting Creek* in *Maryland* in the confines of *Virginia* in lat.  $38 \frac{1}{2}^{\circ}$  at  $5$  in the morning (that is at  $10^h$  at *London*) saw the Comet above *Spica*  $\mathfrak{M}$ , and very nearly join'd with it, the distance between them being about  $\frac{1}{4}$  of one deg. And from these observations compar'd I conclude, that at  $9^h 44'$  at *London*, the Comet was in  $\approx 18^{\circ}. 50'$  with about  $1^{\circ}. 25'$  latitude south. Now by the theory the Comet was at that time in  $\approx 18^{\circ}. 52'. 15''$  with  $1^{\circ}. 26'. 54''$  lat. south.

*Nov. 20.* *Montenari* professor of astronomy at *Padua*, at  $6^h$  in the morning at *Venice* (that is  $5^h. 10'$  at *London*) saw the Comet in  $\approx 23^{\circ}$  with latitude  $1^{\circ}. 30'$  south. The same day at *Boston*, it was distant from *Spica*  $\mathfrak{M}$  by about  $4^{\circ}$  of longitude east, and therefore was in  $\approx 23^{\circ}. 24'$  nearly.

*Nov. 21.* *Ponthaus* and his companions at  $7 \frac{1}{4}^h$  in the morning, observ'd the Comet in  $\approx 27^{\circ}. 50'$  with latitude  $1^{\circ}. 16'$  south. *Cellins* in  $\approx 28^{\circ}$ . *P. Anglo* at  $5^h$  in the morning, in  $\approx 27^{\circ}. 45'$ . *Montenari* in  $\approx 27^{\circ}. 51'$ . The same day in the island of *Jamaica*, it was seen near the beginning of  $\mathfrak{M}$  and of about the same latitude

titude with *Spica ♏*, that is,  $2^{\circ} 2'$ . The same day at  $5^h$  morning at *Ballafore* in the *East-Indies* (that is at  $11^h$ ,  $20'$  of the night preceding at *London*) the distance of the Comet from *Spica ♏* was taken  $7^{\circ} 35'$  to the east. It was in a right line between the spike and the ballance, and therefore was then in  $\approx 26^{\circ} 58'$  with about  $1^{\circ} 11'$  lat. south; and after  $5^h$ ,  $40'$ . (that is at  $5^h$  morning at *London*) it was in  $\approx 28^{\circ} 12'$  with  $1^{\circ} 16'$  lat. south. Now by the theory the Comet was then in  $\approx 28^{\circ} 10' 36''$  with  $1^{\circ} 53' 35''$  lat. south.

*Nov. 22.* The Comet was seen by *Montenari* in  $m 2^{\circ} 33'$ . But at *Boston* in *New-England*, it was found in about  $m 3^{\circ}$ , and with almost the same latitude as before, that is,  $1^{\circ} 30'$ . The same day at  $5^h$  morning at *Ballafore* the Comet was observ'd in  $m 1^{\circ} 50'$ ; and therefore at  $5^h$  morning at *London* the Comet was in  $m 3^{\circ} 5'$  nearly. The same day at  $6\frac{1}{2}^h$  in the morning at *London*, *Dr. Hook* observ'd it in about  $m 3^{\circ} 30'$ ; and that in the right line which passeth through *Spica ♏* and *Cor Leonis*; not indeed exactly, but deviating a little from that line towards the north. *Montenari* likewise observ'd, that this day and some days after, a right line drawn from the Comet through *Spica*, pass'd by the south side of *Cor Leonis*, at a very small distance therefrom. The right line through *Cor Leonis* and *Spica ♏* did cut the ecliptic in  $\text{♏ } 3^{\circ} 46'$  at an angle of  $2^{\circ} 51'$ . And if the Comet had been in this line and in  $m 3^{\circ}$  its latitude would have been  $2^{\circ} 26'$ . But since *Hook* and *Montenari* agree, that the Comet was at some small distance from this line towards the north, its latitude must have been something less. On the 20th, by the observation of *Montenari*, its latitude was almost the same with that of *Spica*, that is about  $1^{\circ} 30'$ . But by the agreement of *Hook*, *Montenari* and *Ange*, the latitude was continually increasing and therefore must now on the 22d, be sensibly greater than

*made*  
 1°. 30'. And taking a mean between the extreme limits but now stated 2°. 26' and 1°. 30', the latitude will be about 1°. 58'. *Hook* and *Montenari* agree that the tail of the Comet was directed towards *Spica* ♀, declining a little from that Star towards the south according to *Hook*, but towards the north, according to *Montenari*. And therefore that declination was scarcely sensible; and the tail lying nearly parallel to the equator, deviated a little from the opposition of the Sun, towards the north.

*made*  
 Nov. 23. O. S. At 5<sup>h</sup> morning at *Nuremberg* (that is at 4<sup>h</sup> $\frac{1}{2}$  at *London*) Mr. *Zimmerman* saw the Comet in ♍ 8°. 8' with 2°. 31' south lat. its place being collected by taking its distances from fixed Stars.

*calculated*  
 Nov. 24. Before Sun-rising the Comet was seen by *Montenari* in ♍ 12°. 52' on the north side of the right line through *Cor Leonis* and *Spica* ♀, and therefore its latitude was something less than 2°. 38'. And since the latitude, as we said, by the concurring observations of *Montenari*, *Ango*, and *Hook*, was continually increasing; therefore it was now on the 24th something greater than 1°. 58'; and, taking the mean quantity, may be reckon'd 2°. 18', without any considerable error. *Ponthaus* and *Galletius* will have it that the latitude was now decreasing; and *Cellius* and the observer in *New-England*, that it continued the same, viz. of about 1°, or 1 $\frac{1}{2}$ °. The observations of *Ponthaus* and *Cellius* are more rude, especially those which were made by taking the azimuths and altitudes; as are also the observations of *Galletius*. Those are better which were made by taking the position of the Comet to the fixt Stars by *Montenari*, *Hook*, *Ango*, and the observer in *New-England*, and sometimes by *Ponthaus* and *Cellius*. The same day, at 5<sup>h</sup> morning at *Ballasore* the Comet was observed in ♍ 11°. 45'; and therefore at 5<sup>h</sup> morning at *London* was in ♍ 13° nearly. And by

by the theory, the Comet was at that time in  $\mathfrak{M}$   $13^{\circ}. 22'. 42''$ .

*Nov.* 25. Before Sun-rise *Montenari* observ'd the Comet in  $\mathfrak{M}$   $17^{\circ} \frac{1}{4}$  nearly; and *Cellius* observ'd at the same time that the Comet was in a right line between the bright Star in the right thigh of Virgo and the southern Icale of Libra; and this right line cuts the Comet's way in  $\mathfrak{M}$   $18^{\circ}. 36'$ . And by the theory the Comet was in  $\mathfrak{M}$   $18^{\circ} \frac{1}{3}$  nearly. Willant:  
muscle  
divisional  
direction

From all this it is plain that these observations agree with the theory, so far as they agree with one another, and by this agreement it is made clear that it was one and the same Comet that appeared all the time from *Nov.* 4. to *Mar.* 9. The path of this Comet did twice cut the plane of the ecliptic, and therefore was not a right line. It did cut the ecliptic, not in opposite parts of the heavens, but in the end of Virgo and beginning of Capricorn, including an arc of about  $98^{\circ}$ . And therefore the way of the Comet did very much deviate from the path of a great circle. For in the month of *Nov.* it declined at least  $3^{\circ}$  from the ecliptic towards the south; and in the month of *Dec.* following it declined  $29^{\circ}$  from the ecliptic towards the north; the two parts of the orbit in which the Comet descended towards the Sun, and ascended again from the Sun, declining one from the other by an apparent angle of above  $30^{\circ}$ , as observ'd by *Montenari*. This Comet travel'd over 9 signs, to wit, from the last deg. of  $\Omega$  to the beginning of  $\Pi$ , beside the sign of  $\mathfrak{Q}$ , thro' which it pass'd before it began to be seen. And there is no other theory by which a Comet can go over so great a part of the heavens with a regular motion. The motion of this Comet was very unequable. For about the 20th of *Nov.* it describ'd about  $5^{\circ}$  a day. Then its motion being retarded, between *Nov.* 26. and *Dec.* 12. to wit, in the space of  $15 \frac{1}{2}$  days, it describ'd only  $40^{\circ}$ . But the motion thereof being afterwards accelerated, it claro  
circle  
camino  
orient  
sun  
circle  
desper

describ'd near  $5^{\circ}$  a day, till its motion began to be again retarded. And the theory which justly corresponds with a motion so unequable, and through so great a part of the heavens, which observes the same laws with the theory of the Planets, and which accurately agrees with accurate astronomical observations, cannot be otherwise than true.

And thinking it would not be improper, I have giv'n  
(Pl. 18.) a true representation of the orbit which this  
Comet describ'd, and of the tail which it emitted in  
several places, in the annexed figure; protracted in the  
plane of the trajectory. In this scheme *ABC* repre-  
sents the trajectory of the Comet, *D* the Sun, *DE* the  
axis of the trajectory, *DF* the line of the nodes, *GH*  
the intersection of the sphere of the *orbis magnus* with  
the plane of the trajectory, *I* the place of the Comet  
*Nov. 4. Ann. 1680*, *K* the place of the same *Nov. 11*,  
*L* the place of the same *Nov. 19*. *M* its place *Dec. 12*.  
*N* its place *Dec. 21*. *O* its place *Dec. 29*. *P* its place  
*Jan. 5. following*. *Q* its place *Jan. 25*. *R* its place  
*Feb. 5*. *S* its place *Feb. 25*. *T* its place *March 5*. and  
*V* its place *March 9*. In determining the length of  
the tail I made the following observations.

Nov. 4. and 6. the tail did not appear; Nov. 11. the tail just begun to shew itself, but did not appear above  $\frac{1}{2}$  deg. long through a 10 foot telescope; Nov. 17. the tail was seen by *Pomhaus* more than  $15^{\circ}$  long; Nov. 18. in *New-England* the tail appear'd  $30^{\circ}$  long, and directly opposite to the Sun, extending itself to the planet Mars, which was then in  $\Upsilon 9^{\circ} 54'$ ; Nov. 19. in *Mary-Land*, the tail was found  $15^{\circ}$  or  $20^{\circ}$  long, Dec. 10. (by the observation of Mr. *Flamsteed*) the tail pass'd through the middle of the distance intercepted between the tail of the Serpent of *Ophiuchus* and the Star  $\delta$  in the south wing of *Aquila*, and did terminate near the Stars *A, w, b*, in *Bayer's* tables. Therefore the end of the tail was in  $\Upsilon 19^{\frac{1}{2}^{\circ}}$ , with latitude about

34½°

*Plate XVIII. Vol. II. Pag. 358.*

G

B

A

B

Yc



34  $\frac{1}{4}^{\circ}$  north; Dec. 11. it ascended to the head of *Sagitta* (Bayer's  $\alpha$ ,  $\beta$ ) terminating in  $\vee$  26°. 43', with latitude 38°. 34' north; Dec. 12. it pass'd through the middle of *Sagitta*, nor did it reach much farther; terminating in  $\approx$  4<sup>th</sup>, with latitude 42  $\frac{1}{2}^{\circ}$  north nearly. But these things are to be understood of the length of the brighter part of the tail. For with a more faint light, observ'd too perhaps in a serener sky, at Rome, Dec. 12. 5<sup>h</sup>. 40'. by the observation of *Pontanus*, the tail arose to 10° above the rump of the swan, and the side thereof towards the west and towards the north was 45' distant from this star. But about that time the tail was 3° broad towards the upper end; and therefore the middle thereof was 2°. 15' distant from that star towards the south, and the upper end was  $\times$  in 22° with latitude 61° north. And thence the tail was about 70° long. Dec. 21. it extended almost to *Cassiopeia's* chair, equally distant from  $\beta$  and from *Schedir*, so as its distance from either of the two was equal to the distance of the one from the other, and therefore did terminate in  $\tau$  24° with latitude 47  $\frac{1}{2}^{\circ}$ . Dec. 29. it reach'd to a contact with *Scheat* on its left, and exactly fill'd up the space between the two stars in the northern foot of *Andromeda*, being 54° in length; and therefore terminated in  $\delta$  19° with 35° of latitude. Jan. 5. it touch'd the Star  $\pi$  in the breast of *Andromeda* on its right side, and the Star  $\mu$  of the girdle on its left; and according to our observations, was 40° long; but it was curved, and the convex side thereof lay to the south. And near the head of the Comet, it made an angle of 4° with the circle which pass'd through the Sun and the Comet's head. But towards the other end, it was inclin'd to that circle in an angle of about 10° or 11°. And the chord of the tail contain'd with that circle an angle of 8°. Jan. 13. the tail terminated between *Alamech* and *Algol*, with a light that was sensible enough; but with a faint light it ended over against the Star  $\kappa$  in

debet

A a 4

contra

Persens's



*Perseus's* side. The distance of the end of the tail from the circle passing through the Sun and the Comet, was  $3^{\circ} 50'$ . And the inclination of the chord of the tail to that circle was  $8\frac{1}{2}^{\circ}$ . *Jan.* 25. and 26. it shone with a faint light to the length of  $6^{\circ}$  or  $7^{\circ}$ . And for a night or two after when there was a very clear sky, it extended to the length of  $12^{\circ}$ , or something more, with a light that was very faint and very hardly to be seen. But the axe thereof was exactly directed to the bright Star in the eastern shoulder of *Auriga*, and therefore deviated from the opposition of the Sun towards the north, by an angle of  $10^{\circ}$ . Lastly, *Feb.* 10. with a telescope I observ'd the tail  $2^{\circ}$  long. For that fainter light which I spoke of, did not appear through the glasses. But *Ponthaus* writes that on *Feb.* 7. he saw the tail  $12^{\circ}$  long. *Feb.* 25. the Comet was without a tail, and so continued till it disappeared.

Now if one reflects upon the orbit describ'd, and duly considers the other appearances of this Comet, he will be easily satisfy'd that the bodies of Comets are solid, compact, fixt and durable, like the bodies of the Planets. For if they were nothing else but the vapours or exhalations of the Earth, of the Sun, and other Planets, this Comet in its passage by the neighbourhood of the Sun, would have been immediately dissipated. For the heat of the Sun is as the density of its rays, that is, reciprocally as the square of the distance of the places from the Sun. Therefore, since on *Dec.* 8. when the Comet was in its perihelion, the distance thereof from the centre of the Sun was to the distance of the Earth from the same as about 6 to 1000, the Sun's heat on the Comet was at that time to the heat of the Summer-Sun with us, as 1000000 to 36, or as 28000 to 1. But the heat of boiling water is about 3 times greater than the heat which dry earth acquires from the Summer-Sun, as I have try'd; and the heat of red-hot iron (if my conjecture is right) is about

about three or four times greater than the heat of boiling water. And therefore the heat, which dry earth on the Comet, while in its perihelion, might have conceived from the rays of the Sun, was about 2000 times greater than the heat of red-hot iron. But by so fierce a heat, vapours and exhalations, and every volatile matter must have been immediately consum'd and dissipated.

This Comet therefore must have conceiv'd an immense heat from the Sun, and retain that heat for an exceeding long time. For a globe of iron of an inch in diameter, expos'd red-hot to the open air, will scarcely lose all its heat in an hour's time; but a greater globe would retain its heat longer in the proportion of its diameter, because the surface (in proportion to which it is cool'd by the contact of the ambient air) is in that proportion less in respect of the quantity of the included hot matter. And therefore a globe of red-hot iron, equal to our Earth, that is, about 40000000 feet in diameter, would scarcely cool in an equal number of days, or in above 50000 years. But I suspect that the duration of heat may, on account of some latent causes, increase in a yet less proportion than that of the diameter; and I should be glad that the true proportion was investigated by experiments.

It is further to be observ'd, that the Comet in the month of *December*, just after it had been heated by the Sun, did emit a much longer tail, and much more splendid, than in the month of *November* before, when it had not yet arriv'd at its perihelion. And universally, the greatest and most fulgent tails always arise from Comets, immediately after their passing by the neighbourhood of the Sun. Therefore the heat received by the Comet conduces to the greatness of the tail. From whence I think I may infer, that the tail is nothing else but a very fine vapour, which the head or nucleus of the Comet emits by its heat.

But

But we have had <sup>hæmor</sup> three several opinions about the tails of Comets. For some will have it, that they are nothing else but the beams of the Sun's light transmitted through the Comet's <sup>man</sup> heads, which they suppose to be transparent; others that they proceed from the refraction which light suffers in passing from the Comet's head to the Earth: and lastly others, that they are a sort of clouds or vapour constantly rising from the Comet's heads, and tending towards the parts opposite to the Sun. The first is the opinion of such, as are yet unacquainted with optics. For the beams of the Sun are seen in a darkened room only in consequence of the light that is reflected from them by the little particles of dust and smook which are always flying about in the air. And for that reason in air impregnated with thick smook, those beams appear with great brightness, and move the sense vigorously; in a yet finer air they appear more faint, and are less easily discerned; but in the heavens, where there is no matter to reflect the light, they can never be seen at all. Light is not seen as it is in the beam, but as it is thence reflected to our eyes. For vision can be no otherwise produced than by rays falling upon the eyes. And therefore there must be some reflecting matter in those parts where the tails of the Comets are seen: for otherwise, since all the celestial spaces are equally illuminated by the Sun's light, no part of the heavens could appear with more splendor than another. The second opinion is liable to many difficulties. The tails of Comets are never seen variegated with those colours which commonly are inseparable from refraction. And the distinct transmission of the light of the fixt Stars and Planets to us, is a demonstration that the æther or celestial medium is not endow'd with any refractive power. For as to what is alledg'd that the fixt Stars have been sometimes seen by the Egyptians, environ'd with a Coma, or Capillitium, because that has but rarely happen'd, it

invidio is

is rather to be ascrib'd to a casual refraction of clouds; and to the radiation and scintillation of the fixt Stars, to the refractions both of the eyes and air. For upon laying a telescope to the eye those radiations and scintillations immediately disappear. By the tremulous agitation of the air and ascending vapours, it happens that the rays of light are alternately turn'd aside from the narrow space of the pupil of the eye; but no such thing can have place in the much wider aperture of the object-glass of a telescope. And hence it is, that a scintillation is occasion'd in the former case, which ceases in the latter. And this cessation in the latter case is a demonstration of the regular transmission of light through the heavens, without any sensible refraction. But to obviate an objection that may be made from the appearing of no tail, in such Comets as shine but with a faint light; as if the secondary rays were then too weak to affect the eyes, and for that reason it is that the tails of the fixt Stars do not appear; we are to consider, that by the means of telescopes the light of the fixt Stars may be augmented above an hundred fold, and yet no tails are seen; that the light of the Planets is yet more copious without any tail; but that Comets are seen sometimes with huge tails, when the light of their heads is but faint and dull. For so it happen'd in the Comet of the year 1680, when in the month of Dec. it was scarcely equal in light to the Stars of the second magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60° or 70°, and upwards; and afterwards on the 27 and 28 of January when the head appear'd but as a Star of the 7<sup>th</sup> magnitude, yet the tail (as was said above) with a light that was sensible enough, though faint, was stretcht out to 6 or 7 degrees in length, and with a languishing light that was more difficultly seen, ev'n to 12°. and upwards. But on the 9 and 10 of February, when to the (naked eye) the head appear'd no more, through

*mejor  
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*colocando*  
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claridad*  
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*hacia arriba*  
*bastante  
amplia  
extensión*  
*desnudo ojo*  
*a teles*

into  
posteriorment
mentis
 a telescope I view'd the tail of  $2^\circ$  in length. But farther, if the tail was owing to the refraction of the celestial matter, and did deviate from the opposition of the Sun, according to the Figure of the heavens; that deviation in the same places of the heavens should be always directed towards the same parts. But the Comet of the year 1680 December  $28^d$ ,  $8^{\frac{1}{2}}^h$ . P. M. at London was seen in  $\kappa$   $8^\circ$ .  $41'$ . with latitude north  $28^\circ$ .  $6'$ ; while the Sun was in  $\vee$   $18^\circ$ .  $26'$ . And the Comet of the year 1577 Dec.  $29^d$ . was in  $\kappa$   $8^\circ$ .  $41'$ , with latitude north  $28^\circ$ .  $40'$ , and the Sun as before in about  $\vee$   $18^\circ$ .  $26'$ . In both cases the situation of the Earth was the same, and the Comet appear'd in the same place of the heavens: Yet in the former case the tail of the Comet (as well by my observations as by the observations of others) deviated from the opposition of the Sun towards the north, by an angle of  $4\frac{1}{2}$  degrees, whereas in the latter, there was (according to the observations of Tycho) a deviation of 21 degrees towards the south. The refraction therefore of the heavens being thus disprov'd, it remains that the *phenomena* of the tails of Comets must be deriv'd from some reflecting matter.

situation
 And that the tails of Comets do arise from their heads, and tend towards the parts opposite to the Sun, is further confirm'd from the laws which the tails observe. As that lying in the planes of the Comet's orbits which pass through the Sun, they constantly deviate from the opposition of the Sun towards the parts which the Comet's heads in their progress along these orbits have left. That to a spectator, plac'd in those planes, they appear in the parts directly opposite to the Sun; but as the spectator recedes from those planes, their deviation begins to appear, and daily becomes greater. That the deviation, *ceteris paribus*, appears less, when the tail is more oblique to the orbit of the Comet, as well as when the head of the Comet approaches nearer

nearer to the Sun, especially if the angle of deviation is  
 estimated near the head of the Comet. That the tails  
 which have no deviation appear straight, but the tails *diver-*  
 which deviate are likewise bended into a certain cur- *emvtrada*  
 vature. That this curvature is greater when the deviation  
 is greater; and is more sensible, when the tail, *ceteris pa-*  
ribus, is longer: for in the shorter tails the curvature *maximas*  
 is hardly to be perceiv'd. That the angle of deviation *difficil*  
 is less near the Comet's head, but greater towards the  
 other end of the tail; and that because the convex side  
 of the tail regards the parts, from which the deviation  
 is made, and which lye in a right line drawn out in- *servia*  
 finitely from the Sun through the Comet's head. And  
 that the tails that are long and broad, and shipe with a *anchan*  
 stronger light, appear more resplendent and more exact-  
 ly defin'd on the convex than on the concave side.  
 Upon which accounts, it is plain that the *phenomena clavo*  
 of the tails of Comets, depend upon the motions of  
 their heads, and by no means upon the places of  
 the heavens in which their heads are seen, and that  
 therefore the tails of Comets do not proceed from the  
 refraction of the heavens, but from their own heads, *caberas*  
 which furnish the matter that forms the tail. For, as *proporciones*  
 in our air, the smoak of a heated body ascends, either *humor*  
 perpendicularly if the body is at rest, or obliquely, if  
 the body is mov'd obliquely; so in the heavens, where *doude*  
 all bodies gravitate towards the Sun, smoak and vapour *humor*  
 must (as we have already said) ascend from the Sun, *ya*  
 and either rise perpendicularly, if the smoaking body *sube*  
 is at rest; or obliquely, if the body, in all the progress *maximas*  
 of its motion, is always leaving those places from which *sube*  
 the upper or higher parts of the vapour had risen be- *minimo*  
 fore. And that obliquity will be least, where the va- *es desist*  
 pour ascends with most velocity, (to wit) near the smoak-  
 ing body, when that is near the Sun. But because the  
 obliquity varies, the column of vapour will be incur-  
 vated; and because the vapour in the preceding side is  
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 something more recent, that is, has ascended something more late from the body, it will therefore be something more dense on that side, and must on that account reflect more light, as well as be better defin'd. I add nothing concerning the sudden uncertain agitation of the tails of Comets, and their irregular figures, which Authors sometimes describe, because they may arise from the mutations of our air, and the motions of our clouds, in part obscuring those tails; or perhaps from parts of the *Via Lactea*, which might have been confounded with and mistaken for parts of the tails of the Comets as they (passed by.) ~~omit~~

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 But that the atmospheres of Comets may furnish a supply of vapour, great enough to fill so immense spaces, we may easily understand from the rarity of our own air. For the air near the surface of our Earth, possesses a space 850 times greater than water of the same weight. And therefore a cylinder of air 850 feet high, is of equal weight with a cylinder of water, of the same breadth and but one foot high. But a cylinder of air, reaching to the top of the atmosphere, is of equal weight with a cylinder of water, about 33 feet high: and therefore, if from the whole cylinder of air, the lower part of 850 feet high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 feet high. And from thence (and by the hypothesis, confirm'd by many experiments, that the compression of air is as the weight of the incumbent atmosphere, and that the force of gravity is reciprocally as the square of the distance from the center of the Earth) raising a calculus, by cor. prop. 22. book 2. I found, that at the height of one semidiameter of the Earth, reckon'd from the Earth's surface, the air is more rare than with us, in a far greater proportion than of the whole space within the orb of Saturn to a spherical space of one inch in diameter. And therefore if a sphere of our air, of but one inch in thick-  
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ness

ness, was equally rarify'd with the air at the height of one semi-diameter of the Earth from the Earth's surface, it would fill all the regions of the Planets to the orb of Saturn and far beyond it. Wherefore since the air at greater distances is immensely rarify'd, and the coma or atmosphere of Comets is ordinarily about ten times higher, reckoning from their centers, than the surface of the nucleus, and the tails rise yet higher, they must therefore be exceedingly rare. And tho' on account of the much thicker atmospheres of Comets and the great gravitation of their bodies towards the Sun, as well as of the particles of their air and vapours mutually one towards another, it may happen that the air in the celestial spaces and in the tails of Comets, is not so vastly rarify'd; yet from this computation it is plain, that a very small quantity of air and vapour is abundantly sufficient to produce all the appearances of the tails of Comets. For that they are indeed of a very notable rarity appears from the shining of the Stars through them. The atmosphere of the Earth, illuminated by the Sun's light, tho' but of a few miles in thickness, quite obscures and extinguishes the light not only of all the Stars, but ev'n of the Moon itself: whereas the smallest Stars are seen to shine through the immense thickness of the tails of Comets, likewise illuminated by the Sun, without the least diminution of their splendor. Nor is the brightness of the tails of most Comets ordinarily greater than that of our air an inch or two in thickness, reflecting in a darken'd room the light of the Sun beams let in by an hole of the window-shut. = ventan curada

And we may pretty nearly determine the time spent during the ascent of the vapour from the Comet's head to the extremity of the tail, by drawing a right line from the extremity of the tail to the Sun, and marking the place where that right line intersects the Comet's orbit. For the vapour that is now in the extremity of the tail,

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- gantab  
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tail, if it has ascended in a right line from the Sun, must have begun to rise from the head, at the time when the head was in the point of intersection. It is true, the vapour does not rise in a right line from the Sun, but retaining the motion which it had from the Comet before its ascent, and compounding that motion with its motion of ascent, arises obliquely. And therefore, the solution of the problem will be more exact, if we draw the line which intersects the orbit parallel to the length of the tail; or rather (because of the curvilinear motion of the Comet,) diverging a little from the line or length of the tail. And by means of this principle I found, that the vapour which Jan. 25. was in the extremity of the tail, had begun to rise from the head before Dec. 11. and therefore had spent in its whole ascent 45 days; but that the whole tail which appear'd on Dec. 10. had finish'd its ascent in the space of the two days then elaps'd from the time of the Comet's being in its perihelion. The vapour therefore, about the beginning and in the neighbourhood of the Sun, rose with the greatest velocity, and afterwards continu'd to ascend with a motion constantly retarded by its own gravity; and the higher it ascended, the more it added to the length of the tail. And while the tail continu'd to be seen, it was made up of almost all that vapour, which had risen since the time of the Comet's being in its perihelion; nor did that part of the vapour which had risen first, and which form'd the extremity of the tail, cease to appear, till its too great distance, as well from the Sun from which it receiv'd its light, as from our eyes, render'd it invisible. Whence also it is, that the tails of other Comets which are short, do not rise from their heads with a swift and continual motion, and (soon after) disappear; but are permanent and lasting columns of vapours and exhalations; which ascending from the heads with a slow motion of many days, and partaking of the motion of the

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the heads which they had from the beginning, continue to go along together with them through the heavens. From whence again we have another argument <sup>not convincing</sup> proving the celestial spaces to be free and without resistance, since in them not only the solid bodies of the Planets and Comets, but also the extremely rare vapours of Comets tails, maintain their rapid motions with great freedom, and for an exceeding long time. <sup>libertad</sup>

*Kepler* ascribes the ascent of the tails of the Comets to the atmospheres of their heads; and their direction towards the parts opposite to the Sun, to the action of the rays of light carrying along with them the matter of the Comet's tails. And without any great incongruity we may suppose, that in so free spaces, so fine a matter as that of the æther may yield to the action of the rays of the Sun's light, though <sup>luz</sup> those rays are not <sup>desarrolla</sup> able <sup>anque</sup> sensibly to move the gross <sup>no del</sup> substances in our parts, which are <sup>interpretado</sup> clogg'd with so palpable a resistance. Another author thinks, that there may be a sort of particles of matter endow'd with a <sup>propiedad</sup> principle of levity, as well as others are with a power of gravity; that the matter of the tails of Comets may be of the former sort, and that its ascent from the Sun, may be owing to its levity. But considering that the gravity of terrestrial bodies is as the matter of the bodies, and therefore can be neither more nor less in the same quantity of matter, I am inclin'd to believe that this ascent may rather proceed from the rarefaction of the matter of the Comet's tails. The ascent of smoak in a chimney is owing to the impulse of the air, with which it is entangled. The air rarefy'd by heat ascends, because its specific gravity is diminish'd, and in its ascent carries along with it the smoak, with which it is engag'd. And why may not the tail of a Comet rise from the Sun after the same manner? For the Sun's rays do not act upon the mediums which they pervade otherwise than

B b *penetrata*

by

by reflection and refraction. And those reflecting particles heated by this action, heat the matter of the æther which is involv'd with them. That matter is rarefied by the heat which it acquires; and because by this rarefaction the specific gravity with which it tended towards the Sun before is diminish'd, it will ascend therefrom, and carry along with it the reflecting particles, of which the tail of the Comet is compos'd. But the ascent of the vapours is further promoted by their circumgyration about the Sun, in consequence whereof they endeavour to recede from the Sun, while the Sun's atmosphere and the other matter of the heavens are either altogether quiescent, or are only mov'd with a slower circumgyration deriv'd from the rotation of the Sun. And these are the causes of the ascent of the tails of the Comets in the neighbourhood of the Sun, where their orbits are bent into a greater curvature, and the Comets themselves are plung'd into the denser, and therefore heavier parts of the Sun's atmosphere; upon which account they do then emit tails of an huge length. For the tails which then arise, retaining their own proper motion, and in the mean time gravitating towards the Sun, must be revolv'd in ellipses about the Sun in like manner as the heads are, and by that motion must always accompany the heads, and freely adhere to them. For the gravitation of the vapours towards the Sun can no more force the tails to abandon the heads, and descend to the Sun, than the gravitation of the heads can oblige them to fall from the tails. They must by their common gravity, either fall together towards the Sun, or be retarded together in their common ascent therefrom. And therefore, (whether from the causes already describ'd, or from any others) the tails and heads of Comets may easily acquire, and freely retain any position one to the other, without disturbance or impediment from that common gravitation.

The

The tails therefore that rise in the perihelion positions of the Comets will go along with their heads into far remote parts, and together with the heads will either return again from thence to us, after a long course of years; or rather, will be there rarefied, and by degrees quite vanish away. For afterwards in the descent of the heads towards the Sun, new short tails will be emitted from the heads with a slow motion; and those tails by degrees will be augmented immensely, especially in such Comets as in their perihelion distances descend as low as the Sun's atmosphere. For all vapour in those free spaces is in a perpetual state of rarefaction and dilatation. And from hence it is, that the tails of all Comets are broader at their upper extremity, than near their heads. And it is not unlikely, but that the vapour, thus perpetually rarefy'd and dilated, may be at last dissipated, and scatter'd through the whole heavens, and by little and little be attracted towards the Planets by its gravity, and mixed with their atmosphere. For as the seas are absolutely necessary to the constitution of our Earth, that from them, the Sun, by its heat, may exhale a sufficient quantity of vapours, which being gather'd together into clouds, may drop down in rain, for watering of the earth, and for the production and nourishment of vegetables; or being condens'd with cold on the tops of mountains, (as some philosophers with reason judge) may run down in springs and rivers; so for the conservation of the seas, and fluids of the Planets, Comets seem to be requir'd, that from their exhalations and vapours condens'd, the wastes of the Planetary fluids, spent upon vegetation and putrefaction, and converted into dry earth, may be continually supplied and made up. For all vegetables entirely derive their growths from fluids, and afterwards in great measure are turn'd into dry earth by putrefaction; and a sort of slime is always found to settle at the bottom of putrified fluids. And hence it is, that the bulk of the solid earth is

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continually increased, and the fluids, if they are not supplied from without, must be in a continual decrease, and quite fail at last. I suspect moreover, that 'tis chiefly from the Comets that spirit comes, which is indeed the smallest, but the most subtle and useful part of our air, and so much required to sustain the life of all things with us.

The atmospheres of Comets, in their descent towards the Sun, by running out into the tails are spent and diminish'd, and become narrower, at least on that side which regards the Sun; and in receding from the Sun, when they less run out into the tails, they are again enlarg'd, if *Hevelius* has justly mark'd their appearances. But they are seen least of all just after they have been most heated by the Sun, and on that account then emit the longest and most resplendent tails; and perhaps at the same time the nuclei are environ'd with a denser and blacker smook, in the lowermost parts of their atmosphere. For smook that is rais'd by a great and intense heat, is commonly the denser and blacker. Thus the head of that Comet which we have been describing, at equal distances both from the Sun and from the Earth, appear'd darker after it had pass'd by its perihelion, than it did before. For in the month of *December* it was commonly compar'd with the Stars of the third magnitude, but in *November*, with those of the first or second. And such as saw both appearances, have describ'd the first, as of another and greater Comet than the second. For *November* 19. this Comet appear'd to a young man at *Cambridge*, though with a pale and dull light, yet equal to *Spica Virginis*; and at that time it shone with greater brightness than it did afterwards. And *Montenari*, *Nov.* 20. st. vet. observed it larger than the Stars of the first magnitude, its tail being then 2 deg. long. And Mr. *Storer*, (by letters which have come into my hands) writes, that in the month of *Dec.* when the tail appear'd of the greatest bulk and

and splendor, the head was but small, and far less than that which was seen in the month of *November* before Sun-rising; and conjecturing at the cause of the appearance, he judg'd it to proceed from there being a greater quantity of matter in the head at first, which was afterwards gradually spent.

And, which further makes for the same purpose, I find, that the heads of other Comets, which did put forth tails of the greatest bulk and splendor, have appeared but obscure and small. For in *Brasile*, *March* 5. 1668. 7<sup>h</sup> P. M. *St. N. P. Valentinus Estancius* saw a Comet near the horizon, and towards the south west, with a head so small as scarcely to be discern'd, but with a tail above measure splendid, so that the reflection thereof from the sea was easily seen by those who stood upon the shoar. And it look'd like a fiery beam extended  $23^{\circ}$  in length from west to south, almost parallel to the horizon. But this excessive splendor continu'd only three days, decreasing apace afterwards; and while the splendor was decreasing, the bulk of the tail increas'd. Whence in *Portugal*, it is said to have taken up one quarter of the heavens, that is, 45 degrees, extending from west to east with a very notable splendor, though the whole tail was not seen in those parts, because the head was always hid under the horizon. And from the increase of the bulk, and decrease of the splendor of the tail, it appears that the head was then in its recess from the Sun, and had been very near to it in its perihelion, as the Comet of 1680 was. And we read, in the *Saxon* chronicle, of a like Comet appearing in the year 1106, *the Star whereof was small and obscure*, (as that of 1680.) *but the splendour of its tail was very bright, and like a huge fiery beam stretch'd out in a direction between the east and north*, as *Hevelius* has it also from *Simeon* the monk of *Durham*. This Comet appear'd in the beginning of *February*, about the evening, and towards the south west part of heaven:

From whence, and from the position of the tail, we infer, that the head was near the Sun. *Matthew Paris* says, *It was distant from the Sun by about a cubit, from three of the clock (rather six) till nine, putting forth a long tail.* Such also was that most resplendent Comet, described by *Aristotle*, lib. 1. *Meteor.* 6. *The head whereof could not be seen, because it had set before the Sun, or at least was hid under the Sun's rays; but next day it was seen as well as might be. For having left the Sun but a very little way, it set immediately after it. And the scatter'd light of the head, obscur'd by the too great splendor (of the tail) did not yet appear. But afterwards (as Aristotle says) when the splendor (of the tail) was now diminish'd (the head of) the Comet recover'd its native brightness; and the splendour (of its tail) reach'd now to a third part of the heavens (that is, to  $60^{\circ}$ .) This appearance was in the winter season, (an. 4. olymp. 101.) and rising to Orion's girdle, it there vanish'd away.* It is true that the Comet of 1618, which came out directly from under the Sun's rays, with a very large tail, seem'd to equal, if not to exceed, the Stars of the first magnitude. But then abundance of other Comets have appear'd yet greater than this, that put forth shorter tails; some of which are said to have appear'd as big as Jupiter; others as big as Venus, or even as the Moon.

We have said, that Comets are a sort of Planets, revolv'd in very eccentric orbits about the Sun. And as in the Planets which are without tails, those are commonly less, which are revolv'd in lesser orbits, and nearer to the Sun; so in Comets it is probable, that those which in their perihelion approach nearer to the Sun, are generally of less magnitude, that they may not agitate the Sun too much by their attractions. But as to the transverse diameters of their orbits, and the periodic times of their revolutions, I leave them to be determin'd by comparing Comets together which after long intervals of time return again in the same orbit,

bit. In the mean time, the following proposition may give some light in that enquiry.

# PROPOSITION XLII. PROBLEM XXII.

*To correct a Comet's trajectory found as above.*  
*corrected*

*Operation 1.* Assume that position of the plane *presumed* of the trajectory which was determin'd according to the preceding proposition. And select three places of the Comet, deduc'd from very accurate observations, and at great distances one from the other. Then suppose A to represent the time between the first observation and the second; and B the time between the second and the third. But it will be convenient that in one of those times the Comet be in its perigeon, or at least not far from it. From those apparent places find by trigonometric operations the three true places of the Comet in that assum'd plane of the trajectory; then through the places found, and about the center of the Sun as the focus, describe a conic section by arithmetical operations, according to prop. 21. book 1. Let the areas of this figure which are terminated by radij drawn from the Sun to the places found, be D and E, to wit, D the area between the first observation and the second, and E the area between the second and third. And let T represent the whole time, in which the whole area  $D + E$  should be described with the velocity of the Comet found by prop. 16. book 1. *minime*

*Oper. 2.* Retaining the inclination of the plane of the trajectory to the plane of the ecliptic, let the longitude of the nodes of the plane of the trajectory be increas'd by the addition of 20 or 30 minutes, which call P. Then from the foresaid three observ'd *conjecture*  
*clamo* *B b 4 antedichan* places



places of the Comet, let the three true places be found (as before) in this new plane, as also the orbit passing through those places, and the two areas of the same describ'd between the two observations, which call  $d$  and  $e$ ; and let  $t$  be the whole time in which the whole area  $d + e$  should be describ'd.

*Oper. 3.* Retaining the longitude of the nodes in the first operation, let the inclination of the plane of the trajectory to the plane of the ecliptic be increas'd by adding thereto  $20'$  or  $30'$ , which call  $Q$ . Then from the fore-said three observ'd apparent places of the Comet, let the three true places be found in this new plane, as well as the orbit passing through them, and the two areas of the same describ'd between the observation, which call  $\delta$  and  $\epsilon$ , and let  $\tau$  be the whole time in which the whole area  $\delta + \epsilon$  should be describ'd.

Then taking  $C$  to  $1$ , as  $A$  to  $B$ ; and  $G$  to  $1$ , as  $D$  to  $E$ ; and  $g$  to  $1$ , as  $d$  to  $e$ ; and  $\gamma$  to  $1$ , as  $\delta$  to  $\epsilon$ ; let  $S$  be the true time between the first observation and the third; and observing well the signs  $+$  and  $-$ , let such numbers  $m$  and  $n$  be found out as will make  $2G - 2C = mG - mg - nG - n\gamma$ ; and  $2T - 2S = mT - mt - nT - n\tau$ . And, if in the first operation  $I$  represents the inclination of the plane of the trajectory to the plane of the ecliptic, and  $K$  the longitude of either node, then  $I - nQ$  will be the true inclination of the plane of the trajectory to the plane of the ecliptic; and  $K - mP$  the true longitude of the node. And lastly, if in the first, second, and third operations, the quantities  $R$ ,  $r$ , and  $\rho$ , represent the parameters of the trajectory, and the quantities  $\frac{1}{L}$ ,  $\frac{1}{l}$ ,  $\frac{1}{\lambda}$ , the transverse diameters of the same; then  $R - mr - mR - n\rho - nR$  will be the true parameter, and  $\frac{1}{L - ml - mL - n\lambda - nL}$  will be the true transverse diameter of the trajectory which

agree with the observations, with approx-  
nexed table, calculated by Dr. Halley.

<i>om</i>	<i>The obser'd Places</i>	<i>The places computed in the orb.</i>
4. 20"	<i>Long.</i> ♈ 7 <sup>d</sup> . 01'. 00"	♈ 7°. 01'. 29"
2. 10	<i>Lat. S.</i> 21. 39. 00	21. 38. 50
2. 45	<i>Long.</i> ♈ 6. 15. 00	♈ 6. 16. 05
2. 40	<i>Lat. S.</i> 22. 24. 00	22. 24. 00
8. 00	<i>Long.</i> ♈ 3. 06. 00	♈ 3. 07. 33
5. 40	<i>Lat. S.</i> 25. 22. 00	25. 21. 40
. 15	<i>Long.</i> ♎ 2. 56. 00	♎ 2. 56. 00
. 30	<i>Lat. S.</i> 49. 25. 00	49. 25. 00
. 50	<i>Long.</i> ♎ 28. 40. 30	♎ 28. 43. 00
. 00	<i>Lat. S.</i> 45. 48. 00	45. 46. 00
. 00	<i>Long.</i> ♎ 13. 03. 00	♎ 13. 05. 00
. 00	<i>Lat. S.</i> 39. 54. 00	39. 53. 00
. 25	<i>Long.</i> ♎ 2. 16. 00	♎ 2. 18. 30
. 00	<i>Lat. S.</i> 33. 41. 00	33. 39. 40
. 00	<i>Long.</i> ♎ 24. 24. 00	♎ 24. 27. 00
. 30	<i>Lat. S.</i> 27. 45. 00	27. 46. 00
. 00	<i>Long.</i> ♎ 9. 00. 00	♎ 9. 02. 28
. 00	<i>Lat. S.</i> 12. 36. 00	12. 34. 13
. 00	<i>Long.</i> ♎ 7. 05. 40	♎ 7. 08. 45
. 00	<i>Lat. S.</i> 10. 23. 00	10. 22. 13

which

which the Comet describes. And from the transverse diameter given the periodic time of the Comet is also given.

*Q. E. I.* But the periodic times of the revolutions of Comets, and the transverse diameters of their orbits, cannot be accurately enough determin'd, but by comparing Comets together which appear at different times. If after equal intervals of time, several Comets are found to have describ'd the same orbit, we may thence conclude, that they are all but one and the same Comet revolv'd in the same orbit. And then from the times of their revolutions, the transverse diameters of their orbits will be given; and from those diameters the elliptic orbits themselves will be determin'd.

To this purpose, the trajectories of many Comets ought to be computed, supposing those trajectories to *necessitate* be parabolic. For such trajectories will always nearly agree with the *phenomena*, as appears not only from the parabolic trajectory of the Comet of the year 1680, which I compar'd above with the observations, but likewise from that of the notable Comet, which appear'd in the years 1664, and 1665, and was observ'd by *Hevelius*; who, from his own observations, calculated the longitudes and latitudes thereof, *quidem* though with *ambigua* little accuracy. But from the same observations *Dr. Halley* did again compute its places; and from those new places determin'd its trajectory; finding its ascending node in  $\Pi$   $21^{\circ}. 13'. 55''$ ; the inclination of the orbit to the plane of the ecliptic  $21^{\circ}. 18'. 40''$ ; the distance of its perihelion from the node, estimated in the Comet's orbit  $49^{\circ}. 27'. 30''$ . its perihelion in  $\Omega$   $8^{\circ}. 40'. 30''$ ; with heliocentric latitude south,  $16^{\circ}. 01'. 45''$ ; the Comet to have been in its perihelion *Nov.* 24<sup>d</sup>. 11<sup>h</sup>. 52'; *P. M.* equal time at *London*, or 13<sup>h</sup>. 8', at *Dantzick*, *O. S.* and that the *latus rectum* of the parabola was 410286 such parts as the Sun's mean distance from the Earth is suppos'd to contain 100000. And how nearly the places of the Comet computed in this orbit agree with the observations, will appear from the annexed table, calculated by *Dr. Halley*. *conversa*

In *February*, the beginning of the year 1665. the 1st Star of Aries, which I shall hereafter call  $\gamma$ , was in  $\Upsilon$   $28^{\circ}.30'.15''$ , with  $7^{\circ}.8'.58''$  north lat. The 2d Star of Aries was in  $\Upsilon$   $29^{\circ}.17'.18''$ , with  $8^{\circ}.28'.16''$  north lat. And another Star of the seventh magnitude which I call A, was in  $\Upsilon$   $28^{\circ}.24'.45''$ , with  $8^{\circ}.28'.33''$ , north lat. The Comet *Feb.*  $7^d.7^h.30'$ , at *Paris* (that is *Feb.*  $7^d.8^h.37'$ , at *Dantzick*) O. S. made a triangle with those Stars  $\gamma$  and A, which was right-angled in  $\gamma$ . And the distance of the Comet from the Star  $\gamma$  was equal to the distance of the Stars  $\gamma$  and A, that is  $1^{\circ}.19'.46''$ , of a great circle; and therefore in the parallel of the latitude of the Star  $\gamma$  it was  $1^{\circ}.20'.26''$ . Therefore if from the longitude of the Star  $\gamma$  there be subducted the longitude  $1^{\circ}.20'.26''$ , there will remain the longitude of the Comet  $\Upsilon$   $27^{\circ}.9'.49''$ . M. *Anzont*, from this observation of his, placed the Comet in  $\Upsilon$   $27^{\circ}.0'$ , nearly. And by the scheme in which Dr. *Hooke* delineated its motion, it was then in  $\Upsilon$   $26^{\circ}.59'.24''$ . I place it in  $\Upsilon$   $27^{\circ}.4'.46''$ , taking the middle between the two extremes.

From the same observation, M. *Anzont* made the latitude of the Comet at that time,  $7^{\circ}$  and  $4'$  or  $5'$  to the north. But he had done better to have made it  $7^{\circ}.3'.29''$ , the difference of the latitudes of the Comet and the Star  $\gamma$  being equal to the difference of the longitude of the Stars  $\gamma$  and A.

*Feb.*  $22^d.7^h.30'$ , at *London*, that is, *Feb.*  $22^d.8^h.46'$ , at *Dantzick*, the distance of the Comet from the Star A, according to Dr. *Hooke*'s observation, as was delineated by himself in a scheme, and also by the observations of M. *Anzont*, delineated in like manner by M. *Petit*, was a 5th part of the distance between the Star A and the first Star of Aries, or  $15'.57''$ ; and the distance of the Comet from a right line joining the Star A and the first of Aries, was a fourth part of the same 5th part, that is  $4'$ . And therefore the Comet was in  $\Upsilon$   $28^{\circ}.29'.46''$ , with  $8^{\circ}.12'.36''$ , north lat.

*Mar.*

*Mar. 1. 7<sup>h</sup>. 0'*, at *London*, that is, *Mar. 1. 8<sup>h</sup>. 16'*, at *Dantzick*, the Comet was observ'd near the 2d Star in Aries, the distance between them being to the distance between the first and second Stars in Aries, that is, to  $1^{\circ}.33'$ , as 4 to 45 according to Dr. *Hooke*, or as 2 to 23 according to M. *Gottignies*. And therefore the distance of the Comet from the 2d Star in Aries was  $8'.16''$ , according to Dr. *Hooke*, or  $8'.5''$ , according to M. *Gottignies*; or taking a mean between both  $8'.10''$ . But according to M. *Gottignies*, the Comet had gone beyond the 2d Star of Aries, about a 4th <sup>marchado =</sup> or a 5th <sup>abotro lado</sup> part of the space, that it commonly went over in a day, to wit, about  $1'.35''$ ; (in which he agrees very well with M. *Anzoni*) or according to Dr. *Hooke*, not quite so much, as perhaps only  $1'$ . Wherefore if to the longitude of the 1st Star in Aries, we add  $1'$ , and  $8'.10''$ , to its latitude, we shall have the longitude of the Comet  $\Upsilon 29^{\circ}.18'$ , with  $8^{\circ}.36'.26''$ , north lat.

*Mar. 7. 7<sup>h</sup>. 30'*, at *Paris* (that is, *Mar. 7. 8<sup>h</sup>. 37'*, at *Dantzick*) from the observations of M. *Anzoni*, the distance of the Comet from the 2d Star in Aries, was equal to the distance of that Star from the Star A, that is,  $52'.29''$ ; and the difference of the longitude of the Comet and the 2d Star in Aries was  $45'$ , or  $46'$ , or taking a mean quantity  $45'.30'$ . And therefore the Comet was in  $\odot 0^{\circ}.2'.48''$ . From the scheme of the observations of M. *Anzoni*, constructed by M. *Petit*, *Hevelius* collected the latitude of the Comet  $8^{\circ}.54'$ . But the engraver did not rightly trace the curvature <sup>gravado</sup> of the Comet's way toward the end of the motion: and <sup>distorsion</sup> *Hevelius* in the scheme of M. *Anzoni*'s observations which he constructed himself, corrected this irregular <sup>el mismo</sup> curvature, and so made the latitude of the Comet  $8^{\circ}.55'.30''$ . And by farther correcting this irregularity the latitude may become  $8^{\circ}.56'$ , or  $8^{\circ}.57'$ .

This Comet was also seen *Mar.* 9, and at that time its place must have been in  $\odot$   $0^{\circ}.18'$  with  $9^{\circ}.3'\frac{1}{2}$  north lat. nearly.

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This Comet appeared three months together, in which space of time it travell'd over almost <sup>year</sup> six signs, and in one of the days thereof describ'd almost 20 deg. Its course did very much deviate from a great circle, bending towards the north, and its motion towards the end from retrograde became direct. And notwithstanding its course was so uncommon, yet by the table it appears that the theory, from beginning to end, agrees with the observations no less accurately than the theories of the Planets usually do with the observations of them. But we are to subduct about  $2'$  when the Comet was swiftest, which we may effect by taking off  $12''$  from the angle between the ascending node and the perihelion, or by making that angle  $49^{\circ}.27'.18''$ . The annual parallax of both these Comets (this and the preceding) was very conspicuous, and by its quantity demonstrates the annual motion of the Earth in the *orbis magnus*.

This theory is likewise confirm'd by the motion of that Comet, which in the year 1683 appear'd retrograde, in an orbit whose plane contain'd almost a right angle with the plane of the ecliptic, and whose ascending node (by the computation of Dr. *Halley*) was in  $\text{III}$   $23^{\circ}.23'$ ; the inclination of its orbit to the ecliptic  $83^{\circ}.11'$ ; its perihelion in  $\text{II}$   $25^{\circ}.29'.30''$ ; its perihelion distance from the Sun 56020 of such parts as the radius of the *orbis magnus* contains 100000; and the time of its perihelion *July* 2<sup>d</sup>. 3<sup>h</sup>. 50'. And the places thereof computed by Dr. *Halley* in this orbit, are compar'd with the places of the same observ'd by Mr. *Flamsteed*, in the following table.

1683

1683 Eq. Time	Sun's Place.		Comet's Long.comp.		Lat.Nor. comput.		Comet's Long.obser.		Lat.Nor. observ'd		Diff. Long.		Diff. Lat.	
	d.	h.	°	'	"	°	'	"	°	'	"	'	"	'
Jul.	13.12.55		Ω	1.02.30		☿	13.05.42		29.28.13		☿	13.06.42		29.28.20
	15.11.15			2.53.12			11.37.48		29.34.00			11.39.43		29.34.50
	17.10.20			4.45.45			10.07.06		29.33.30			10.08.40		29.34.00
	23.13.40			10.38.21			5.10.27		28.51.42			5.11.30		28.50.28
	25.14.5			12.35.28			3.27.53		24.24.47			3.27.00		28.23.40
Aug.	31.9.42			18.09.22		☿	27.55.03		26.22.52		☿	27.54.24		26.22.25
	31.14.55			18.21.53		☿	27.41.07		26.16.57		☿	27.41.08		26.14.50
	2.14.56			20.17.16			25.29.32		25.16.19			25.28.46		25.17.28
	4.10.49			22.02.50			23.18.20		24.10.49			23.16.55		24.12.19
	6.10.9			23.56.45			20.42.23		22.47.05			20.40.32		22.49.05
	9.10.26			26.50.52			16.07.57		20.06.37			16.05.55		20.06.10
	15.14.1		☿	2.47.13			3.30.48		11.37.33			3.26.18		11.32.01
	16.15.10			3.48.02			0.43.07		9.34.16			0.41.55		9.34.13
	18.15.44			5.45.33		☿	24.52.53		5.11.15		☿	24.49.05		5.09.11
									South					South
	22.14.44			9.35.49			11.07.14		5.16.58			11.07.12		5.16.58
	23.15.52			10.36.48			7.02.18		8.17.09			7.01.17		8.16.41
	26.16.2			13.31.10		☿	24.45.31		16.38.00		☿	24.44.00		16.38.20

This theory is yet further confirm'd by the motion of that retrograde Comet, which appear'd in the year 1682. The ascending node of this (by Dr. Halley's computation) was in ☿ 21°. 16'. 30"; the inclination of its



its orbit to the plane of the ecliptic  $17^{\circ}.56'.00''$ ; its perihelion in  $\approx 2^{\circ}.52'.50''$ ; its perihelion distance from the Sun 58328 parts, of which the radius of the *orbis magnus* contains 100000; the equal time of the Comet's being in its perihelion *Sept.* 4<sup>d</sup>. 7<sup>h</sup>. 39'. And its places, collected from Mr. *Flamsteed*'s observations, are compar'd with its places computed from our theory, in the following table.

1682 App. Time	Sun's Place.	Comet's Lon. comp.	Lat. comp.	Lat. Nor. comp.	Com. Long. observ'd.	Lat. Nor. observ'd.	Diff. Longit.	Diff. Latitude
d h m	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "	' "	' "
Aug. 19. 16. 38	♊ 7.00.07	♊ 18.14.28	♊ 25.50.07	♊ 25.50.07	♊ 18.14.40	♊ 25.49.55	- 0.12	+ 0.12
20. 15. 38	♊ 7.55.52	♊ 24.46.23	♊ 26.14.42	♊ 26.14.42	♊ 24.46.22	♊ 26.12.52	+ 0.01	+ 1.50
21. 08. 21	♊ 8.36.14	♊ 29.37.15	♊ 26.20.03	♊ 26.20.03	♊ 29.38.02	♊ 26.17.37	- 0.47	+ 2.26
22. 08. 08	♊ 9.33.55	♊ 6.29.53	♊ 26.08.42	♊ 26.08.42	♊ 6.30.03	♊ 26.07.12	- 0.10	+ 1.30
29. 08. 20	♊ 16.22.40	♊ 12.37.54	♊ 18.37.47	♊ 18.37.47	♊ 12.37.49	♊ 18.34.05	+ 0.05	+ 3.42
30. 07. 45	♊ 17.19.41	♊ 15.36.01	♊ 17.26.43	♊ 17.26.43	♊ 15.35.18	♊ 17.27.17	+ 0.43	- 0.34
Sept. 1. 07. 33	♊ 19.16.09	♊ 20.30.53	♊ 15.13.00	♊ 15.13.00	♊ 20.27.04	♊ 15.09.49	+ 3.49	+ 3.11
4. 07. 22	♊ 22.11.28	♊ 25.42.00	♊ 12.23.48	♊ 12.23.48	♊ 25.40.58	♊ 12.22.00	+ 1.02	+ 1.48
5. 07. 32	♊ 23.10.29	♊ 27.00.46	♊ 11.33.08	♊ 11.33.08	♊ 26.59.24	♊ 11.33.51	+ 1.22	- 0.43
8. 07. 16	♊ 26.05.58	♊ 29.58.44	♊ 9.26.46	♊ 9.26.46	♊ 29.58.45	♊ 9.26.43	- 0.01	+ 0.03
9. 07. 26	♊ 27.05.09	♊ 0.44.10	♊ 8.49.10	♊ 8.49.10	♊ 0.44.04	♊ 8.48.25	+ 0.06	+ 0.45

This theory is also confirmed by the retrograde motion of the Comet that appeared in the year 1723. The ascending node of this Comet (according to the computation of Mr. Bradley, Savilian Professor of Astronomy at Oxford) was in  $\Upsilon$   $14^{\circ} 16'$ . The inclination of the orbit to the plane of the ecliptic  $49^{\circ} 59'$ . Its perihelion was in  $\delta$   $12^{\circ} 15' 20''$ . Its perihelion distance from the Sun 998651 parts, of which the radius of the *orbis magnus* contains 1000000, and the equal time of its perihelion September 16<sup>d</sup>. 16<sup>h</sup>. 10'. The places of this Comet computed in this orbit by Mr. Bradley, and compared with the places observed by himself, his uncle Mr. Pound, and Dr. Halley, may be seen in the following table.

1723. Eq. Time.	Comet's Long. obs.	Lat. Nor. obs.	Comet's Lon. com.	Lat. Nor. comp.	Diff. Lon.	Diff. Lat.
<i>a h m</i>	<i>° ' "</i>	<i>° ' "</i>	<i>° ' "</i>	<i>° ' "</i>	<i>" "</i>	<i>" "</i>
<i>Oct.</i> 9.8.5	7.22.15	5. 2. 0	7.21.26	5. 2.47	+49	-47
10.6.21	6.41.12	7.44.13	6.41.42	7.43.18	-50	+55
12.7.22	5.39.58	11.55. 0	5.40.19	11.54.55	-21	+ 5
14.8.57	4.59.49	14.43.50	5. 0.37	14.44. 1	-48	-11
15.6.35	4.47.41	15.40.51	4.47.45	15.40.55	- 4	- 4
21.6.22	4. 2.32	19.41.49	4. 2.21	19.42. 3	+11	-14
22.6.24	3.59. 2	20. 8.12	3.59.10	20. 8.17	- 8	- 5
24.8. 2	3.55.29	20.55.18	3.55.11	20.55. 9	+18	+ 9
29.8.56	3.56.17	22.20.27	3.56.42	22.20.10	-25	+17
30.6.20	3.58. 9	22.32.28	3.58.17	22.32.12	- 8	+16
<i>Nov.</i> 5.5.53	4.16.30	23.38.33	4.16.23	23.38. 7	+ 7	+26
8.7. 6	4.29.36	24. 4.30	4.29.54	24. 4.40	-18	-10
14.6.20	5. 2.16	24.48.46	5. 2.51	24.48.16	-35	+30
20.7.45	5.42.20	25.24.45	5.43.13	25.25.17	-53	-32
<i>Dec.</i> 7.6.45	8. 4.13	26.54.18	8. 3.55	26.53.42	+18	+36

From these examples it is abundantly evident, that the motions of Comets are no less accurately represented by our theory, than the motions of the Planets commonly are by the theories of them. And therefore, by means of this theory, we may enumerate the orbits of Comets, and so discover the periodic time of a Comet's revolution in any orbit; whence at last we shall

shall have the transverse diameters of their elliptic orbits and their aphelion distances.

That retrograde Comet which appear'd in the year 1607, describ'd an orbit whose ascending node (according to Dr. *Halley's* computation) was in  $820^{\circ}.21'$ ; and the inclination of the plane of the orbit to the plane of the ecliptic  $17^{\circ}.2'$ ; whose perihelion was in  $\approx 2^{\circ}.16'$ ; and its perihelion distance from the Sun 58680 of such parts as the radius of the *orbis magnus* contains 100000. And the Comet was in its perihelion October 16<sup>d</sup>. 3<sup>h</sup>. 50'. Which orbit agrees very nearly with the orbit of the Comet which was seen in 1682. If these were not two different Comets, but one and the same, that Comet will finish one revolution in the space of 75 years. And the greater axe of its orbit will be to the greater axe of the *orbis magnus*, as  $\sqrt{3}:75 \times 75$  to 1, or as 1778 to 100, nearly. And the aphelion distance of this Comet from the Sun will be to the mean distance of the Earth from the Sun as about 35 to 1. From which data it will be no hard matter to determine the elliptic orbit of this Comet. But these things are to be supposed, on condition, that after the space of 75 years the same Comet shall return again in the same orbit. The other Comets seem to ascend to greater heights, and to require a longer time to perform their revolutions.

But because of the great number of Comets, of the great distance of their aphelions from the Sun, and of the slowness of their motions in the aphelions, they will, by their mutual gravitations, disturb each other: so that their eccentricities and the times of their revolutions will be sometimes a little increased, and sometimes diminished. Therefore we are not to expect that the same Comet will return exactly in the same orbit, and in the same periodic times. It will be sufficient if we find the changes no greater, than may arise from the causes just spoken of.

And

And hence a reason may be assign'd why Comets are not comprehended within the limits of a zodiac as the Planets are; but, being confin'd to no bounds, <sup>limites</sup> are with various motions dispers'd all over the heavens; <sup>permeant</sup> namely, to this purpose, that in their aphelions, where their motions are exceeding slow, receding to greater distances one from another they may suffer less disturbance from their mutual gravitations. And hence it is, that the Comets which descend the lowest, and therefore move the slowest in their aphelions, <sup>mesita</sup> ought also to ascend the highest. = la mai grande

The Comet which appear'd in the year 1680. was in its perihelion less distant from the Sun than by a sixth part of the Sun's diameter: and because of its extreme velocity in that proximity to the Sun, and some density of the Sun's atmosphere, it must have suffer'd some resistance and retardation; and therefore, being attracted something nearer to the Sun in every revolution will at last fall down upon the body of <sup>hacia abajo</sup> the Sun. <sup>no</sup> Nay in its aphelion, where it moves the slowest, it may sometimes happen to be yet farther retarded by the attractions of other Comets, and in consequence of this retardation descend to the Sun. So fixed Stars that have been gradually wasted by the light and vapours emitted from them for a long time, may be recruited by Comets that fall upon them; and <sup>reclutad</sup> from this fresh supply of new fewel, those old Stars, <sup>recruited</sup> acquiring new splendor, may pass for new Stars. Of this kind are such fixed Stars as appear on a sudden and shine with a wonderful brightness at first, and afterwards vanish by little and little. Such was that Star which appeared in *Cassiopeias* chair; which *Cornelius Gemma* did not see upon the 8th of *November* 1572, though he was observing that part of the heavens upon that very night, and the skie was perfectly serene; but the next night (*Nov. 9.*) he saw it shining much brighter than any of the fixed Stars, <sup>maravillosa</sup> and

and scarcely inferiour to *Venus* in splendor. *Tycho Brahe* saw it upon the 11th of the same month when it shone with the greatest lustre; and from that time he observ'd it to decay by little and little; and in 16 months time it entirely disappear'd. In the month of *November*, when it first appeared, its light was equal to that of *Venus*. In the month of *December* its light was a little diminished, and was now become equal to that of *Jupiter*. In *January* 1573. it was less than *Jupiter* and greater than *Sirius*, and about the end of *February* and the beginning of *March* became equal to that Star. In the months of *April* and *May* it was equal to a Star of the 2d magnitude. In *June*, *July* and *August* to a Star of the 3d magnitude. In *September*, *October* and *November* to those of the 4th magnitude, in *December* and *January* 1574. to those of the 5th, in *February* to those of the 6th magnitude, and in *March* it entirely vanished. Its colour at the beginning was clear, bright and inclining to white, afterwards it turned a little yellow, and in *March* 1573. it became ruddy like *Mars* or *Aldebaran*; in *May* it turned to a kind of dusky whiteness like that we observe in *Saturn*, and that colour it retained ever after, but growing always more and more obscure. Such also was the Star in the right foot of *Serpentarius*, which *Kepler's* scholars first observed *September* 30. O. S. 1604, with a light exceeding that of *Jupiter*, tho' the night before it was not to be seen. And from that time it decreas'd by little and little, and in 15 or 16 months entirely disappeared. Such a new Star, appearing with an unusual splendor, is said to have moved *Hipparchus* to observe, and make a catalogue of, the fixed Stars. As to those fixed Stars that appear and disappear by turns, and encrease slowly and by degrees, and scarce ever exceed the Stars of the 3d magnitude, they seem to be of another kind, which revolve about their axes, and having a light and a dark side, shew those two different

different sides by turns. The vapours which arise from the Sun, the fixed Stars, and the tails of the Comets, may meet at last with, and fall into, the atmospheres of the Planets by their gravity; and there be condensed and turned into water and humid spirits, and from thence by a slow heat pass gradually into the form of salts, and sulphurs, and tinctures, and mud, and clay, and sand, and stones, and coral, and other terrestrial substances. *arise* *in edra* *sales*

## GENERAL SCHOLIUM:

The hypothesis of Vortices is press'd with many difficulties. That every Planet by a radius drawn to the Sun may describe areas proportional to the times of description, the periodic times of the several parts of the Vortices should observe the duplicate proportion of their distances from the Sun. But that the periodic times of the Planets may obtain the sesquuplicate proportion of their distances from the Sun, the periodic times of the parts of the Vortex ought to be in the sesquuplicate proportion of their distances. That the smaller Vortices may maintain their lesser revolutions about *Saturn*, *Jupiter*, and other Planets, and swim quietly and undisturb'd in the greater Vortex of the Sun, the periodic times of the parts of the Sun's Vortex should be equal. But the rotation of the Sun and Planets about their axes, which ought to correspond with the motions of their Vortices, recede far from all these proportions. The motions of the Comets are exceeding regular, are govern'd by the same laws with the motions of the Planets, and can by no means be accounted for by the hypothesis of Vortices. For Comets are carry'd with very eccentric motions through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a Vortex.

C c 2

Bodies

Bodies, projected in our air, suffer no resistance but from the air. Withdraw the air, as is done in Mr. Boyle's vacuum, and the resistance ceases. For in this void a bit of fine down and a piece of solid gold descend with equal velocity. And the parity of reason must take place in the celestial spaces above the Earth's atmosphere; in which spaces, where there is no air to resist their motions, all bodies will move with the greatest freedom; and the Planets and Comets will constantly pursue their revolutions in orbits given in kind and position, according to the laws above explain'd. But though these bodies may indeed persevere in their orbits by the mere laws of gravity, yet they could by no means have at first deriv'd the regular position of the orbits themselves from those laws.

The six primary Planets are revolv'd about the Sun, in circles concentric with the Sun, and with motions directed towards the same parts and almost in the same plane. Ten Moons are revolv'd about the Earth, Jupiter and Saturn, in circles concentric with them, with the same direction of motion, and nearly in the planes of the orbits of those Planets. But it is not to be conceived that mere mechanical causes could give birth to so many regular motions: since the Comets range over all parts of the heavens, in very eccentric orbits. For by that kind of motion they pass easily through the orbs of the Planets, and with great rapidity; and in their aphelions, where they move the slowest, and are detain'd the longest, they recede to the greatest distances from each other, and thence suffer the least disturbance from their mutual attractions. This most beautiful System of the Sun, Planets and Comets, could only proceed from the counsel and dominion of an intelligent and powerful being. And if the fixed Stars are the centers of other like systems, these being form'd by the like wise counsel, must be all subject to the dominion

of One; especially, since the light of the fixed Stars is of the same nature with the light of the Sun, and from every system light passes into all the other systems. And lest the systems of the fixed Stars should, by their gravity, fall on each other mutually, he hath placed those Systems at immense distances one from another.

This Being governs all things, not as the soul of the world, but as Lord over all: And on account of his dominion he is wont to be called *Lord God παντοκράτωρ*, or *Universal Ruler*. For *God* is a relative word, and has a respect to servants; and *Deity* is the dominion of God, not over his own body, as those imagine who fancy God to be the soul of the world, but over servants. The supreme God is a Being eternal, infinite, absolutely perfect; but a being, however perfect, without dominion, cannot be said to be Lord God; for we say, my God, your God, the God of *Israel*, the God of Gods, and Lord of Lords; but we do not say, my Eternal, your Eternal, the Eternal of *Israel*, the Eternal of Gods; we do not say, my Infinite, or my Perfect: These are titles which have no respect to servants. The word *God* usually signifies *Lord*; but every lord is not a God. It is the dominion of a spiritual being which constitutes a God; a true, supreme or imaginary dominion makes a true, supreme or imaginary God. And from his true dominion it follows, that the true God is a Living, Intelligent and Powerful Being; and from his other perfections, that he is Supreme or most Perfect. He is Eternal and Infinite, Omnipotent and Omniscient; that is, his duration reaches from Eternity to Eternity; his

<sup>a</sup> Dr. Pocock derives the Latin word *Deus* from the Arabic *du*, (in the oblique case *di*), which signifies *Lord*. And in this sense Princes are called *Gods*, *Psal.* lxxxii. ver. 6. and *John* x. ver. 35. And *Moses* is called a *God* to his brother *Aaron*, and a *God* to *Pharaoh* (*Exod.* iv. ver. 16. and vii. ver. 8. And in the same sense the souls of dead Princes were formerly, by the Heathens, called *gods*, but falsely, because of their want of dominion.



presence from Infinity to Infinity; he governs all things, and knows all things that are or can be done. He is not Eternity or Infinity, but Eternal and Infinite; he is not Duration or Space, but he endures and is present. He endures for ever, and is every where present; and by existing always and every where, he constitutes Duration and Space. Since every particle of Space is *always*, and every indivisible moment of Duration is *every where*, certainly the Maker and Lord of all things cannot be *never* and *no where*. Every soul that has perception is, though in different times and in different organs of sense and motion, still the same indivisible person. There are given successive parts in duration, co-existent parts in space, but neither the one nor the other in the person of a man, or his thinking principle; and much less can they be found in the thinking substance of God. Every man, so far as he is a thing that has perception, is one and the same man during his whole life, in all and each of his organs of sense. God is the same God, always and every where. He is omnipresent, not *virtually* only, but also *substantially*; for virtue cannot subsist without substance. In him <sup>b</sup> are all things contained and moved; yet neither affects the other: God suffers nothing from the motion of bodies; bodies find no resistance from the omnipresence of God. 'Tis allowed by all that the supreme God exists necessarily; and by the same necessity he exists

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<sup>b</sup> This was the opinion of the Ancients. So *Pythagoras* in *Cicero de Nat. Deor.* lib. i. *Thales*, *Anaxagoras*, *Virgil*, *Georg.* lib. iv. ver. 220. and *Æneid.* lib. vi. ver. 721. *Philo Allegor.* at the beginning of lib. i. *Aratus* in his *Phænomena*. at the beginning. So also the sacred Writers, as *St. Paul*, *Acts* xvii. ver. 27, 28. *St. John's Gospel.* chap. xiv. ver. 2. *Moses* in *Deut.* iv. ver. 39. and x. ver. 14. *David*, *Psal.* cxxxix. ver. 7, 8, 9. *Solomon*, *1 Kings* viii. ver. 27. *Job* xxii. ver. 12, 13, 14. *Jeremiah* xxiii. ver. 23, 24. The Idolaters supposed the Sun, Moon and Stars, the Souls of Men, and other parts of the world, to be parts of the supreme God, and therefore to be worshipped: but erroneously.

*always* and *every where*. Whence also he is all similar, all eye, all ear, all brain, all arm, all power to perceive, to understand, and to act; but in a manner not at all human, in a manner not at all corporeal, in a manner utterly unknown to us. As a blind man has no idea of colours, so have we no idea of the manner by which the all-wise God perceives and understands all things. He is utterly void of all body and bodily figure, and can therefore neither be seen, nor heard, nor touched; nor ought he to be worshipped under the representation of any corporeal thing. We have ideas of his attributes, but what the real substance of any thing is, we know not. In bodies we see only their figures and colours, we hear only the sounds, we touch only their outward surfaces, we smell only the smells, and taste the savours; but their inward substances are not to be known, either by our senses, or by any reflex act of our minds; much less then have we any idea of the substance of God. We know him only by his most wise and excellent contrivances of things, and final causes; we admire him for his perfections; but we reverence and adore him on account of his dominion. For we adore him as his servants; and a God without dominion, providence, and final causes, is nothing else but Fate and Nature. Blind metaphysical necessity, which is certainly the same always and every where, could produce no variety of things. All that diversity of natural things which we find, suited to different times and places, could arise from nothing but the ideas and will of a Being necessarily existing. But by way of allegory, God is said to see, to speak, to laugh, to love, to hate, to desire, to give, to receive, to rejoice, to be angry, to fight, to frame, to work, to build. For all our notions of God are taken from the ways of mankind, by a certain similitude which, though not perfect, has some likeness however. And thus much concerning God; to dis-

course of whom from the appearances of things, does certainly belong to Natural Philosophy.

Hitherto we have explain'd the phænomena of the heavens and of our sea, by the power of Gravity, but have not yet assign'd the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centers of the Sun and Planets, without suffering the least diminution of its force; that operates, not according to the quantity of the surfaces of the particles upon which it acts, (as mechanical causes use to do,) but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides, to immense distances, decreasing always in the duplicate proportion of the distances. Gravitation towards the Sun, is made up out of the gravitations towards the several particles of which the body of the Sun is compos'd; and in receding from the Sun, decreases accurately in the duplicate proportion of the distances, as far as the orb of Saturn, as evidently appears from the quiescence of the aphelions of the Planets; nay, and even to the remotest aphelions of the Comets, if those aphelions are also quiescent. But hitherto I have not been able to discover the cause of those properties of gravity from phænomena, and I frame no hypotheses. For whatever is not deduc'd from the phænomena, is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferr'd from the phænomena, and afterwards render'd general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered. And to us it is enough, that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

And

And now we might add something concerning a certain most subtle Spirit, which pervades and lies hid in all gross bodies; by the force and action of which Spirit, the particles of bodies mutually attract one another at near distances, and cohere, if contiguous; and electric bodies operate to greater distances, as well repelling as attracting the neighbouring corpuscles; and light is emitted, reflected, refracted; inflected, and heats bodies; and all sensation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this Spirit, mutually propagated along the solid filaments of the nerves, from the outward organs of sense to the brain, and from the brain into the muscles. But these are things that cannot be explain'd in few words, nor are we furnish'd with that sufficiency of experiments which is required to an accurate determination and demonstration of the laws by which this electric and elastic spirit operates.

**I N D E X.**

# I N D E X.

*The first number denotes the Volume, the second the Page; unless where a section is referred to.*

## A.

**Æ** Quinoxes, their precession

the cause of that motion shewn II, 252

the quantity of that motion computed from the causes II, 320

Air,

its density at any height, collected by Prop. 22. Book. II.

and its density at the height of one semidiameter of the Earth shewn II, 366

its elastic force, what cause it may be attributed to II, 77

its gravity compared with that of water II, 366

its resistance, collected by experiments of pendulums II, 99; the same more accurately by experiments of falling bodies, and a Theory II, 160

Angles of contact not all of the same kind, but some infinitely less than others I, 53

Apfides, their motion shewn I, Sect. 9. p. 177

Areas which revolving bodies, by radii drawn to the centre of force, describe, compared with the times of description I, 57, 60, 62, 220, 231

Attraction of all bodies demonstrated II, 225

the certainty of this demonstration shewn II, 203; the cause or manner thereof nowhere defined by the Author II, 392

*As*, the mathematical signification of this word defined I, 50

## C.

Centre,

the common center of gravity of many bodies does not alter its state of motion or rest by the actions of the bodies among themselves I, 27

the

# I N D E X.

- the common centre of gravity of the Earth, Sun, and all the Planets is at rest II, 232 ; confirmed by Cor. 2. Prop. 14. Book 3.
- the common centre of gravity of the Earth and Moon goes round the *orbis magnus* II, 235 ; its distance from the Earth and from the Moon II, 311
- Centre of the forces by which revolving bodies are retained in their orbits, how indicated by the description of areas I, 63 ; how found by the given velocities of the revolving bodies I, 67
- Circle, by what law of centripetal force tending to any given point, its circumference may be described I, 64, 70, 73
- Conic sections, by what law of centripetal force tending to any given point they may be described by revolving bodies I, 93
- the geometrical description of them when the foci are given I, Sect. 4.
- when the foci are not given I, Sect. 5.
- when the centres or asymptotes are given I, 132
- Comets
- a sort of Planets, not meteors II, 331, 360
- higher than the Moon, and in the planetary regions II, 323
- their distance how collected very nearly by observations II, 324
- more of them observed in the hemisphere towards the Sun, than in the opposite hemisphere ; and how this comes to pass II, 330
- shine by the Sun's light reflected from them II, 330
- surrounded with vast atmospheres II, 327, 331
- those which come nearest to the Sun probably the least, II, 374
- why they are not comprehended within a zodiack, like the Planets, but move differently into all parts of the Heavens II, 385
- may sometimes fall into the Sun, and afford a new supply of fire II, 385
- the use of them hinted II, 371
- move in conic sections, having their foci in the Sun's centre, and by radij drawn to the Sun describe areas proportional to the times. Move in ellipses if they come round again in their orbits, but these ellipses will be near to parabolas II, 332
- Comet's parabolic trajectory found from three observations given II, 340 ; corrected when found II, 375
- Comet's place in a parabola found to a given time II, 333 ; I, 143
- Comet's velocity compared with the velocity of the Planets II, 332
- Comets Tails
- directed from the Sun II, 364
- brightest

# I N D E X.

· brightest and largest immediately after their passage thro' the neighbourhood of the Sun II, 361  
 · their wonderful rarity II, 367  
 · their origine and nature II, 327, 509  
 · in what space of time they ascend from the heads II, 367  
 Comet of the years 1664 and 1665  
   the observations of its motion compared with the theory II, 377  
 Comet of the years 1680 and 1681  
   observations of its motion II, 344  
   its motion computed in a parabolic orbit II, 350; in an elliptic orbit II, 352  
   its trajectory, and its tail in the several parts of its orbit, delineated II, 358  
 Comet of the year 1682  
   its motion compared with the theory II, 382  
   seems to have appeared in the year 1607, and likely to return again after a period of 75 years II, 384  
 Comet of the year 1683  
   its motion compared with the theory II, 381  
 Comet of the year 1723  
   its motion compared with the theory II, 383  
 Curves distinguished into geometrically rational and geometrically irrational I, 148  
 Curvature of figures how estimated II, 32; II, 267  
 Cycloid or epicycloid,  
   its rectification I, 199, 200  
   its *evoluta* I, 204

Cylinder, the attraction of a  
 Cylinder composed of attracting particles, whose forces are reciprocally as the square of the distances I, 302

## D.

Descent of heavy bodies in vacuo, how much it is II, 240  
 Descent or ascent rectilinear, the spaces described, the times of description, and the velocities acquired in such ascent or descent, compared, on the supposition of any kind of centripetal force I, Sect. 7.  
 Descent and ascent of bodies in resisting mediums II, 4, 22, 24, 47, 50, 145

## E.

Earth,  
   its dimension by *Norwood*, by *Picart*, and by *Cassini* II, 240  
   its figure discovered, with the proportion of its diameters, and the measure of the degrees upon the meridian II, 239, 245  
   the excess of its height at the equator above its height at the poles II, 243, 251  
   its greatest and least semidiameter II, 243; its mean semidiameter *ibid.*  
   the globe of Earth more dense than if it was entirely water II, 230  
   the nutation of its axis II, 252  
   the

# I N D E X.

the annual motion thereof in the *orbis magnus* demonstrated II, 380  
 the eccentricity thereof how much II, 299  
 the motion of its aphelion how much II, 237  
 Ellipsis,  
   by what law of centripetal force tending to the centre of the figure it is described by a revolving body I, 75  
   by what law of centripetal force tending to the focus of the figure it is described by a revolving body I, 79.

## F.

Fluid, the definition thereof I, 64  
 Fluids, the laws of their density and compression shewn II, Sect. 5.  
   their motion in running out at an hole in a vessel determined II, 124  
 Forces  
   their composition and resolution I, 22  
   attractive forces of sphaerical bodies, composed of particles attracting according to any law, determined I, Sect. 12.  
   attractive forces of bodies not sphaerical composed of particles attracting according to any law determined I, Sect. 13.  
 Force  
   centrifugal force of bodies on the Earth's æquator, how great II, 240  
   centripetal force defined I, 4.

the absolute quantity of centripetal force defined I, 6  
 the accelerative quantity of the same defined *ib.*  
 the motive quantity of the same defined I, 7  
 the proportion thereof to any known force how collected I, 66  
 the invention of the centripetal forces, when a body is revolved in a non-resisting space 'about an immoveable centre, in any orbit I, Sect. 2. and 3.  
 the centripetal forces tending to any point by which any figure may be described by a revolving body, being given; the centripetal forces tending to any other point, by which the same figure may be described in the same periodic time, are also given I, 72  
 the centripetal forces by which any figure is described by a revolving body, being given; there are given the forces by which a new figure may be described, if the ordinates are augmented or diminished in any given ratio, or the angle of their inclination be any how changed, the periodic time remaining the same I, 77  
 centripetal forces decreasing in the duplicate proportion of the distances, what figures may be described by them I, 85; 222  
 a centripetal force that is reciprocally as the cube of the ordinate



# I N D E X.

ordinate tending to a vastly remote centre of force will cause a body to move in any given conic section I, 74

- a centripetal force that is as the cube of the ordinate tending to a vastly remote centre of force will cause a body to move in an hyperbola I, 310

## G.

God, his Nature II, 389

Gravity,

of a different nature from magnetical force II, 225  
mutual between the Earth and its parts I, 37  
the cause of it not assigned II, 392

tends towards all the Planets II, 219; from the surfaces of the Planets upwards decreases in the duplicate ratio of the distances from the centre II, 229; from the same downwards decreases nearly in the simple ratio of the same II, 229

tends towards all bodies, and is proportional to the quantity of matter in each II, 225

is the force by which the Moon is retained in its orbit II, 215

the same proved by an accurate calculus II, 311, 312.

is the force by which the primary Planets and the Satellites of Jupiter and Saturn are retained in their orbits II, 219

## H.

Heat, an iron rod increases in length by heat II, 250  
of the Sun, how great at different distances from the Sun II, 360  
how great in Mercury II, 229  
how great in the Comet of 1680, when in its perihelion II, 360

Heavens

are void of any sensible resistance II, 231, 331, 369, and therefore of almost any corporeal fluid whatever 161, 162

suffer light to pass through them without any refraction, II, 362

Hydrostatics, the principles thereof delivered II, Sect. 5.

Hyperbola

by what law of centrifugal force tending from the centre of the figure it is described by a revolving body I, 77

by what law of centrifugal force tending from the focus of the figure it is described by a revolving body I, 82

by what law of centripetal force tending to the focus of the figure it is described by a revolving body I, 81

Hypotheses of what kind soever rejected from this philosophy II, 393

I, 14:

# I N D E X.

## I.

### Jupiter,

- its periodic time II, 210
- its distance from the Sun II, 211
- its apparent diameter II, 207
- its true diameter II, 228
- its attractive force how great II, 227
- the weights of bodies on its surface II, 228
- its density *ib.*
- its quantity of matter *ib.*
- its perturbation by Saturn how much II, 234
- the proportion of its diameters exhibited by computation II, 244; and compared with observations *ib.* and 245
- its rotation about its axis in what time performed II, 244
- the cause of its belts hinted at II, 331

## L.

### Light,

- its propagation not instantaneous I, 316; not caused by the agitation of any ethereal medium II, 181
- its velocity different in different mediums I, 313
- a certain reflection it sometimes suffers explained I, 314
- its refraction explained I, 311
- refraction is not made in the single point of incidence I, 317

an incurvation of light about the extremities of bodies observed by experiments I, 316

## M.

Magnetic force I, 37; II, 79, 225, 313

### Mars,

- its periodic time II, 210
- its distance from the Sun II, 211
- the motion of its aphelion II, 237

### Matter,

quantity of matter defined I, 1

its *vis insita* defined I, 2

its impressed force defined I, 3

its extension, hardness, impenetrability, mobility, *vis inertiae*, gravity, how discovered II, 203

subtle matter of *Des-Cartes* enquired into II, 107

Mechanical Powers explained and demonstrated I, 38

### Mercury,

- its periodic time II, 210
- its distance from the Sun II, 211
- the motion of its aphelion II, 237

### Method

of first and last ratios I, Sect. 1.

of transforming figures into others of the same analytical order I, 121

of fluxions II, 17

differential II, 333

of finding the quadratures of all curves very nearly true II, 335

of

# I N D E X.

of converging series applied to the solution of difficult problems I, 187, 189, 302, II, 32, 286

## Moon

the figure of its body collected by calculation II, 314

its librations explained II, 238

its mean apparent diameter II, 311

its true diameter, *ibid.*

weight of bodies on its surface *ibid.*

its density *ibid.*

its quantity of matter *ibid.*

its mean distance from the Earth, how many greatest semidiameters of the Earth contained therein *ibid.* how many mean semidiameters II, 313

its force to move the Sea how great II, 306; not perceptible in experiments of pendulums or any statical or hydrostatical observations II, 310

its periodic time II, 312

the time of its synodical revolution II, 266

its motions and the inequalities of the same derived from their causes II, 252, 298

revolves more slowly, in a dilated orbit, when the Earth is in its perihelion; and more swiftly in the aphelion the same, its orbit being contracted II, 252, 298, 299

revolves more slowly in a dilated orbit when the apogæon is in the syzygies with the Sun; and more swiftly in a contracted orbit when the apogæon is in the quadratures II, 300

revolves more slowly, in a dilated orbit, when the node is in the syzygies with the Sun; and more swiftly, in a contracted orbit, when the node is in the quadratures II, 301

moves slower in its quadratures with the Sun, swifter in the syzygies; and by a radius drawn to the Earth describes an area, in the first case less in proportion to the time, in the last case greater II, 252; the inequality of those areas computed II, 263; its orbit is more curve, and goes farther from the Earth in the first case; in the last case its orbit is less curve, and comes nearer to the Earth II, 252; the figure of this orbit and the proportion of its diameters collected by computation II, 267; a method of finding the Moon's distance from the Earth by its horary motion *ibid.*

its apogæon moves more slowly when the Earth is in its aphelion, more swiftly in the perihelion II, 253, 299

its apogæon goes forward most swiftly when in the syzygies with the Sun; and goes backward in the quadratures II, 253, 301

its eccentricity greatest when the apogæon is in the syzygies with the Sun; least when the same is in the quadratures II, 253, 301

its nodes move more slowly when the Earth is in its aphelion

# I N D E X.

- aphelion and more swiftly  
in the perihelion II, 253,  
299
- its nodes are at rest in their  
syzygies with the Sun, and  
go back most swiftly in the  
quadratures II, 253
- the motions of the nodes and  
the inequalities of its mo-  
tions computed from the  
theory of gravity II, 273,  
278, 283, 287; the same  
from a different principle  
II, 289
- the inclination of its orbit to  
the ecliptic greatest in the  
syzygies of the node with  
the Sun, and least in  
the quadratures I, 245
- the variations of the inclina-  
tion computed from the  
theory of gravity II, 293,  
296
- the equations of the Moon's  
motions for astronomical  
uses II, 299, &c.
- the annual equation of the  
Moon's mean motion II,  
299
- the first semi-annual equation  
of the same II, 300
- the second semi-annual equa-  
tion of the same II, 301
- the first equation of the Moon's  
centre II, 302, I, 149, &c.
- the second equation of the  
Moon's centre II, 303
- Moon's first variation II, 271
- the annual equation of the  
mean motion of its apogee  
II, 299
- the semi-annual equation of  
the same II, 301
- the semi-annual equation of  
its eccentricity *ibid.*
- the annual equation of the  
mean motion of its nodes  
II, 299
- the semi-annual equation of  
the same II, 288
- the semi-annual equation of  
the inclination of the orbit  
to the ecliptic II, 298
- the method of fixing the the-  
ory of the Lunar motions  
from observations II, 304
- Motion, its quantity defined I, 2
- Motion absolute and relative I,  
10; the separation of one  
from the other possible; de-  
monstrated by an example  
I, 17
- Motion, laws thereof I, 19
- Motions, composition and reso-  
lution of them I, 22
- Motion of concurring bodies  
after their reflexion, by  
what experiments collected  
I, 33
- Motion of bodies.  
in eccentric sections Sect. 3.  
in moveable orbits Sect. 9.  
in given superficies, and of the  
reciprocal motion of pen-  
dulum Sect. 10.
- Motion of bodies tending to  
each other with centripetal  
forces Sect. 11.
- Motion of very small bodies a-  
gitated by centripetal for-  
ces tending to each part of  
some very great body Sect.  
14.
- Motion of bodies resisted in the  
ratio of the velocities II,  
Sect. 1.
- in the duplicate ratio of the  
velocity II, Sect. 2.

D d partly

# I N D E X.

partly in the simple, and partly in the duplicate ratio of the same II, Sect. 3.

## Motion

of bodies proceeding by their *vis insita* alone in resisting mediums II, 1, 2, 12, 15, 44, 46, 122

of bodies ascending or descending in right lines in resisting mediums, and acted on by an uniform force of gravity II, 4, 22, 24, 47, 50

of bodies projected in resisting mediums, and acted on by an uniform force of gravity II, 6, 28

of bodies revolving in resisting mediums II, Sect. 4.

of funependulous bodies in resisting mediums II, Sect. 6.

Motion and resistance of fluids II, Sect. 7.

Motion propagated through fluids II, Sect. 8.

Motion of fluids after the manner of a vortex, or circular II, Sect. 9.

## O.

Ovals for optic uses, the method of finding them, which *Cartesius* concealed I, 317; a general solution of *Cartesius's* problem I, 319

## Orbits,

the invention of those which are described by bodies going off from a given place with a given velocity, according to a given right line; when the centripetal force is reciprocally as

the square of the distance and the absolute quantity of that force is known I, 90  
of those which are described by bodies when the centripetal force is reciprocally as the cube of the distance I, 74, 174, 184  
of those which are described by bodies agitated by any centripetal forces whatever I, Sect. 8.

## P.

Parabola, by what law of centripetal force tending to the focus of the figure the same may be described I, 84

Pendulums, their properties explained I, 803, 212; II, Sect. 6.

the diverse lengths of isochronous pendulums in different latitudes compared among themselves, both by observations, and by the theory of gravity II, 246 to 251

Place defined, and distinguished into absolute and relative I, 10

Places of bodies moving in conic sections found to any assigned time I, Sect. 6.

## Planets,

not carried about by corporeal vortices II, 197.

## Planets primary,

surround the Sun II, 209  
move in ellipses whose focus is in the Sun's centre II, 234.

by radij drawn to the Sun describe areas proportional to the times II, 211, 234  
revolve

# I N D E X.

- revolve in periodic times that are in the sesquiplicate proportion of the distances from the Sun II, 210
- are retained in their orbits by a force of gravity, which respects the Sun, and is reciprocally as the square of the distance from the Sun's centre II, 214, 219
- Planets secondary,  
move in ellipses having their focus in the centre of the primary II, 252  
by radij drawn to their primary describe areas proportional to the times II, 206, 208, 212  
revolve in periodic times that are in the sesquiplicate proportion of their distances from the primary II, 206, 208
- Planets,  
their periodic times II, 210  
their distances from the Sun II, 211  
the aphelia and nodes of their orbits do almost rest II, 236  
their orbits determined II, 237  
the way of finding their places in their orbits II, 148 to 153  
their density suited to the heat they receive from the Sun II, 229  
their diurnal revolutions equable II, 238  
their axes less than the diameters that stand upon them at right angles II, 239
- Problem *Keplerian*, solved by the trochoid, and by approximations I, 148 to 153
- Problem of the ancients, of four lines, related by *Pappus*, and attempted by *Cartesius* by an algebraic calculus, solved by a geometrical composition I, 110
- Projectiles move in parabola's when the resistance of the medium is taken away I, 32 77, 310 II, 34
- Projectiles, their motions in resisting mediums II, 6, 28
- Pulses of the air, by which sounds are propagated, their intervals or breadths determined II, 180, 183; these intervals in sounds made by open pipes probably equal to twice the length of the pipes II, 183
- Q.
- Quadratures general of oval figures not to be obtained by finite terms I, 145.
- Qualities of bodies how discovered, and when to be supposed universal II, 203
- R.
- Resistance,  
the quantity thereof in mediums not continued II, 120  
in continued mediums II, 243  
in mediums of any kind whatever II, 123  
D d 2                      Resist-

# I N D E X.

- Resistances**, the theory thereof confirmed by experiments of pendulums II, 95 to 108  
by experiments of falling bodies II, 145 to 162
- Resistance of mediums**,  
is as their density *cæteris paribus* II, 106, 107, 113, 121, 143, 160  
is in the duplicate proportion of the velocity of the bodies resisted, *cæteris paribus* II, 11, 96, 113, 121, 143, 154  
is in the duplicate proportion of the diameters of spherical bodies resisted, *cæteris paribus* II, 101, 103, 113, 143
- Resistance of fluids** threefold ; and arises either from the inactivity of the fluid matter, or the tenacity of its parts, or friction II, 54 ; the resistance found in fluids almost all of the first kind II, 107, 159 and cannot be diminished by the subtilty of the parts of the fluid, if the density remain II, 161
- Resistance of a globe** what proportion it bears to that of a cylinder, in mediums not continued II, 117  
in compressed mediums II, 141
- Resistance of a globe** in mediums not continued II, 120 ; in compressed mediums II, 143 ; how found by experiments II, 145 to 160
- Resistance to a frustum** of a cone, how made the least possible II, 119
- Resistance**, what kind of solid it is that meets with the least II, 120
- Rest**, true and relative I, 10
- Rules of philosophy** II, 202
- S.
- Satellites**,  
the greatest heliocentric elongation of Jupiter's Satellites II, 207  
the greatest heliocentric elongation of the *Hugenian* Satellit from Saturn's centre II, 227  
the periodic times of Jupiter's Satellites, and their distances from his centre II, 206, 207  
the periodic times of Saturn's Satellites and their distances from his centre II, 208, 209  
the inequalities of the motions of the Satellites of Jupiter and Saturn derived from the motions of the Moon II, 252
- Saturn**,  
its periodic time II, 210  
its distance from the Sun II, 211  
its apparent diameter II, 209  
its true diameter II, 228  
its attractive force how great II, 227  
the weight of bodies on its surface II, 228  
its density *ib.*  
its quantity of matter *ib.*  
its perturbation by the approach of Jupiter how great II, 235  
the apparent diameter of its ring II, 209
- Sesqui-

# I N D E X.

- Sesquiplicate proportion defined**  
I, 52
- Shadow of the Earth to be augmented in lunar eclipses, because of the refraction of the atmosphere** II, 304
- Sun,**  
moves round the common centre of gravity of all the Planets II, 232.  
the periodic time of its revolution about its axis II, 238  
its mean apparent diameter II, 311  
its true diameter II, 228  
its horizontal parallax *ibid.*  
has a menstrual parallax II, 235  
its attractive force how great II, 227  
the weight of bodies on its surface II, 228  
its density *ibid.*  
its quantity of matter *ibid.*  
its force to disturb the motions of the Moon II, 215, 262  
its force to move the Sea II 305
- Sounds,**  
their nature explained II, 167, 173, 176, 178, 180, 181, 182  
not propagated *in directum* II, 166; caused by the agitation of the air II, 181  
their velocity computed II, 181, 182; some what swifter by the theory in summer than in winter II, 183  
cease immediately, when the motion of the sonorous body ceases II, 176  
how augmented in speaking-trumpets II, 183
- Space,**  
absolute and relative I, 9, 10, 11  
not equally full II, 224
- Sphæroid, the attraction of the same when the forces of its particles are reciprocally as the squares of the distances**  
I, 303
- Spiral cutting all its radij in a given angle, by what law of centripetal force, tending to the centre thereof, it may be described by a revolving body** I, 74; II, 56, 61
- Spirit, pervading all bodies and concealed within them, hinted at as required to solve a great many phenomena of Nature** II, 393
- Stars**  
the fixed Stars demonstrated to be at rest II, 236  
their twinkling what to be ascribed to II, 363  
new Stars, whence they may arise II, 385
- Substances of all things unknown**  
II, 391
- T.
- Tides of the Sea derived from their cause** II, 255, 305, 306
- Time absolute and relative** I, 9, 11.
- Time, the astronomical equation thereof proved by pendulum-clocks and the eclipses of Jupiter's satellites,** I, 12
- V.



# I N D E X.

## V.

- A Vacuum** proved, or that all spaces (if said to be full) are not equally full II, 224
- Velocity**, the greatest that a globe falling in a resisting medium can acquire II, 143
- Velocities** of bodies moving in conic sections where the centripetal force tends to the focus I, 87, 88, 89
- Venus**,  
     its periodic time II, 210  
     its distance from the Sun II, 211  
     the motion of its aphelion II, 237
- Vortices**, their nature and constitution examined II, Sect. 9. 387

## W.


- Waves**, the velocity with which they are propagated on the superficies of stagnant water II, 171
- Weights** of bodies towards the Sun, the Earth, or any Planet, are, at equal distances from the centre, as the quantities of matter in the bodies II, 220  
     they do not depend upon the forms and textures of bodies II, 223
- Weights** of bodies in different regions of the Earth found out and compared together II, 245

## A P P E N.

# APPENDIX.

*Among the Explications, (given by a Friend,) of some Propositions in this Book, not demonstrated by the Author, the Editor finding these following, has thought it proper to annex them. Thus,*

*To Cor. 2. Prop. 91. Book 1. Pag 303.*

1.  O find the force whereby a sphere (*AdBg*.) on the diameter *AB*, attracts the body *P*. (*Pl. 19. Fig. 1.*)

Let  $SA = SB = r$ ,  $PS = d$ ,  $PE = x$ ,  $PB = a = d + r$ ,  $PA = a = d - r$ ; Theref.  $aa = dd - rr$ ; also  $a + a = 2d$ ,  $a - a = 2r$ ; Therefore  $aa - aa = 4dr$ : And  $SE = d - x$ ,  $AE = x - a$ ,  $BE = a - x$ .

Now the force whereby the circle, whose radius is *Ed*, attracts the body *P*, is as  $1 - \frac{PE}{Pd}$  (by Cor. 1. Prop. 90.)

a

And

ii

# A P P E N D I X.

And  $\overline{Ed}^2 = (AE \times EB = \overline{SA}^2 - \overline{SE}^2 = rr - dd - 2dx - xx) = aa - 2dx - xx$ . Also  $\overline{Pd}^2 = (\overline{Ed}^2 + \overline{EP}^2 = 2dx - aa - xx + xx =) 2dx - aa$ : Th.  $\frac{PE}{Pd} = \frac{x}{\sqrt{-aa + 2dx}}$ . Therefore

$x - \frac{x}{\sqrt{-aa + 2dx}}$  or  $\dot{x} - \frac{\dot{xx}}{\sqrt{-aa + 2dx}}$  is the fluxion of the attractive force of the sphere on the body P, or the ordinate of a curve whose area represents that force.

But the fluent of  $\dot{x}$  is  $x$ ; and the fluent of  $\frac{\dot{xx}}{\sqrt{-aa + 2dx}}$  is  $\frac{aa + dx}{3dd} \sqrt{-aa + 2dx}$  (by Tab. 1. Form 4. Caf. 2. *Quadr. of Curv.*)

Therefore  $x - \frac{aa + dx}{3dd} \sqrt{-aa + 2dx}$  is the general expression of the area of the curve.

Now let  $x=a$ , then area =  $(a - \frac{aa + da}{3dd} \sqrt{-aa + 2da}) = \frac{d^3 - r^3}{3dd} = A$ .

Also let  $x=a$ , then area =  $(a - \frac{aa + da}{3dd} \sqrt{-aa + 2da}) = \frac{d^3 - r^3}{3dd} = B$ .

And the force whereby the sphere attracts the body P is as  $(A - B$  or as  $\frac{2r^3}{3d^2} =) \frac{2\overline{SA}^3}{3\overline{PS}^2}$ .

22

2. The

2. The force whereby the spheroid *ADBG*, attracts the body *P*, may, in the same manner, be found thus. Let  $SC = c$ ,

The force of a circle whose radius is *ED*, to attract *P*, is as  $1 - \frac{PE}{PD}$ , (by Cor. 1. Prop. 90.) Now

$$\overline{ED}^2 = \frac{\overline{SE}^2}{\overline{SA}^2} \times AEB = \frac{cc}{rr} \times \overline{aa + 2dx - xx}$$

(by the Conics;) and  $\overline{PD}^2 = (\overline{ER}^2 = \overline{ED}^2 + \overline{EP}^2$   
 $= \frac{-aacc + 2dccx - ccxx}{rr} + xx = )$   
 $\frac{-aacc + 2dccx + rr - ccxx}{rr}$ . Therefore  $(1 -$

$$\frac{PE}{PD} = 1 - \frac{x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr - cc}{rr}xx}}$$

or)  $\dot{x} = \frac{xx}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr - cc}{rr}xx}}$  is

the fluxion of the attractive force of the spheroid on the body *P*, or the ordinate of a curve whose area is the measure of that force.

Now the fluent of  $\dot{x}$  is *x*; and (by *Cas. 2. Form 8. Tab. 2.*

*Quad. Cur.*) the fluent of  $\frac{xx}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr - cc}{rr}xx}}$

$$\text{is } \left( \frac{-\frac{8dcc}{rr}s + \frac{4dcc}{rr}xv - \frac{4aacc}{rr}v}{-\frac{4aacc}{rr}x + \frac{rr - cc}{rr} - \frac{4ddc^2}{r^4}} = \frac{-2drrs + drrxv - aarrv}{aa \times cc - rr - ddcc} = \frac{-2ds + dxv - axv}{-cc - dd + rr} = \right)$$

$= \frac{2ds - dxv + axv}{cc + dd - rr}$ . Therefore  $x + \frac{dxv - axv - 2ds}{cc + dd - rr}$  is the general expression for the area of the curve.

$$\text{But } v = PD = ER = \sqrt{-\frac{aacc}{rr} - \frac{2dcc}{rr}x - \frac{cc - rr}{rr}x^2}$$

is an ordinate to a conic section, whose abscissa is  $x$ ; and  $s$ ,  $\sigma$ , the areas  $NMB$ ,  $NKA$ , adjacent to the ordinates  $BM$ ,  $AK$ : Put  $D = s - \sigma$ .

Let  $x = a$ , or  $PE = PB = BM$ ; then  $v = a$ , or  $PD = PB = BM$ , and the area  $= a + \frac{daa - aas - 2ds}{cc + dd - rr} = A$ .

And let  $x = a$ , or  $PE = PA = AK$ ; then  $v = a$ , or  $PD = PA = AK$ , and the area  $= a + \frac{daa - aas - 2ds}{cc + dd - rr} = B$ .

And the attractive force of the spheroid on  $P$ , is as

$$(A - B = a - a + \frac{d \times aa - a \times a - a \times a - a - 2dxs - \sigma}{cc + dd - rr}) = 2r + \frac{2ddr + 2r^3 - 2dD}{cc + dd - rr} = \frac{2rcc + 2d \times 2dr - D}{cc + dd - rr}$$

But  $2d = (a - a) = BM + AK$ , therefore  $2dr =$  trapezium  $ABMK$ ; and  $D = (s - \sigma) =$  area  $AKRMB$ ; therefore  $D - 2dr =$  mixtilinear area  $KRMLK = C$ ; consequently  $2dr - D = -C$ ; therefore  $2d \times 2dr - D = -2dC$ ; therefore the attractive force of the spheroid on  $P$ , is as  $\frac{2rcc - 2dC}{cc + dd - rr} =$

$$\frac{2AS \times \overline{SC}^2 - 2PS \times \overline{KRMK}}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2}. \text{ Consequently, the attractive force of the spheroid upon the body } P \text{ will be}$$

to the attractive force of a sphere, whose diameter is  $AB$ , upon the same body  $P$ , as  $\frac{rcc - dC}{cc + dd - rr}$  to  $\frac{r^3}{3dd}$ , or

$$\text{as } \frac{AS \times \overline{SC}^2 - PS \times \overline{KRMK}}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2} \text{ to } \frac{\overline{AS}^3}{3\overline{PS}^2}. \quad \text{To}$$

*To Schol. Prop. 34. Book 2. p. 119. l. 20.*

For let it be proposed to find the vertex of the cone, a frustum of which has the describ'd property.

Let  $CFGB$  be the frustum, and  $S$  the vertex required. (*Pl. 19. Fig. 2.*)

Now conceive the medium to consist of particles which strike the surface of a body (moving in it) in a direction opposite to that of the motion; then the resistance will be the force which is made up of the efficacy of the forces of all the strokes.

In any line  $Pp$ , parallel to the axis of the cone, and meeting its surface in  $p$ , take  $pm$  of a given length, for the space describ'd by each point of the cone in a given time: Draw  $mq$  perpendicular to the side ( $CF$ ) of the cone, and  $qn$  perpendicular to  $pm$ .

Therefore the line  $pm$  will represent the velocity, or force, with which a particle of the medium strikes the surface of the cone obliquely in  $p$ .

But the force  $mp$  is equivalent to two forces, the one ( $mq$ ) perpendicular, the other ( $pq$ ) parallel to the side of the cone; which last is therefore of no effect.

And the perpendicular force  $mq$  is equivalent to two forces, the one ( $mn$ ) parallel to the axis of the cone, the other ( $qn$ ) perpendicular to it; which also is destroy'd by the contrary action of another particle on the opposite side of the cone.

There remains only the force  $mn$ , which has any effect in resisting or moving the cone in the direction of its axis.

Therefore the whole force of a single particle, or the effect of the perpendicular stroke of a particle, upon the base of a circumscribing cylinder, is to the effect of the oblique stroke upon the surface of the cone (in  $p$ ) as  $mp$  to  $mn$ , or as  $\overline{mp}^2$  to  $(mp \times mn =) \overline{mq}^2$ , or as  $\overline{CF}^2$  to  $\overline{CH}^2$ .

2

Now

Now the number of particles striking in a parallel direction on any surface, is as the area of a plane figure perpendicular to that direction, and that would just receive those strokes.

Therefore, the number of particles striking against the frustum, that is, against the surfaces describ'd by the rotation of  $FD$ , and  $CF$ , each particle with the forces  $mp$ , and  $mn$  respectively, is as the circle describ'd by ( $FD$  or)  $OH$ , and the annulus described by  $CH$ , that is, as  $\overline{OH}^2$  to  $\overline{CO}^2 - \overline{OH}^2$ .

But the whole force of the medium in resisting, is the sum of the forces of the several particles.

Therefore, the resistance of the medium, or the whole efficacy of the force of all the strokes against the end  $FG$  of the frustum, is to the resistance against the convex surface thereof, as  $(mp \times \overline{OH}^2$  to  $mn \times \overline{CO}^2 - \overline{OH}^2$  or as  $\overline{CF}^2 \times \overline{OH}^2$  to  $\overline{CH}^2 \times \overline{CO}^2 - \overline{OH}^2$  or as)  $\overline{OH}^2$  to  $\frac{\overline{CH}^2 \times \overline{CO}^2 - \overline{OH}^2}{\overline{CF}^2}$ .

Therof. the whole resistance of the medium against the frustum may be represented by  $(\overline{OH}^2 + \frac{\overline{CH}^2 \times \overline{CO}^2 - \overline{OH}^2}{\overline{CF}^2})$

$$= \frac{\overline{CF}^2 \times \overline{OH}^2 - \overline{CH}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}$$

$$=) \frac{\overline{HF}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}, \text{ which call } z;$$

that is, (putting  $OC = r$ ,  $OD = 2a$ ,  $OS = y$ , then  $CH = (\frac{OC \times FH}{OS} =) \frac{2ar}{y}$ , and  $OH = \frac{y - 2ar}{y}$ ),

$$z = \frac{r^4 + r^2 y^2 - 4ar^2 y + 4a^2 r^2}{r^2 + y^2}; \text{ therefore } r^4 + r^2 y^2 - 4ar^2 y + 4a^2 r^2 = r^2 z + y^2 z: \text{ Consequently}$$

$2r^2yy - 4ar^2y = 2yz + y^2z + r^2z$ ; But  $z$  is a minimum; therefore  $rry - 2arr = zy$ ; consequently  $(z =) \frac{r^2y - 2ar^2}{y} = \frac{r^4 + r^2y^2 - 4ar^2y + 4a^2r^2}{rr + yy}$ .

Hence  $yy - 2ay = rr$ ; and making  $OQ = QD = a$ ; then  $(y - a)QS = (\sqrt{rr + aa})QC$ .

*To the same Schol. p. 120. l. 10.*

On the right-line  $BC$ , (*Pl. 19. Fig. 3.*) suppose the parallelograms  $BG\gamma b$ ,  $MN\nu m$ , of the least breadth, to be erected, whose heights  $BG$ ,  $MN$ , their distance  $Mb$ , and half the sum of their bases  $\frac{1}{2}Mm + \frac{1}{2}Bb = a$ , are given: Let half the difference of the bases  $\frac{1}{2}Mm - \frac{1}{2}Bb$  be called  $x$ : Let  $G$  and  $N$  be points in the curve  $GND$ ; and producing  $b\gamma$ , and  $m\nu$  to  $g$  and  $n$ , (so that  $\gamma g = \nu n = b$ ;) the points  $g$  and  $n$  may also be in the same curve.

Now if the figure  $CDNGB$ , revolving about the axis  $BC$ , generates a solid, and that solid moves forwards in a rare and elastic medium from  $C$  towards  $B$ , (the position of the right-line  $BC$  remaining the same;) then will the sum of the resistances against the surfaces generated by the lineolæ  $Gg$ ,  $Nn$ , be the least possible, when  $\overline{Gg}^4$  is to  $\overline{Nn}^4$  as  $BG \times Bb$  to  $MN \times Mm$ .

For the force of a particle on  $Gg$  and  $Nn$ , to move them in the direction  $BC$ , is as  $\frac{1}{\overline{Gg}^2}$  and  $\frac{1}{\overline{Nn}^2}$ ; and the number of particles that strike in the same time on the surfaces generated by  $Gg$  and  $Nn$ , are as (the annuli describ'd by  $g\gamma$  and  $n\nu$ , that is, as  $BG \times g\gamma$  and  $MN \times n\nu$ , or as)  $BG$  and  $MN$ ; therefore the resistances against those surfaces are as  $\frac{BG}{\overline{Gg}^2}$  to  $\frac{MN}{\overline{Nn}^2}$ , that is (putting  $y$  for  $\overline{Gg}^2$ , and  $z$  for  $\overline{Nn}^2$ ;) as  $\frac{BG}{y}$  to  $\frac{MN}{z}$ .



But the sum of these resistances ( $\frac{BG}{y} + \frac{MN}{z}$ ) is a minimum. Therefore— $BG \times \frac{y}{yy} - MN \times \frac{z}{zz} = 0$ , or

$$MN \times \frac{z}{zz} = -BG \times \frac{y}{yy} : \text{But } y = (\overline{Gg}^2 = \overline{Bb}^2 + \overline{\gamma g}^2 =) aa - 2ax + xx + bb; \text{ and } z = (\overline{Nn}^2 = \overline{Mm}^2 + \overline{vn}^2 =) aa + 2ax + xx + bb; \text{ therefore } y = 2xx - 2ax, \text{ and } z = 2ax + 2xx : \text{ consequently } \frac{MN}{zz} \times z \dot{x}$$

$$\times \overline{a+x} = \frac{BG}{yy} \times 2 \dot{x} \times \overline{a-x}; \text{ or } \left( \frac{MN}{zz} \times \overline{a+x} = \right)$$

$$\frac{MN}{zz} \times Mm = \left( \frac{BG}{yy} \times \overline{a-x} = \right) \frac{BG}{yy} \times Bb. \text{ Therefore}$$

$$(yy) \overline{Gg}^4 : (zz) \overline{Nn}^4 :: BG \times Bb : MN \times Mm.$$

Consequently, that the sum of the resistances against the surfaces generated by the lineolæ  $Gg$  and  $Nn$ , may be the least possible,  $\overline{Gg}^4$  must be to  $\overline{Nn}^4$  as  $GBb$  to  $NMm$ .

Wherefore, if  $\gamma g$  be made equal to  $\gamma G$ , so that the angle  $\gamma Gg$  may be  $45^\circ$ , and the angle  $BGg$   $135^\circ$ ; also  $\overline{Gg}^2 = 2 \overline{\gamma g}^2$ , and  $\overline{Gg}^4 = 4 \overline{\gamma g}^4$ ; then  $4 \overline{\gamma g}^4 : \overline{Nn}^4 :: GBb : NMm$ ; and since  $GR$  is parallel to  $Nn$ , and  $BG, BR$  parallel to  $nv, Nv$ ; also  $nv = \gamma \gamma = \gamma G$ ; it follows that  $(nv = \gamma G =) Bb : (Nv =) Mm :: BG : BR$ ; therefore  $Bb = \frac{BG \times Mm}{BR}$ ;

also  $(nv =) \gamma G : Nn :: BG : GR$ . Consequently  $\frac{4 \overline{\gamma g}^4}{\overline{Nn}^4} = \frac{4 \overline{BG}^4}{\overline{GR}^4} = \left( \frac{GBb}{NMm} = \right) \frac{\overline{BG}^2}{MN \times BR}$ . Therefore

$4 \overline{BG}^2 \times BR$  is to  $\overline{GR}^2$  as  $GR$  to  $MN$ .

F I N I S.







THE  
L A W S  
OF THE  
MOON'S  
M O T I O N  
According to  
GRAVITY.





# The LAWS of the M O O N's M O T I O N.

¶ justice to the editor of this translation of *Sir Isaac Newton's Principia*, it is proper to acquaint the reader, that it was with my consent, he published an advertisement, at the end of a volume of miscellanies, concerning a small tract which I intended to add to his book by way of appendix ; my design in which was to deliver some general elementary propositions, serving, as I thought, to explain and demonstrate the truth of the rules in *Sir Isaac Newton's Theory of the Moon*.

THE occasion of the undertaking was merely accidental; for he shewing me a paper which I communicated to the author, in the year 1717, relating to the motion of the nodes of the Moon's orbit; I recollected, that the method made use of in settling the Equation for that motion, was equally applicable to any other motion of revolution. And therefore I thought that it would not be unacceptable to a reader of the *Principia*, to see the uses of the said method explained in the other Equations of the Moon's motion: Especially since the greatest part of the Theory of the Moon is laid down without any proof; and since those propositions relating to the Moon's motion, which are demonstrated in the *Principia*, do generally depend upon calculations very intricate and abstruse, the truth of which is not easily examined, even by those that are most skilful; and which however might be easily deduced from other principles.

But in my progress in this design, happening to find several general propositions relating to the Moon's motions, which serve to determine many things, which have hitherto been taken from the observations of Astronomers: And  
 having

having reason to think, that the Theory of the Moon might by these means, be made more perfect and compleat than it is at present ; I retarded the publication of the book, 'till I could procure due satisfaction by examining observations on places of the Moon. But finding this to be a work requiring a considerable time, not only in procuring such places as are proper, but also in performing calculations, upon a new method, not yet accommodated to practise by convenient rules, or assisted by tables ; I thought it therefore more convenient for the Bookseller, not to stop the publication of his impression any longer upon this account. But that I may in some measure, satisfy those who are well conversant in Sir *Isaac Newton's Principia*, (and I could wish that none but such would look over these papers,) that the said advertisement was not without some foundation ; and that I may remove any suspicion that the design is entirely laid aside, I have put together, altho' in no order, as being done upon a sudden resolution, some of the Propositions, among many others, that I have by me, which seem chiefly to be wanting in a Theory of the Moon, as it is a speculation



founded on a physical cause; and those are what relate to the stating of the mean motions. For altho' it be of little or no use in Astronomy to know the rules for ascertaining the mean motions of the Node or Apogee, since the fact is all that is wanting, and that is otherwise known by comparing the observations of former ages with those of the present; yet in matter of speculation, this is the chief and most necessary thing required: since there is no other way to know that the cause is rightly assigned, but by shewing that the motions are so much and no more than what they ought to be.

But that it may not be altogether without its use, I have added all the rules for the equation of the Moon's motion, except two; one of which is a monthly equation of the variation depending on the Moon's anomaly; and the other an equation arising from the Earth's being not in the focus of the Moon's orbit, as it has been supposed to be, in all the modern theories since *Horrox*.

For not having had time to examine over the observations which are necessary, but being oblig'd instead thereof, to take Sir *Isaac Newton's* theory for my chief guide and direction, I cannot venture

venture to depart from it too far, in establishing equations entirely new; since I am well assured, upon the best authority, that it is never found to err more than seven or eight minutes.

And therefore, hoping that the reader, who considers the sudden occasion and necessity of my publishing these Propositions at this time, will make due allowance for the want of order and method, and look upon them only as so many distinct Rules and Propositions not connected: I shall begin, without any other preface, with shewing the origine of that inequality, which is called the Variation or Reflection of the Moon.

THE variation or reflection is that monthly inequality in the Moon's mo-

The Variation of the Moon.

tion, wherein it more manifestly differs from the laws of the motion of a planet in an elliptic orbit. *Tycho Brahe* makes this inequality to arise from a kind of libratory motion backwards and forwards, whereby the Moon is accelerated and retarded by turns, moving swifter in the first and third quarter, and slower in the second and fourth, which inequality is principally observed in the octants.

Sir *Isaac Newton* accounts for the

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variation

variation from the different force of gravity of the Moon and Earth to the Sun, arising from the different distances of the Moon in its several aspects.

The mean gravity of the Moon to the Sun, he supposes, is satisfied by the annual motion of the Moon round the Sun; the gravity of the Moon to the Earth, he supposes, is satisfied by a revolution of the Moon about the Earth. But the difference of the Moon's gravity to the Sun more or less than the Earth's gravity, he supposes, produces two effects; for as this difference of force may be resolved into two forces, one acting in the way, or contrary to the way, of the Moon about the Earth, and the other acting in the line to or from the Earth: the first causes the Moon to describe a larger or smaller area in the same time about the Earth, according as it tends to accelerate or retard it; the other changes the form of the lunar orbit from what it ought to be merely from the Moon's gravity to the Earth, and both together make up that inequality which is called the variation.

But since the real motion of the Moon, tho' a simple motion, caused by a continual deflection from a streight line, by  
the

the joint force of its gravity to the Sun and Earth, thereby describing an orbit, which incloses not the Earth but the Sun, is yet considered as a compound motion, made from two motions, one about the Sun, and the other about the Earth ; because two such motions are requisite to answer the two forces of its gravity, if separately considered : For the very same reason, the Moon's motion ought to be resolved into a third motion of revolution, since there remains a third force to be satisfied, and that is the force arising from the alteration of the Moon's gravity to the Sun. And this when considered, will require a motion in a small ellipsis, in the manner here described.

THE circle *ADFH* represents the orbit of the Moon about the Earth in the center *T*, as it would be at a mean distance, supposing the Moon had no gravity to any other body but the Earth. The diameter *ATF* divides that part of the orbit which is towards the Sun, suppose *ADF*, from the part opposite to the Sun, suppose *AHF*. The diameter at right angles *HTD*, is the line of the Moon's conjunction with or opposition to the Sun. The figure *PQLK* is an Ellipsis, whose center

ter is carried round the Earth in the orbit  $ABDEFH$ , having its longer axis  $PL$  in length double of the shorter axis  $QK$ , and lying always parallel to  $TD$ , the line joining the centers of the Earth and Sun. Whilst the said figure is carried from  $A$  to  $B$ , the Moon revolves the contrary way from  $Q$  to  $N$ , so as to describe equal areas in equal times about the centre of it; and to perform its revolution in the same time as the center of the said Elliptic epicycle (if it may be so called,) performs its revolution; the Moon being always in the remoter extremity of its shorter axis in  $Q$  and  $K$  when it is in the quarters, and in the nearest extremity of its longer axis at the time of the new and full Moon.

THE shorter semiaxis of this Ellipsis  $AQ$  is to the distance of its center from the Earth  $AT$ , in the duplicate proportion of the Moon's periodical time about the Earth to the Sun's periodical time; Which proportion, if there be 2139 revolutions of the Moon to the Stars in 160 sydercal years, is that of 47 to 8400.

THE figure which is described by this compound motion of the Moon in the Elliptic epicycle, whilst the center of it is carried round the Earth, very nearly represents the form of the Lunar orbit; supposing it without eccentricity, and that the

the plane was coincident with the plane of the ecliptic, and that the Sun continu'd in the same place during the whole revolution of the Moon about the Earth.

FROM the above construction it appears, that the proportion between the mean distance of the Moon and its greatest or least distances, is easily assigned; being something larger than that which is assigned by Sir *Isaac Newton* in the 8th proposition of his third book. But as the computation there given, depends upon the solution of a biquadratic equation, affected with numeral coefficients; which renders it impossible to compare the proportions with each other, so as to see their agreement or disagreement, except in a particular application to numbers; I shall therefore set down a rule, in general terms, derived from his method, which will be exact enough, unless the periods of the Sun and Moon should be much nearer equal than they are. Let  $L$  be the periodical time of the Moon,  $S$  the period of the Sun,  $M$  the synodical period of the Moon to the Sun, and  $D$  be the difference of the periods of the Sun and Moon; then, according to Sir *Isaac Newton's* method, the difference of the two axes of the Moon's elliptic orbit, as it is contracted by the  
action

action of the Sun, is to the sum of the said axes as  $3L \times \frac{M+L}{2}$  to  $4DD-SS$ . But according to the construction before laid down, the said proportion is as  $3LL$  to  $2SS-LL$ .

By Sir *Isaac Newton's* rule, the difference will be to the sum, nearly as 5 to 694; and consequently the diameters will be nearly as 689 to 699, or 69 to 70: But by the latter rule, the difference will be to the sum, nearly as 1 to 119; and the diameters or distances of the Moon, in its conjunction and quadrature with the Sun, will be as 59 to 60. Dr. *Halley*, (who in his remarks upon the Lunar theory, at the end of his catalogue of the Southern stars, first took notice of this contraction of the Lunar orbit in the Syzygies from the phenomena of the Moon's motion) makes the difference of the diameters to the sum, as 1 to 90; and consequently the greater axis to the lesser, as  $45\frac{1}{2}$  to  $44\frac{1}{2}$ .

BUT the difference, in these proportions of the extream distances, tho' it may appear considerable, is not, however, to be distinguish'd by the observations on the diameters of the Moon, whilst the variations of the diameters, from

from this cause, are intermixt with the other much greater variations, arising from the eccentricity of the orbit.

THE angle of the Moon's elongation Fig. 1. from the center, designed by  $BTN$ , is properly the variation or reflection of the Moon. The properties of which are evident from the description.

FIRST, It is as the sine of the double distance of the Moon from the quadrature or conjunction with the Sun: For it is the difference of the two angles  $BT A$  and  $NT A$ , whose tangents, by the construction, are in a given proportion.

SECONDLY, The variation is, *cæteris paribus*, in the duplicate proportion of the synodical time of the Moon's revolution to the Sun. For the variation is in proportion to the mean diameter of the epicycle, and that is in the duplicate proportion of the synodical time of revolution.

THE greatest variation is an angle, whose sine is to the radius, as the difference of the greatest and least distances  $TQ$  and  $TL$ , that is  $3AQ$ , to their sum. According to the proportion of the lines before described, this rule makes the elongation near 29 minutes ; which would  
be



be the variation, supposing the Moon perform'd its revolution to the Sun in the time of its revolution round the Earth. But if that elongation of 29 Minutes be increased in the duplicate proportion of the synodical time to the periodical time of revolution, it will produce near 34 minutes for the variation.

It is to be noted, that what is said of the epicycle, is upon supposition, that the Earths orbit round the Sun is a circle; if the eccentricity of the annual orbit be considered, the mean diameter of the epicycle must increase or diminish reciprocally in the triplicate proportion of the Sun's distance.

The method of finding the inequalities in any Revolution.

THE construction which I communicated to Sir *Isaac Newton*, for the annual motion of the nodes of the Moon's orbit, (which is printed in the scholium to the 33d proposition of his 3d book) is a case of a general method, for shewing the inequality of any motion round a center, when the hourly motion or velocity of the object varies, according to any rule, depending on its aspect to some other object. For in any revolution, the mean motion and inequality are to be assigned by means of a curvilinear figure

figure, wherein equal areas are described about the center in equal times; the property of which figure is, that the rays from the center, are always reciprocally in the subduplicate proportion of the hourly motion or velocity about the center.

Thus in the figure described in my Fig. 2. construction, where  $TN$  is the line of the nodes,  $TA$  the line drawn to the Sun, is supposed to revolve round the center  $T$ , with the velocity of the Sun's motion from the node; and the ray  $TB$ , which is taken always in the subduplicate proportion of that velocity, will describe equal areas in equal times; so that the sector  $NTB$  will be the mean motion of the Sun; the sector  $NTA$  the motion of the Sun from the node; and consequently the area  $NAB$  the motion of the node; which will be a retrograde motion if the area be within the circle, and direct if it falls without. From whence it follows,

1. That the periodical time of the Sun's revolution to the node, will be to the periodical time of the Sun's revolution, as the area of the curvilinear figure, to the area of the circle.

2. That if a circle be described, whose area is equal to the area of the curvilinear

near

near figure, it will cut that figure in the place where the Sun has the mean motion from the node.

3. If an angle  $NTF$  be made, which shall comprehend an area in the said circle, equal to the sector  $NTB$  in the figure, that angle will be the mean motion of the Sun from the node. And consequently,

4. The angle  $FTB$ , which is the difference between the Sun's true motion from the node, designed by  $ATN$ , and the Sun's mean motion from the node, designed by  $FTN$ , will be the equation for the Sun's motion from the node, when the Sun's position to the node is designed by the angle  $ATN$ .

FROM all which it appears, that what is said of the Sun's motion from the node, will hold as to any other motion round a center; as of the Sun from the Moon, or the Moon from the node or apogee. In any such revolution, a curvilinear figure may be described about the center, by the areas of which, the relation between the mean and true motion may be shewn; and consequently the inequality or equation of the motion.

Thus

And as in every revolution there is a certain figure which is proper to shew this relation, such a figure may be call'd an Equant for that motion or revolution.

And in every revolution where the Equant is a figure of the same property, the inequalities or equations will alter according to the same rule.

Thus, if the Equant be an ellipsis about the center, as in that for the motion of the Sun from the node,

*First*, The mean motion in the whole revolution, will be a geometrical mean proportional, between the greatest motion in the extremity of the lesser axis, and the least motion in the extremity of the longer axis: For the radius of the circle, which is equal to an ellipsis, is a mean proportional between the two semiaxes.

*Secondly*, The tangents of the angles of the mean and true motion, are in the given proportion of the two axes of the ellipsis. Thus the tangents of the angles of the true and mean motion of the Sun from the node, *viz.* the tangents of the angles *ATN* and *ETN*, are in proportion as the ordinates *BG* and *FG*, that is, as the semiaxes *TH* and *TN*. Fig. 2.

*Thirdly*, THE sine of the angle of the greatest inequality in the octants is

B to

to the radius, as half the sum of the axes to half their difference.

It is to be noted, that the equant is an ellipsis about the center, in every motion, where the excess of the velocity about the center above the least velocity, is always in the duplicate proportion of the sine of the angle of the true motion, from the place where the velocity about the center is least. From which remark, upon examination it will appear, that the following motions are to be reduced to an Elliptic equant described about the center.

The monthly motion of the Moon from the node.

THE annual motion of the Sun from the node.

THE motion of the Moon from the Sun, as it is accelerated or retarded, by the alteration of the area describ'd about the Earth, according to Sir *Isaac Newton's* 26th prop. 3d book.

AND the annual Motion of the Sun from the apogee. How these several equants are determin'd will appear by what follows.

THE node is in its swiftest retrograde motion, when the Sun and Moon are in conjunction or oppo-

The motion of the Nodes.

opposition, and in a quadrature with the line of the nodes. According to Sir *Isaac Newton's* method, (explain'd at the end of the thirtieth proposition of the third book) the force of the Sun to produce a motion in the node, at this time, is equal to three times the mean Solar force; that is, by the construction of the elliptic epicycle, equal to a force, which is to the force of gravity, as  $3 A Q$  to  $AT$ , or three times the lesser semi-axis of the ellipsis to the distance of its center from the center of the Earth. But if the Moon revolve in the elliptic epicycle as before described, the force to make a motion in the node at the time mention'd, will be to the force of gravity, as  $3 DL$  to  $DT$ , or three times the longer semi-axis to the distance of the center; which is the double of the former force. But then, according to Sir *Isaac's* method, the motion of the node at this time, is to the Moon's motion, as the solar force to create a motion in the node is to the force of gravity. But if the Moon be conceived as revolving in a circle, with the velocity of its motion from the node at this time, when the node moves swiftest, and the plane of the said circle be supposed to have a rotation

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upon an axis perpendicular to the plane of the ecliptic, and the contrary way to the motion of the Moon, so as to produce the motion of the node, and leave the Moon to move with its own motion about the Earth; the force to make a motion in the node seems to be the difference of the forces to retain it with the velocity of its motion in the moveable and immoveable planes: But the velocities of bodies revolving in circles are in the subduplicate proportion of the central forces. From whence it follows, that

*The motion of the Moon from the node at this time, when the node moves swiftest, is to the motion of the Moon, in the subduplicate proportion of the sum of the forces to the force of gravity, or as the sum of TD and  $\frac{3}{2}$ DL to TD.*

And this would be the greatest motion of the node, upon supposition that the plane of the Moon's orbit was almost co-incident with the plane of the ecliptic; but if the inclination be considered, the motive force for the node must be diminished, in the proportion of the sine-complement of the inclination to the radius. How much  
this

this motion is, will appear by the following short calculation.

The distance  $TD$  being as before equal to 8400, and  $3DL$  being 282; the inclination of the plane in this position is  $4^{\circ}. 59'. 35''$ ; the sine-complement of which is to the radius, as 525 to 527 nearly; therefore the force of gravity is to the motive force for the node thus diminished, in the compound proportion of 8400 to 282, and of 527 to 525; that is, in the proportion of 4216 to 141. So that the greatest motion of the Moon from the node is to the motion of the Moon, in the subduplicate proportion of 4357 to 4216, that is, in the proportion nearly of 613 to 603. According to which calculation, the greatest hourly motion of the node ought to be  $32''$ .  $47'''$ . By Sir *Isaac Newton's* method, it amounts to  $33''$ .  $10\frac{1}{2}'''$ .

This is the swiftest retrograde motion of the node, when the line of the nodes is in a quadrature with the Sun, and the Moon is in its greatest latitude in conjunction or opposition to the Sun. But the equant for the motion of the Moon from the node in this month, when the line of the nodes is in quadrature with the Sun, is an ellipsis about the center; and therefore the

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mean



mean motion in this month will be known by the following rule:

*The mean motion of the Moon from the node, in that month when the line of the nodes is in a quadrature with the Sun, is a geometrical mean proportional, between the greatest motion of the Moon from the node and the motion of the Moon.*

AND therefore this mean motion, will be to the motion of the Moon, in the subduplicate proportion of 613 to 603, that is, nearly in the proportion of 1221 to 1211. So that the mean motion of the node in this month, will be to the motion of the Moon, as 10 to 1211, which makes the mean hourly motion  $16''. 19''' \frac{1}{40}$ . According to Sir *Isaac Newton* it amounts to  $16''. 35'''$ ; but, by the corrections which he afterwards uses, it is reduced to  $16''. 16''' \frac{2}{3}$ .

BUT the equant for the annual motion of the Sun from the node being also an ellipsis, it follows, that

*The mean motion of the Sun from the node, is a geometrical mean proportional, between the motion of the Sun and the mean motion of the Sun from the node, in the month when the line of the nodes is in quadrature with the Sun.*

How

How near this rule agrees with the observations, will appear by this calculation.

Since the mean motion of the node in that month, when the line of nodes is in quadrature to the Sun, was before shewn to be to the Moon's mean motion, as 10 to 1211; and the motion of the Sun is to the motion of the Moon, as 160 to 2139: it follows, that the motion of the node and the motion of the Sun will be in the proportion of 154 and 1395; and therefore, by the rule, the Sun's mean motion from the node, is to the Sun's mean motion, in the subduplicate proportion of 1549 to 1395, that is, nearly as 98 to 93. Which corresponds with the observations; there being 98 revolutions of the Sun to the node in 93 revolutions of the Sun. The subduplicate proportion taken more nearly, is as 941 to 893, which will produce  $19^{\circ} . 21' . 3''$ , for the motion of the node from the fix'd Stars, in a sydercal year. The motion (as observ'd) is  $19^{\circ} . 21' . 22''$ .

Had the calculation from the rule, been more exactly made in large numbers, the annual motion produced would be  $19^{\circ} . 21' . 07''\frac{1}{2}$ , which is  $14''$

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less

less than the motion, as observed by the Astronomers.

Which difference may very probably arise from the Sun's parallax; and if so, it may perhaps furnish the best and most certain method of adjusting and fixing the true distance of the Sun. For the Sun's force being something more on that half of the orb which is towards the Sun, than what it is on the other half, the elliptic epicycle is accordingly larger in the first case, than in the latter. And by calculation, I find that the mean motion of the node, arising after consideration is had of this difference, is more than the mean motion from the mean magnitude of the epicycle, by near 2" in the year, for every minute in the parallactic angle of the orbit of the Moon, or for every second of the Sun's parallax. And by the best computation I have yet made, this difference of 14", in the annual motion of the node, will arise from about 8" of parallax; which will make the Sun's distance above 25000 semi-diameters of the Earth.

IN like manner as the equant for the motion of the node, in that month when the line of the nodes is in quadrature with the Sun, is an ellipsis; so in any other

other month it is also an ellipsis : the motion of the node being direct and retrograde by turns, in the Moon's passing from the quadrature to the Sun to the place of its node, and from the place of its node to the quadrature.

BUT these elliptic equants do not only serve to shew the inequality of the motion of the node, but also the inclination of the plane of the Moon's orbit to the plane of the ecliptic. Thus the rays in the elliptic equants, for the motion of the Moon from the node in each month, design the inclinations of the plane of its orbit to the plane of the ecliptic, in the several respective positions of the Moon to the line of the nodes. And the rays of the elliptic equant for the annual motion of the Sun from the node, in my Construction, (in the schol. to prop. 33. book 3. of Sir *Isaac Newton's Principia*) design the different mean inclinations of the said plane, to the plane of the ecliptic in each month, when the Sun is in each respective aspect to the line of the nodes.

The Inclination of the Plane of the Moon's orbit to the Plane of the Ecliptic.

THUS if *NT* (the semi-transverse axis of the elliptic equant for the motion of the Sun from the node,) design the

Fig. 2.

the mean inclination of the plane, or, which is the same thing, if it represent the mean distance between the pole of the ecliptic and the pole of the Moon's orbit, in that month when the Sun is in the line of the nodes ;  $TH$ , the semiconjugate axis of the said ellipsis, will design the mean inclination or mean distance of the poles in that month when the line of nodes is in quadrature to the Sun ; and  $TB$ , any other semidiameter of the said ellipsis, will represent the mean distance between the said poles, when the Sun is in that aspect to the line of the nodes, which is designed by the angle  $NTA$ . For example, if the least inclination, designed by the shorter semiaxis  $TH$  be  $5^{\circ}.00'.00''$  ; since  $TH$  is to  $TK$  as the motion of the Sun to the mean motion of the Sun from the node, by the property of this equant ; and since there are 98 revolutions of the Sun to the node in 93 revolutions of the Sun ; it follows, that  $HK$ , the difference between the greatest and least of the mean inclinations in the several months of the year, is to  $TH$  the least, as 5 to 93 ; by which proportion, the said difference will amount to  $16'.10''$ . According to Sir *Isaac Newton's* computation in the 35th prop. of the third book, it is

16'. 23'' $\frac{1}{2}$ . But if the said number be lessened in the proportion of 69 to 70, according to the author's note at the end of the 34th prop. the said difference will become 16'. 9".

AND in like manner, the inclinations of the plane of the Moon's orbit, in that month when the motion of the node is swiftest, (being situated in the line of quadratures with the Sun,) are determined by the equant for the motion of the Moon from the node, in that month.

THUS, let  $TH$  be to  $TN$  in the sub- Fig. 2.  
duplicate proportion of the Moon's motion, to its greatest motion from the node, when the Moon is in the conjunction in  $TH$ ; that is, (as was before determined) let  $TH$  be to  $TN$  in the proportion of 1211 to 1221; and the ellipsis described on the semiaxes  $TH$  and  $TN$ , will be the equant for the motion of the Moon from the node in that month. And the rays of the said equant will design the inclinations of the plane in the several aspects of the Moon to the line of the nodes. That is, if  $TN$  be the inclination of the plane, or the distance of the pole of the ecliptic from the pole of the Moon's orbit, when the Moon  
is

is in  $TN$  the line of the nodes, the ray  $TB$  will represent the distance of the said poles, or the inclination of the plane, in that aspect which is designed by the angle  $NTB$ .

WHICH being laid down, it follows that the whole variation of the inclination, in the time the Moon moves from the line of the nodes to its quadrature in  $THK$ , is to the least inclination, as  $KH$  to  $TH$ , that is, as 10 to 1211. Wherefore if the least inclination be  $4^{\circ}. 59'. 35''$ , the whole variation will be  $2'. 29''$ . This is upon supposition that the Sun continued in the same position to the line of the nodes, during the time that the Moon moves from the node to its quadrature. But the Sun's motion protracting the time of the Moon's period to the Sun, in the proportion of 13 to 12; the variation must be increased in the same proportion, and will therefore be  $2'. 41''$ . According to Sir *Isaac Newton's* computation, as delivered in the corollaries to the 34th prop. of the 3d book, for stating this greatest variation, (the intermediate variations in this or any other month not being computed or shewn by any method) it amounts to  $2'. 43''$ . But if the said quantity be diminish'd in the

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the proportion of 70 to 69, according to his note at the end of the said proposition, it will become the same precisely as it is here deriv'd from the equant.

THE motion of the Moon from the Sun, as it is accelerated or retarded by the increment of the area described about the Earth, (according to the 26th prop. of the 3d book) is also to be reduced to an elliptic equant; by taking the shorter axis to the longer axis, in the subquadruplicate proportion of the force of the Moon's gravity to the Earth, to the said force added to three times the mean Solar force, that is, as  $TA$  to the first of three mean proportionals between  $TA$  and  $TA + 3AQ$ . And in the same proportion is the area described by the Moon about the Earth, when in quadrature with the Sun, to the mean area, or as the mean area to the area described in the syzygies: So that the greatest area in the syzygies is to the least in the quadratures, in the subduplicate proportion of  $TA + 3AQ$  to  $TA$ , or as  $\sqrt{8541}$  to  $\sqrt{8400}$ . This is upon supposition, that the Moon revolves to the Sun in the same time as it revolves about the Earth; which will be found to agree

The Variation of the Area described by the Moon about the Earth.

Fig. 1.



gree very nearly with Sir *Isaac Newton*'s computation, in the before-cited proposition.

The Motion of the Apogee. AND after the same manner an elliptic equant might be constructed, which would very nearly shew the mean motion of the apogee, according to the rules deliver'd by Sir *Isaac Newton* (in the corollaries of the 45th prop. of the first book) for stating the motion of the apogee, namely, by taking the greatest retrograde motion of the apogee, from the force of the Sun upon the Moon in the quarters ; and the greatest direct motion, from the force of the Sun upon the Moon when in the conjunction or opposition ; each according to his rule, deliver'd in the second corollary to the said proposition. And if an ellipsis be made whose axes are in the subduplicate proportion of the Moon's motion from the apogee, when in the said swiftest direct and retrograde motions, the said ellipsis will be nearly the equant for the motion of the Moon from the apogee, and will be found to be nearly of the form of that above for the increment of the area.

BUT the motion of the apogee, according to this method, will be found to

to be no more than  $1^{\circ}. 37'. 22''$ , in the revolution of the Moon from apogee to apogee, which ( according to the observations ) ought to be  $3^{\circ}. 4'. 7''\frac{1}{2}$ .

So that it seems there is more force necessary to account for the motion of the Moon's apogee, than what arises from the variation of the Moon's gravity to the Sun, in its revolution about the Earth.

BUT if the cause of this motion be supposed to arise from the variation of the Moon's gravity to the Earth, as it revolves round in the elliptic epicycle, this difference of force, which is near double the former, will be found to be sufficient to account for the motion; but not with that exactness as ought to be expected. Neither is there any method that I have ever yet met with upon the commonly received principles, which is perfectly sufficient to explain the motion of the Moon's apogee.

The rules which follow concerning the motion of the apogee, and the alteration of the eccentricity, are founded upon other principles, which I may have occasion hereafter to explain, it being, as I apprehend, impossible to derive these, and many other such propositions

positions from the laws of centripetal forces.

Fig. 1.

LET  $TC$  (in the above construction of the Lunar orbit) be the mean distance of the Moon, or half the sum of its greatest and least distances, *viz.*  $TQ$  and  $TL$ ; and let  $CL$  be the mean semidiameter of the elliptic epicycle, or half the sum of the semiaxes; and take a distance  $LM$ , on the other side towards the centre, equal to  $CL$ ; then,

*The mean motion of the Moon from its apogee, is to the mean motion of the Moon, in the subduplicate proportion of  $TM$  to  $TC$ .*

FOR example, Half the shorter axis or  $DC$  is  $23\frac{1}{2}$ ; therefore  $TC$  the mean distance is  $8376\frac{1}{2}$ ;  $CM$  or  $2CL$ , the sum of the semiaxes, is  $141$ ; so that  $TM$  is  $8235\frac{1}{2}$ . Wherefore the motion of the Moon from the apogee is to the motion of the Moon, in the subduplicate proportion of  $8235\frac{1}{2}$  to  $8376\frac{1}{2}$ , or of  $16471$  to  $16753$ , that is, nearly as  $117$  to  $118$ , or more nearly, as  $352$  to  $355$ ; or yet more nearly, as  $1877$  to  $1893$ ; so that there ought to be about  $16$  revolutions of the apogee in  $1893$  revolutions of the Moon; which agrees to great preciseness with the most modern numbers of Astronomy; according to which proportion,

portion, the mean motion of the apogee, in a sydercal year, ought to be  $40^{\circ}.40'.40\frac{1}{2}''$ . But by the numbers in Sir *Isaac Newton's* theory of the Moon, the said motion is  $40^{\circ}.40'.43''$ . According to the numbers of *Tycho Brahe*, it ought to be  $40^{\circ}.40'.47''$ .

THE mean motion of the apogee being stated, I find the following rule for the alteration of the eccentricity.

The Variation of  
the Eccentricity.

*The least eccentricity is to the mean eccentricity, in the duplicate proportion of the Sun's mean motion from the apogee of the Moon's orbit, to the Sun's mean motion. Or in the duplicate proportion of the periodical time of the Sun's revolution, to the mean periodical time of its revolution to the Moon's apogee.*

By the foregoing rule for the mean motion of the apogee, there are 16 revolutions of the apogee in 189; revolutions of the Moon; but there being 254 revolutions of the Moon in 19 revolutions of the Sun; there must be about 7 revolutions of the apogee in about 62 revolutions of the Sun, or rather about 20 in 177. So that the periods of the Sun to the Stars, and of the Sun to the Moon's apogee, are in proportion

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portion nearly as the numbers 157 and 177. The duplicate of which proportion is that of 107 to 136; which, according to the rule, ought to be the proportion of the least eccentricity to the mean eccentricity.

So that by this rule, the mean eccentricity, (or half the sum of the greatest and least,) ought to be to the difference of the mean from the least, (or half the difference of the greatest and the least,) as 136 to 29.

How near this agrees with the Observations, will appear from the numbers of Mr. *Horrox* or Mr. *Flamsted*, and of Sir *Isaac Newton*.

THE mean eccentricity according to Mr. *Flamsted* or Mr. *Horrox* is 0.055236, half the difference between the greatest and least is 0.011617; which numbers are in the proportion of  $135\frac{1}{2}$  to  $28\frac{1}{2}$  nearly.

ACCORDING to Sir *Isaac Newton*, the mean eccentricity is 0.05505, half the difference of the greatest and least is 0.01173; which numbers are in proportion nearly as  $135\frac{47}{8}$  to  $28\frac{47}{8}$ , each of which proportions is very near that above assigned.

BUT it is to be noted, that the rule, which is here laid down, is true only upon supposition that the eccentricity is exceeding

exceeding small. There is another rule derived from a different method, which presupposes the knowledge of the quantity of the mean eccentricity ; and which will not only determine the variation of the eccentricity according to the laws of gravity, with greater exactness, but serve also to correct an hypothesis in the modern theories of the Moon, in which their greatest error seems to consist ; and that is, in placing the earth in the focus of that ellipsis, which is described on the extreme diameters of the lunar orbit ; whereas it ought to be in a certain point nearer the perigee, as I may have occasion to explain more fully hereafter.

THE greatest and least eccentricity being determined ; the equant for the motion of the Sun from the apogee is an ellipsis, whose greater and lesser axes are the greatest and least eccentricities : and therefore, by the property of such an equant as before laid down,

The Equation of the Apogee.

*The sine of the greatest equation of the apogee will be to the radius, as the difference of the axes of the equant is to their sum ; that is, as the difference of the greatest and least eccentricities to their sum.*

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FOR example, since the difference is to the sum as 29 to 136, by what was determined in the foregoing article, the greatest equation of the apogee will be about  $12^{\circ}. 18'. 40''$ . Sir *Isaac Newton* has determined it from the observations to be  $12^{\circ}. 18'$ .

THE greatest and least eccentricities being determined; the eccentricity and equation of the apogee, in any given aspect of the Sun, are determined by the equant, in the following manner.

Fig 2. LET  $TN$  be the greatest eccentricity,  $TH$  the least, the ellipsis on the semi-axes  $TN$  and  $TH$ , the equant for the motion of the apogee.

THEN if the angle  $NTF$ , be made equal to the mean distance or mean motion of the Sun from the apogee, the angle  $NTB$  will be the true distance or motion of the Sun from the apogee; the difference  $BTF$ , the equation of the apogee; and the ray  $TB$ , the eccentricity of the orbit, in that aspect of the Sun to the apogee designed by the angle  $NTB$ . Hence arises this rule.

*The tangent of the mean distance, viz.  $NTF$ , is to the tangent of the true distance  $NTB$ , in the given proportion of the*

*the greatest eccentricity TN to the least TH, that is, as 165 to 107.*

FROM what has been laid down concerning the general property of an equant, that it is a curve line described about the center, whose rays are reciprocally in the subduplicate proportion of the velocity at the center, or the velocity of revolution, it will not be difficult to describe the proper curve for any motion that is proposed; and where the inequality of the motion throughout the revolution is but small, there is no need of any nice or scrupulous exactness in the quadrature of the curve for shewing what the equation is. Thus all the small annual equations of the Moon's motion arising from the different distances of the Sun, at different times of the year, may be reduced to one rule exact enough for the purpose.

FOR since the Sun's force to create these annual alterations, is reciprocally in the triplicate proportion of the distance; the rays of the equant for such a motion, will be in the sesquiplicate proportion of the distance. From whence it will not be difficult to prove, that if the revolution of the motion to be equated, were performed in the time of the Sun's revolution, the equation would be to the

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equation of the Sun's center, nearly as 3 to 2 : and so if the force decreased as any other power of the Sun's distance, suppose that whose index is  $m$ , the equation would be to that of the Sun's center as  $m$  to 2. But if the motion be performed in any other period, the equation will be more or less, in the proportion of the period of the revolution to the Sun, to the period of the revolution of the motion to be equated. Thus if it were the node or apogee of the Moon's orbit, the equation is to the former as the period of the Sun to the node or apogee, to the period of the node or apogee. Which rule makes the greatest equation for the node about  $8'. 56''$ , being a small matter less than that in *Sir Isaac Newton's* theory ; and the greatest equation for the apogee about  $21'. 57''$ , being something larger than that in the same theory.

THE like rule will serve for the annual equation of the Moon's mean motion. If instead of the equation for the Sun's center, another small equation be taken in proportion to it as the force, by *Sir Isaac Newton* called the mean solar force, to the force of the Moon's gravity, or as 47 to 8400 ; the said equation increased in the proportion of the

Sun's

Sun's period to the mean fynodical period of the Moon to the Sun, or of 99 to 8, will be the annual equation of the Moon's mean motion. According to this, the equation, when greatest, will be 12'. 5".

WHAT is said may be sufficient for the present purpose, which is only to lay down the principal laws and rules of the several motions of the Moon, according to gravity. Some other propositions, which seem no less necessary than the former, for compleating the theory of the Moon's motion, as to its astronomical use, I reserve to another time.

BUT to make some amends for the shortness and confusedness of the preceding propositions, I shall add one example to shew the use of the equant more at large, in what is commonly called the solution of the *Keplerian* problem; that being one of the things which I proposed to explain, when the elements for the theory of the Moon were advertised.

*An example of the use of the equant? in  
finding the equation of the center.*

Fig. 3.

**L**ET the figure  $ADP$  be the orbit in which a body revolves, describing equal areas in equal times by lines drawn from a given point  $S$ ; and let it be propos'd to find the equant for the apparent motion of the said body, about any other place within the orbit, suppose  $F$ .

LET there be a line  $FR$  indefinitely produc'd, which revolves with the body as it moves through the arch  $AR$ ; and in the said line take a distance  $Fp$ , which shall be to  $FR$ , the distance of the body from the given point  $F$ , in the subduplicate proportion of the perpendicular let fall upon the tangent of the orbit at  $R$  from the point  $S$ , to the perpendicular on the said tangent let fall from the given point  $F$ ; and the curvilinear figure, describ'd by the point  $p$ , so taken every where, will be the equant for the motion of the body about the point  $F$ .

FOR since the areas described at the distances  $Fp$  and  $FR$  are in the duplicate

cate proportion of those lines, that is, by the construction, in the proportion of the perpendiculars on the tangents let fall from  $S$  and  $F$ ; the areas which the body describes, in moving through the arch  $AR$  about the points  $S$  and  $F$ , are in the proportion of the same perpendiculars. And therefore the area described by the revolution of the line  $Fp$  in the figure, will be equal to that which is described by the revolution of the line  $SR$  in the orbit. So that the areas described in the figure will be equal in equal times, as they are in the orbit. And consequently the rays  $Fp$  of the figure will constantly be in the subduplicate proportion of the velocity of the motion, as it appears at the center  $F$ , which is the property of the equant.

FROM which construction, it will be easy to shew, that in the case where a body describes equal areas in equal times about a fixed point, there may be a place found out within the orbit, about which the body will appear to revolve with a motion more uniform than about any other place.

THUS suppose the orbit  $ADP$  was a figure, wherein the remotest and nearest apsis  $A$  and  $P$  were diametrically opposite, in a line passing through the point  $S$ ,

*S*, viz. the point about which the equal areas are described; then if the point *F* be taken at the same distance from the remotest apsis *A*, as the point *S* is from the nearest apsis *P*, the said center *F* will be the place, about which the body will appear to have the most uniform motion. For in this case the point *F* will be in the middle of the figure *LpDl*, which is the equant for the motion about that point. So that the body will appear to move about the center *F*, as swift when it is in its slowest motion in the remoter apsis *A*, as it does when it is in its swiftest motion in the nearest apsis *P*.

FOR by the construction, when the body is at *A*, the ray of the equant *FL* is a mean proportional between *AF* and *AS*; and when the body is at *P*, the ray of the equant *Fl* is a mean proportional between the two distances *PS* and *PF*, which are respectively equal to the former,

AND in like manner in an orbit of any other given form, a place may be found about which the motion is most regular.

IF what has been said be applied to the case of a body revolving in an elliptic orbit, and describing equal areas  
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in equal times about one of the foci, as is the case of a planet about the Sun, and a secondary planet about the primary one; it will serve to shew the foundation of the several hypotheses and rules which have been invented by the modern Astronomers, for the equating of such motions; and likewise shew how far each of them are deficient or imperfect.

For if the ellipsis  $ADP$  be the orbit of a planet describing equal areas about the Sun in the focus  $S$ , the other focus, suppose  $F$ , will be the place about which the motion is most regular, from what has been already said; that focus being at the same distance from the aphelion  $A$ , as the Sun at  $S$  is from the perihelion  $P$ . And by the construction, each ray ( $Fp$ ) of the equant will always be a mean proportional between  $FR$  and  $RS$ , the two distances of the planet from the two foci in that place where the ray  $Fp$  is taken. For the rays  $SR$  and  $RF$ , making equal angles with the tangent at  $R$ , by the property of the ellipsis, are in the proportion of the perpendiculars from  $S$  and  $F$ , let fall on those tangents. And therefore  $Fp$  being to  $FR$  in the subduplicate proportion

portion of  $SR$  to  $FR$ , it will be a mean proportional between those distances.

1. HENCE when the planet is in the aphelion  $A$ , or perihelion  $P$ , the rays of the equant  $FL$  and  $FL$  are the shortest, each being equal to  $CD$ , the lesser semi-axis of the orbit : For by the property of the ellipsis, the rectangle of the extremum distances from the focus is equal to the square of the lesser semi-axis.

2. WHEN the planet is at its mean distance from the Sun in  $D$  or  $d$ , the extremities of the lesser axis, the equant cuts the orbit in the same place ; the rays of the equant being then the longest, being each equal to the greater semi-axis  $CA$ . For in those points of the orbit, the distances from the foci and the mean proportional are the same.

FROM which form of the equant, it appears,

1. THAT the velocity of the revolution about the focus  $F$  diminishes, in the motion of the planet from the aphelion or perihelion to the mean distance ; and increases in passing from the mean distance to the perihelion or aphelion. For the rays of the equant increase in the first case, and diminish in the latter ; and the velocity of revolution increases in

in the duplicate proportion, as the rays diminish.

2. IN any place of the orbit, suppose  $R$ , the velocity of the revolution about the focus  $F$ , is in proportion to the mean velocity, as the rectangle of the semi-axes of the orbit  $CD$  and  $CA$ , to the rectangle of the focal distances  $RF$  and  $RS$ . For the equant and the orbit, being figures of the same area, are each equal to a circle, whose radius is a mean proportional between the two semi-axes  $CD$  and  $CA$ . But the mean motion about the focus  $F$ , is in those places, where the said circle cuts the equant; and in other places, the velocity of the revolution is reciprocally as the square of the distance, that is, reciprocally as the rectangle of the focal distances  $RF$  and  $RS$ .

3. So that the planet is in its mean velocity of revolution about the focus  $F$ , in four places of the orbit, that is, where the rectangle of the focal distances is equal to the rectangle of the semi-axes; which places in orbits nearly circular, such as those of the planets, are about 45 degrees from the aphelion or perihelion; but may be assigned in general, if need be, by taking a point in the orbit, suppose  $R$ , whose nearest distance from the lesser axis of the orbit  $CD$  is to the longer



longer semi-axis  $CA$ , in the subduplicate proportion of the longer axis to the sum of the two axes; as may be easily proved.

WHAT has been said, may be enough to shew the form of the equant, and the manner of the motion about the upper focus in general. But the precise determination of the inequality of the motion, requires the knowledge of the quadrature of the several sectors of the equant, or at least, if any other method be taken, of that which is equivalent to such a quadrature.

There are divers methods for shewing the relation between the mean and true motion of a planet round the Sun, or round the other focus, some more exact than others. But the following seems the most proper for exhibiting in one view, all the several hypotheses, and rules, which are in common use in the modern Astronomy, whereby it may easily appear, how far they agree or differ from each other, and how much each of them errs from the precise determination of the motion, according to the true law of an equal description of areas about the Sun.

UPON the center  $F$  describe the ellipsis  $LNl$ , equal and similar to the elliptic orbit  $ADP$ ; but having its axes  $FN$

$FN$  and  $FL$  contrarily posited, that is, the shorter axis  $LF$  lying in the longer axis of the orbit  $AP$ , and the longer axis  $FN$  parallel to the shorter  $CD$ . Let the focus of the said ellipsis be in  $f$ . And suppose two other ellipsis  $LB\ell$  and  $Lf\ell$ , to be drawn upon the common axis  $L\ell$ , one passing through the point  $B$ , where the perpendicular  $FN$  intersects the orbit, and the other through the focus  $f$ . Let the line  $FR$ , revolving with the planet in the orbit, be indefinitely produced, till it intersect the first ellipsis  $LN\ell$  (which was similar to the orbit) in  $Q$ , the equant in  $p$ , and the ellipsis  $LB\ell$  (drawn through the intersection  $B$ ,) in  $K$ . From the point  $K$  let fall  $KH$  perpendicular to the line of apsides  $AP$ , and let it be produced till it intersect the first ellipsis  $LN\ell$  in  $O$ , and the ellipsis  $Lf\ell$  (passing through the focus  $f$ ) in  $E$ . And lastly, in the ellipsis  $LN\ell$ , let  $GM$  be an ordinate equal and parallel to  $EH$ . In which construction it is to be noted, that the ellipsis  $Lf\ell$  and  $LB\ell$  are supposed as drawn only to divide the line  $OKH$  in given proportions, that  $KH$  may be to  $OH$ , as the latus rectum of the orbit to the transverse axis; and that  $EH$  or  $GM$ , the base of the elliptic segment  $GLM$ ,  
may

may be to  $OH$ , as the distance of the foci to the transverse axis.

WHICH being premised, it will be easy to prove, that the sector  $pFL$  in the equant, or, which is the same thing, the sector  $RSA$  in the orbit, is equal to the curvilinear area  $OKFMG$ , that is, equal to the elliptic sector  $QFL$ , deducting the segment  $LMG$ , and adding or subtracting the trilinear space  $QKO$ , according as the angle  $RFA$  is less or greater than a right angle. Wherein it is to be noted, that these signs of addition and subtraction are to be used in general, if the angle  $AFR$  is taken from the aphelion in the first semi-circle, but towards the aphelion in the latter semi-circle. But if the angle  $AFR$  be taken the same way throughout the whole revolution, as is the method in Astronomical calculations, then the segment and the trilinear space in the latter semi-circle must be taken with the contrary signs to what are laid down.

HENCE it appears, that the inequality in the motion of a planet about the upper focus  $F$ , consists of three parts.

I. THE first and principal of which is the inequality in the alteration of the angle  $QFL$ , in making equal areas in the ellipsis

ellipfis *LN*. For if a circle equal to the ellipfis be described upon the center *F*, since the radius (being a mean proportional between the two semi-axes) will fall without the ellipfis about the line of apsides, and within it about the middle distances, the angle  $\angle QFL$ , which is proportional to the area described in the circle, will therefore increase faster about the line of apsides, and slower about the middle distances, in describing equal areas in the ellipfis, than it ought to do in the hypothesis of Bishop *Ward*, who makes the planet revolve uniformly about the focus. The equation to rectify this inequality is determined by the following rule.

Fig. 3.

THE tangent of the angle  $\angle QFL$ , is to the tangent of the angle in the circle including the same area, as the longer axis of the ellipfis to the shorter axis; and the difference of the angles, whose tangents are in this proportion, is the equation; as is manifest from what was before said on the properties of an elliptic equant. From the same it also follows, that

1. THE greatest equation is an angle, whose sine is to the radius as the difference of the axes to their sum, or, which is the same thing, as the square of the distance  
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of the foci, to the square of half the sum of the axes. So that in ellipsis nearly circular, of different eccentricities, this greatest equation will vary nearly in the duplicate proportion of the eccentricity.

2. IN ellipsis nearly circular, the equation at any given angle  $QFL$ , is to the greatest equation, nearly as the sine of the double of the given angle to the radius; which follows from hence, that the equation is the difference of two angles, whose tangents are in a given proportion, and nearly equal.

3. THIS equation adds to the mean motion in the first and third quadrant of mean anomaly, and subducts in the second and fourth; as will easily appear from that the line  $QF$ , in describing equal areas in the ellipsis, makes the angle to the line of the apsides, less acute than it would be in an uniform revolution.

THIS is the equation which is accounted for in the hypothesis of *Bullialdus*. For he supposes the motion of the planet in its orbit to be so regulated about the upper focus, that the tangents of the angles, from the lines of apsides, shall always be to the tangents of the angles answering to the mean anomaly, in the proportion of the ordinates  
in

in the ellipsis to the ordinates in the circle circumscribed; which in effect is the same, as if he had made the true equant for its motion about the focus  $F$ , to be the ellipsis as above described.

THE same equation is also used by Sir *Isaac Newton*, in his solution of the *Keplerian* problem, in the scholium to the 31st prop. of the 1st book, and is there designed by the letter  $V$ .

BUT since the true equant  $LDI$  coincides with the elliptic equant in the extremities of the shorter axis at  $L$  and  $I$ , and falls within the same at its intersection with the longer axis  $FN$ , it follows, that the motion of the planet in the semi-circle about the aphelion, is swifter than according to the hypothesis of an equal description of areas in the ellipsis  $LN I$ , and for the same reason slower in the other semi-circle about the perihelion; the velocity about the center  $F$  being always reciprocally in the duplicate proportion of the distance.

WHICH leads to the second part of the inequality of the motion about the focus.

II. THE equation to rectify this inequality, is an angle answering to the segment  $GLM$ ; which angle is to be added to the mean anomaly, to make the area of the elliptic sector  $QFL$ .

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THIS angle or equation is determined by the following rule. Let  $R$  be an angle subtended by an arch equal in length to the radius of the circle, *viz.* 57,29578 degrees; and let  $A$  be an angle, whose sine is to the radius as  $GM$ , the base of the segment, to  $FN$  the semi-transverse axis; also let  $B$  be an arch in proportion to  $R$ , as the sine of the double of the angle  $A$  to the radius: Then the equation for the segment will be equal to  $A - \frac{1}{2}B$ .

THIS equation is at its maximum, when the angle  $LFQ$  is a right angle; the base of the segment becoming equal to  $Ff$ , half the distance of the foci, and the angle  $A$ , being in this case half the angle  $F'D'S'$  formed at the extremity of the lesser axis, and subtended by  $FS$ , the distance of the foci; which is commonly called the greatest equation of the center. And consequently the arch  $B$ , in this case, is to  $R$ , as the sine of the said greatest equation of the center, is to the radius. So that according to this rule, for the measure of the segment, it will follow, That

1. THIS greatest equation is in proportion to the greatest equation of *Bullialdus*, as found in the preceding article for the elliptic equant, nearly

nearly as three times the transverse axis, to eight times the distance of the foci. Or, otherwise, the greatest equation is to the angle designed by  $R$ , as twice the cube of the distance between the foci, to three times the cube of the transverse axis. Either of which rules may be derived from the true angle, as before determined; or by taking  $\frac{2}{3}$  of the rectangle of  $GM$  and  $LM$ , the base and height of the segment, for the measure of that segment.

So that in elliptic orbits nearly circular, this greatest equation for the segment is in the triplicate proportion of the eccentricity.

2. THIS equation at any given angle  $QFL$ , is to the greatest equation, in the triplicate proportion of the ordinate  $OH$  to the semi-transverse; that is, nearly as the cube of the sine of the mean anomaly joined to the double of *Bullialdus's* equation to the cube of the radius. For the segment  $GML$ , which is proportional to the equation, is in the triplicate proportion of its base nearly; and the base is proportional to the ordinate  $OH$ , by the construction.

BUT the ordinate  $OH$  (in a circle described upon the radius  $FN$ ), becomes the sine of an angle, whose tan-

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gent is to the tangent of the angle  $QFL$ , in the proportion of the transverse axis to the conjugate; but the tangent of the same angle  $QFL$ , is to the tangent of the mean motion, answering to the area of the elliptic equant  $QFL$  in the same proportion. So that the ordinate  $OH$  is to the sine of that angle of mean motion, in the duplicate of the said proportion; and consequently the ordinate  $OH$ , in the circle on the radius  $FN$ , is the sine of an angle, nearly equal to the mean anomaly joined to the double of *Bullialdus's* equation.

3. THIS equation adds to the mean motion in passing from the aphelion to the perihelion, and subducts in passing from the perihelion to the aphelion; as is evident from the transit of the point of intersection  $E$  round the periphery of the ellipsis  $Lfl$ .

IN Sir *Isaac Newton's* rule (in the before-cited scholium to the 31st prop. 1st book,) the angle  $X$  answers to this equation for the segment; excepting that it is there taken in the triplicate proportion of the sine of the mean anomaly, instead of the triplicate proportion of the ordinate  $OH$ . The error of this rule makes

III. THE

III. THE third part of the inequality, answering to the trilinear space  $OKQ$ , being the difference of the elliptic sector  $OFQ$  and the triangle  $OFK$ .

THE sector  $OFQ$  is proportional to an angle, which is the difference of two angles, whose tangents are in the given proportion of the semi-latus rectum  $FB$  and the semi-transverse  $FN$ , or in the duplicate proportion of the lesser axis to the axis of the orbit. So that this sector, when at a maximum, is as an angle, whose sine is to the radius, as the difference of the latus rectum and transverse to their sum; or as the difference of the squares of the semi-axes to their sum.

THE triangle  $OFK$  is proportional to the rectangle of the co-ordinates  $OH$  and  $HF$ ; that is, as the rectangle of the sine  $OH$  and its cosine, in the circle on the radius  $FN$ ; or as the sine of the double of that angle, whose sine is  $OH$ ; that is, the double of the angle, whose tangent is to the tangent of the angle  $QFL$ , in the given ratio of the greater to the lesser axis; or whose tangent is the tangent of the angle of mean motion answering to the elliptic sector  $QFL$ , in the duplicate of the said

D 4 ratio.

ratio. But this triangle  $OFK$ , when at a maximum, makes an angle of mean motion, which is to the angle called  $R$ , as  $BN$ , half the difference between the latus rectum and transverse axis, is to the double of the transverse axis.

So that the sector or triangle in orbits nearly circular, is always nearly equal to the double of *Bullialdus's* equation.

THE triangle and sector being thus determined, the equation for the trilinear space is accordingly determined. From what has been said, it appears, that

1. THIS equation for the trilinear space  $OKQ$ , is to that for the triangle  $OKF$ , in a ratio compounded of  $BN$ , the difference between the semi-transverse and semi-latus rectum to the semi-latus rectum, and of the duplicate proportion of the sine  $OH$  to the radius; or  $OKQ$  is to  $OKF$ , in a proportion compounded of the duplicate proportion of the distance of the foci to the square of the lesser axis, and the duplicate proportion of the sine  $OH$  to the radius. For the trilinear figure  $OKQ$  and the triangle  $OKF$ , are nearly as  $OK$  and  $KH$ , which are in that proportion; and consequently it holds in this proportion to the double of *Bullialdus's* equation.

2. THIS

2. THIS equation, in different angles, is as the content under the sine complement and the cube of the sine. For the triangle  $OKF$ , is as the rectangle of the sine and the sine complement.

3. IT is at a maximum, at an angle whose sine complement is to the radius, as the square of the greater axis is to the sum of the squares of the two axes; which in orbits nearly circular, is about 60 degrees of mean anomaly.

4. IN orbits of different eccentricities, it increases in the quadruplicate proportion of the eccentricity.

5. IT observes the contrary signs to that for the elliptic equant, called *Bullialdus's* equation; subducting from the mean motion in the first and third quadrants, and adding in the second and fourth, if the motion is reckoned from the aphelion.

THE use of these equations, in finding the place of a planet from the upper focus, will appear from the following rules, which are easily proved from what has been said.

LET  $t$  be equal to  $CA$  the semi-transverse,  $c$  equal to  $FC$  the distance of the center from the focus,  $b$  equal to  $CD$  the semi-conjugate, and  $R$  an angle subtended by an arch equal to the

the radius, viz.  $57^{\circ}. 17'. 44''. 48'''$ , or 57, 2957795 degrees. Take an angle

$$T = \frac{cc}{2tt} R; \quad E = \frac{b}{2t} T; \quad S = \frac{4c}{3b} T.$$

The angle  $T$  be will the greatest equation for the triangle  $OFK$ ; the angle  $S$  will be the greatest equation for the segment  $LMG$ ; and the angle  $E$  will be the greatest equation for the area  $OKFL$ . Which greatest equations being found, the equations at any angle of mean anomaly, will be determined by the following rules.

LET  $M$  be the mean anomaly; and let  $\tau$  be to  $T$  as the sine of the angle  $2M$  to the radius: In which proportion, as also in the following, there is no need of any great exactness, it being sufficient to take the proportions in round numbers.

TAKE  $e$  to  $E$  as the sine of  $2M \pm 2\tau$  to the radius; and  $s$  to  $S$  as the cube of the sine of  $M \pm \tau$  to the cube of the radius.

THEN the angle  $QFL$  is equal to  $M + e + s$ , in the first quadrant  $LN$ , or  $M - e + s$ , in the second quadrant  $NL$ , or  $M + e - s$  in the third quadrant, or  $M - e - s$  in the fourth quadrant.

NOTE, That the small equation  $\tau$  is always of the same sign with the equation  $e$ ; and

and in the case of the planets, always near the double of that equation.

THE angle  $RFA$  at the upper focus  $F$  being known, the angle  $RSA$  at the Sun in the other focus, is found by the common rule of Bishop *Ward*; viz. the tangent of half the angle  $RSA$ , is to be to the tangent of half the angle  $RFA$ , always in the given proportion of the perihelion distance  $SP$  to the aphelion distance  $SA$ . How these equations are in the several eccentricities of the Moon's orbit, will appear by the following Table.

Eccentr.	E.	S.
	"	"
0.040	1.23	09
0.045	1.45	13
0.050	2.09	17
0.055	2.36	23
0.060	3.06	30
0.065	3.38	38
0.070	4.14	47

To add one example; suppose the eccentricity 0.060, the mean anomaly  $30^\circ$ . The sine of the double of the mean anomaly, that is, the sine of  $60$  is to the radius, nearly as 87 to 100; whence, if the equation  $E = 3'.06''$ , be divided in that proportion

proportion, it will produce  $2'.40''$  nearly, for the equation  $e$ : the sine of  $M$  is, in this case, equal to  $\frac{1}{2}$  the radius, the cube is  $\frac{1}{8}$  of the cube of the radius; whence if the equation  $S=30''$  be divided in the same proportion, it will produce near  $4''$  for the equation  $s$ . Therefore the angle  $RFA$ , which is  $M - e + s$ , will be  $30^\circ.2'.44''$ ; and the half is  $15^\circ.1'.22''$ ; wherefore if the tangent of this angle be diminished, in the proportion of  $1.06$ , the aphelion distance, to  $94$  the perihelion distance, it will produce the tangent of  $13^\circ.23'.13''$ ; the double of which  $26^\circ.46'.26''$ , is the true anomaly or angle at the Sun  $RSA$ . And consequently, the equation of the center is  $3^\circ.13'.34''$  to be subducted, at  $30$  degrees mean anomaly.

WHEN the place of a planet is found by this, or any other method; the place may be corrected to any degree of exactness by the common property of the equant, *viz.* that the rays are reciprocally in the duplicate proportion of the velocity about the center. For in this case, if there be a difference between the mean motion belonging to the angle assumed at the upper focus, and the given mean motion, the error of the angle assumed is to the difference, as the rectangle of the semi-axes to the rectangle

angle

angle of the distances from the foci. But in orbits like those of the planets, the rules as they are delivered above are sufficient of themselves without further correction.



## POSTSCRIPT. = *prolata*

UPON reviewing these few sheets after they were printed off, which happened a little sooner than I expected, I fear the apology I have offered for delivering the propositions relating to the Moon's motion, in this rude manner, without giving any proof of them, or so much as mentioning the fundamental principles of their demonstration, will scarcely pass as a satisfactory one ; especially since there are among these propositions, some which, I am apt to think, cannot easily be proved to be either true or false, by any methods which are now in common use.

WHEREFORE to render some satisfaction in this article, I shall add a few words concerning the principles from whence these propositions, and others of the like nature



nature are derived: and also take the opportunity to subjoin a few remarks, which ought to have been made in their proper places.

*First*, THERE is a law of motion, which holds in the case where a body is deflected by two forces, tending constantly to two fixed points.

WHICH is, *That the body, in such a case, will describe, by lines drawn from the two fixt points, equal solids in equal times, about the line joining the said fixt points.*

THE law of *Kepler*, that bodies describe equal areas in equal times, about the center of their revolution, is the only general principle, in the modern doctrine of centripetal forces.

BUT since this law, as *Sir Isaac Newton* has proved, cannot hold, whenever a body has a gravity or force to any other than one and the same point; there seems to be wanting some such law as I have here laid down, that may serve to explain the motions of the Moon and Satellites, which have a gravity towards two different centers.

IT follows as a corollary to the law here laid down, that if a body, gravitating towards two fixt centers, be supposed, for given small intervals of time,

as

as moving in a plane passing through one of the fixt centers, the inclination of the said plane, to the line joining the centers, will vary according to the area described; that is, if the area be greater, the inclination will be less; and if the area be less, the inclination will be greater, in order to make the solids equal.

THIS corollary, when rightly applied, will serve to explain the variation of the inclination of the plane of the Moon's orbit to the plane of the ecliptic.

AND how extremely difficult it is to compute the variation of the inclination in any particular case, without the knowledge of some such principle as this is, will best appear, if any one consider the intricacy of the calculations, used in the corollaries to the 34 prop. of the third book of the *Principia*, in order to state the greatest quantity of variation, in that month, when the line of the nodes is in quadrature with the Sun, and that only in particular Numbers, whereby it is determined to be  $2'.43''$ .

WHEREAS, there is a plain and general rule in this case, which follows from what is laid down, though not immediately; namely, that the greatest variation in the said position of the Moon's orbit, is  
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to the mean inclination of the plane as the difference of the greatest and least areas described in the same time by the Moon about the earth, when in the conjunction and in the quarters to the mean area.

WHEREFORE, if  $S$  be to  $L$ , as the Sun's period to the Moon's period: The greatest area is to the least, as  $\sqrt{SS+3LL}$  to  $S$ , or as  $S + \frac{3LL}{2S}$  to  $S$  nearly, by what

is said on this article in the 29th page. So that the difference of areas is to the mean area, as  $\frac{1}{2}LL$  to  $SS + \frac{1}{4}LL$ ; and in the same proportion is the greatest variation of the inclination of the plane in this month to the mean inclination, which agrees nearly with Sir *Isaac's* computation.

*Secondly*, THERE is a general method for assigning the laws of the motion of a body to and from the center, abstractly consider'd, from its motion about the center.

THE motion to and from the center is called by *Kepler* a Libratory motion; the knowledge of which seems absolutely requisite, to define the laws of the revolution of a body, in respect of the apses of its orbit.

FOR the revolution of a body, from apsis to apsis, is performed in the time of

of the whole libratory motion; the ap-sides of the orbit being the extreme points, wherein the libratory motion ceases.

So that, according to this method, the motion of a body round the center, is not consider'd as a continued deflection from a streight line; but as a motion compounded of a circulatory motion round the center, and a rectilinear motion to or from the center.

Each of which motions require a proper *Equant*. Of the equant for the motion round the center, I have already given several examples. And in the case of all motions, which are governed by a gravity or force tending to a fixt point; the real orbit in which the body moves, is the equant for this motion. In all other cases it is a different figure.

The *Equant* for the libratory motion, is a curve line figure, the areas of which serve to shew the time wherein the several spaces of the libration are performed.

Which figure is to be determined, by knowing the law of the gravity to the center: For the libratory force, to accelerate or retard the motion to or from the center, is the difference between the gravity of the body to the center, and the centrifugal force arising from  

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the

the circulatory motion. But the latter is always under one rule: For in all revolutions round a center, in any curve line, whether described by a centripetal : centrifugal force is duplicate proportion of the a given small time, and he triplicate proportion which is an immediate a known proposition of The like proportion also

holds as to the centripetal force in all circular motions, from a known proposition of Sir *Isaac Newton*. But what is true of the centripetal force in circles, is universally true of the other force in orbits of any form.

So that by knowing the gravity of the body, since the other force is always known, the difference, which is the absolute force to move the body to or from the center, will be known; and from thence the velocity of the motion, and the space described in any given time, may be found, and the equant described. These hints may be sufficient to shew what the method is.

To add an example. If the gravity be reciprocally as the square of the distance; the equant for the libratory motion, will be found to be an ellipsis  
similar

similar to the orbit, whose longer axis is the double of the eccentricity; the center of the libratory motion, that is the place where it is swiftest, will be in the focus; the time of the libration, through the several spaces, is to be measured by sectors of the said ellipsis, similar to those described by the body round the focus of the orbit; and the period of the libratory motion will be the same with the period of the revolution.

In any other law of gravity, the equant for the libratory motion, will either be of a form different from the orbit, or if it be of the same form, it must not be similarly divided.

I may just mention, that the equant for the libratory motion, in the case of the Moon, is a curve of the third kind, or whose equation is of four dimensions; but is to be described by an ellipsis, the center of the libration not being in the focus.

From this method of resolving the motion, it will not be difficult to shew the general causes of the alteration of the eccentricity and inequality in the motion of the apogee. For when the line of apsides is moving towards the Sun, it may be easily shewn, that since the external force in the apsides, is then centri-

E .2

fugal,

fugal, it will contribute to lengthen the space and time of the libration; by lengthening the space, it increases the eccentricity; and by lengthening the time of the libration, it protracts the time of the revolution to the apsis, and causes what is improperly called a motion of the apsis forward. But when the line of apsides is moving to the quadratures, the external force in the apsides, is at that time centripetal; which will contribute to shorten the space and time of libration; and by shortening the space will thereby lessen the eccentricity, and by shortening the time of libration, will thereby contract the time of the revolution to the apsis; and cause what is improperly called a retrograde motion of the apsis.

I shall only add a few remarks, which ought to have been made in their proper places.

As to the motion of the Moon in the elliptic epicycle (page 9.) it should have been mentioned, that there is no need of any accurate and perfect description of the curve called an ellipsis, it being only to shew the elongation of the Moon, from the center of the epicycle; which doth not require any such accurate description.

It

It should have been said, that when Fig. 1.  
the Moon is in any place of its orbit,  
suppose somewhere at  $N$ , in that half  
of the orbit which is next the Sun, it  
then being nearer the Sun than the Earth,  
has thereby a greater gravity to the Sun  
than the Earth; which excess of gravity,  
according to Sir *Isaac Newton's* method,  
consists of two parts; one acting in the  
line  $NV$ , parallel to that which joins  
the Earth and Sun; and the other acting  
in the line  $VB$  directed to the Earth;  
and these two forces, being compounded  
into one, make a force directed in the  
line  $NB$ ; which is in proportion to the  
force of gravity, as that line  $NB$  is  
to  $TB$  nearly. Wherefore, as there is  
a force constantly impelling the Moon  
somewhere towards the point  $B$ , this  
force is supposed to inflect the motion  
of the Moon into a curve line about  
that point; for the same reason as the  
gravity of it to the Earth, is supposed  
to inflect its motion into a curve line  
about the Earth: not that the Moon  
can actually have so many distinct mo-  
tions, but the one simple motion of the  
Moon round the Sun is supposed to arise  
from a composition of these several  
motions.

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In the last article on the small annual equations, (page 38.) these rules ought to have been added.

Let  $\mathcal{A}$  be the equation of the Sun's center;  $P$  the mean periodical time of the node or apogee;  $S$  the mean synodical time of the Sun's revolution to the node or apogee: Then will  $\frac{3S}{2P} \mathcal{A}$  be the annual equation of the node or apogee, according as  $S$  and  $P$  are expounded.

The like rule will serve for the annual equation of the Moon's mean motion. If  $S$  be put for the Sun's period;  $P$  for the mean synodical period of the Moon to the Sun; and  $L$  for the Moon's period to the Stars: The annual equation of the Moon's mean motion will be  $\frac{3LL}{2PS} \mathcal{A}$ .

According to these rules when expounded, the equation for the node will be found to be always in proportion to the equation of the Sun's center, nearly as 1 to 13.

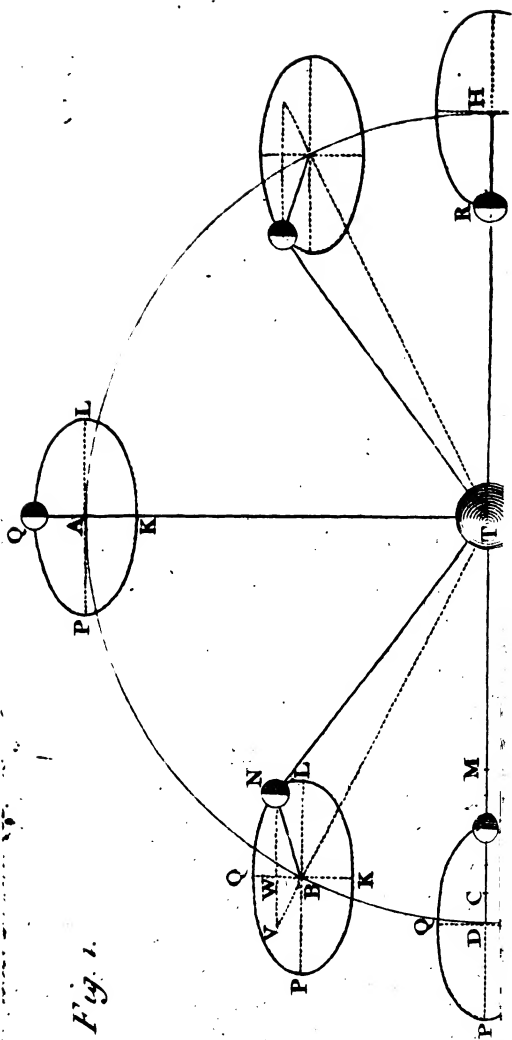
The equation of the apogee to the equation of the Sun's center, as 10 to 53.

And the equation of the Moon's mean motion to the same, as 8 to 77.

It



Fig. 1.



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It may be throughout observed, that the propositions are in general terms, so as to serve, *mutatis mutandis*, for any other satellite, as well as the Moon.

There might have been several other observations and remarks made in many other places, had there been sufficient time for it. But perhaps what I have already said may be too much, considering the manner in which it is delivered.

### ERRATUM.

Page 11. l. 11. for 8th, read 28th.

### F I N I S.



# ERRATA.

## VOLUME I.

PAGE 117. for *drawing*, read *draw*. p. 156. f. 2 *AB*,  
 r.  $\frac{1}{2}AB$ . p. 164. l. 7. 10, 20. p. 165. l. 9. p. 171.  
 l. 27. f. *right line whose power is the area* &c. r. *right line*  
*whose square is equal to the area* &c. p. 166. l. 25, 29. f.  
*right line whose power is the rectangle* &c. r. *right line whose*  
*square is equal to the rectangle* &c. p. 192. l. 23, 24.  
 r.  $A^{\frac{1}{2}-3}$ , or  $A^{\frac{1}{4}-3}$ , or  $A^{\frac{1}{8}-3}$ , or  $A^{\frac{1}{16}-3}$ , l. 29. r.  
 $A^{\frac{1}{32}-3}$ , p. 203. dele *if*. p. 229. l. penult. dele *is*. p.  
 240. l. 26. dele *near*. p. 243. l. 21. f. *when*. r. *because*.  
 p. 272. l. 3. r. *is in the same ratio*.

## VOLUME II.

PAGE 6. Line 21. for *its*, read *the*. p. 24. l. 21.  
 dele *Fig. 2*. p. 50. l. 7. from the bottom. f. *Fig.*  
 5, 6, 7. r. *Fig. 6, 7, 8.* and so in page following. p.  
 95. l. 4. from the bottom, and p. 100. l. 5. f. *Averdu-*  
*pois*, r. *Troy*. p. 130. l. 28. r. *and the water*, &c. p. 140.  
 l. 14. f. *and the*, r. *and whose*. p. 144. l. ult. f. *but*, r.  
*this*. p. 161. l. 2. f. *may*, r. *will*. p. 169. l. 6. f. *leave for*  
*some time*, r. *would otherwise leave*. l. 21. r. *receding from*  
*the parts of the body where it is pressed*, &c. p. 338. l. 7.  
 f. *Fig. 1*. r. *Fig. 2*. p. 341. l. 1. f. *Fig. 2*. r. *Fig. 3*. p.  
 352. l. *elliptic*.











